Efficient Permutation Correlations and Batched Random Access for Two-Party Computation

Stan Peceny

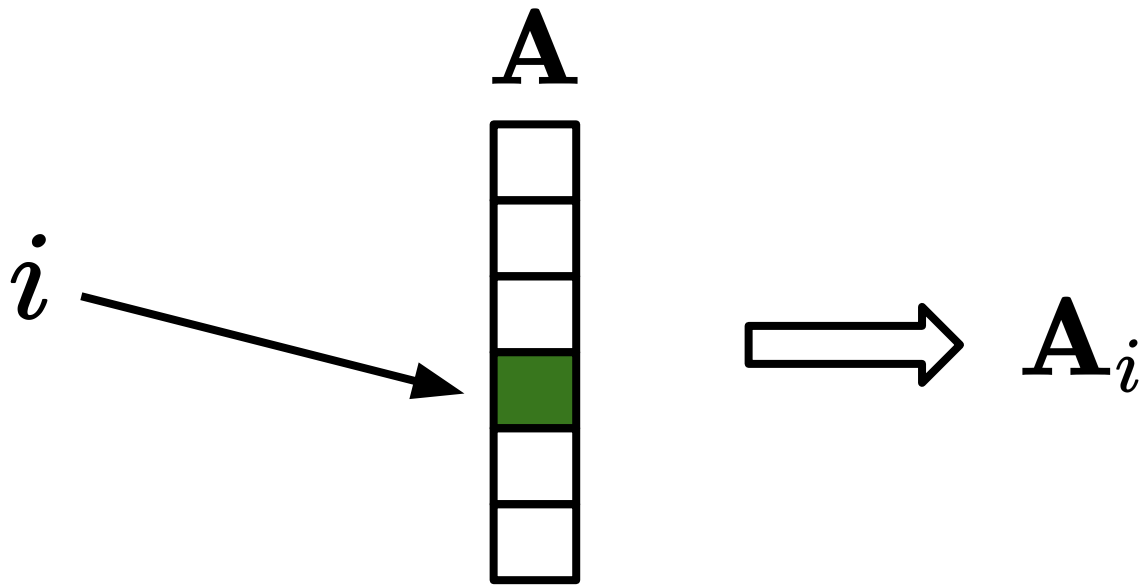
Joint work with

Srini Raghuraman, Peter Rindal, Harshal Shah

Georgia Institute of Technology

VISA

Array Access



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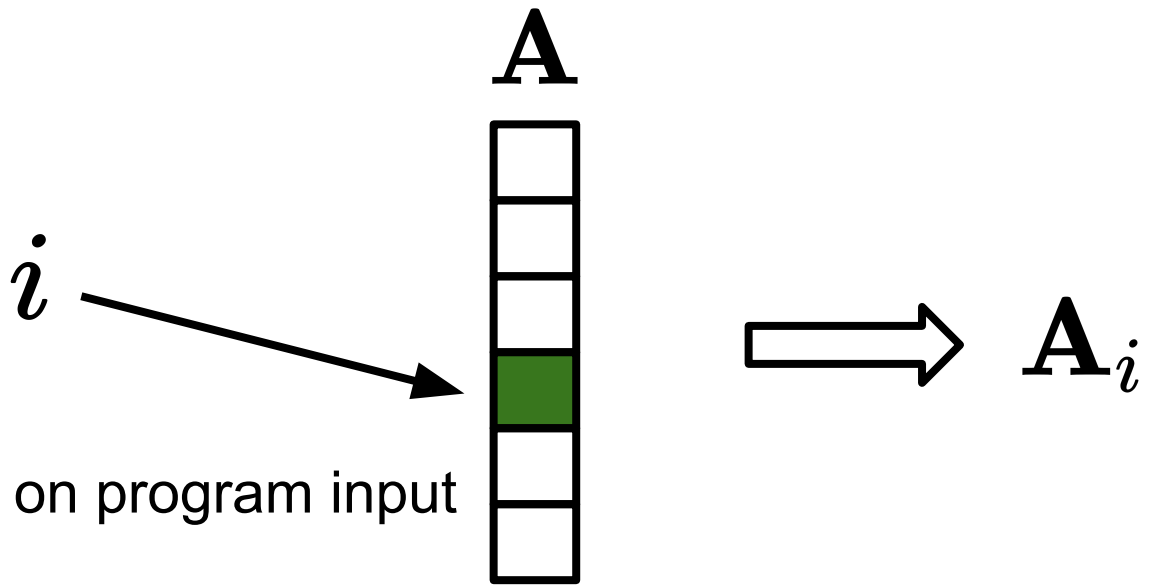
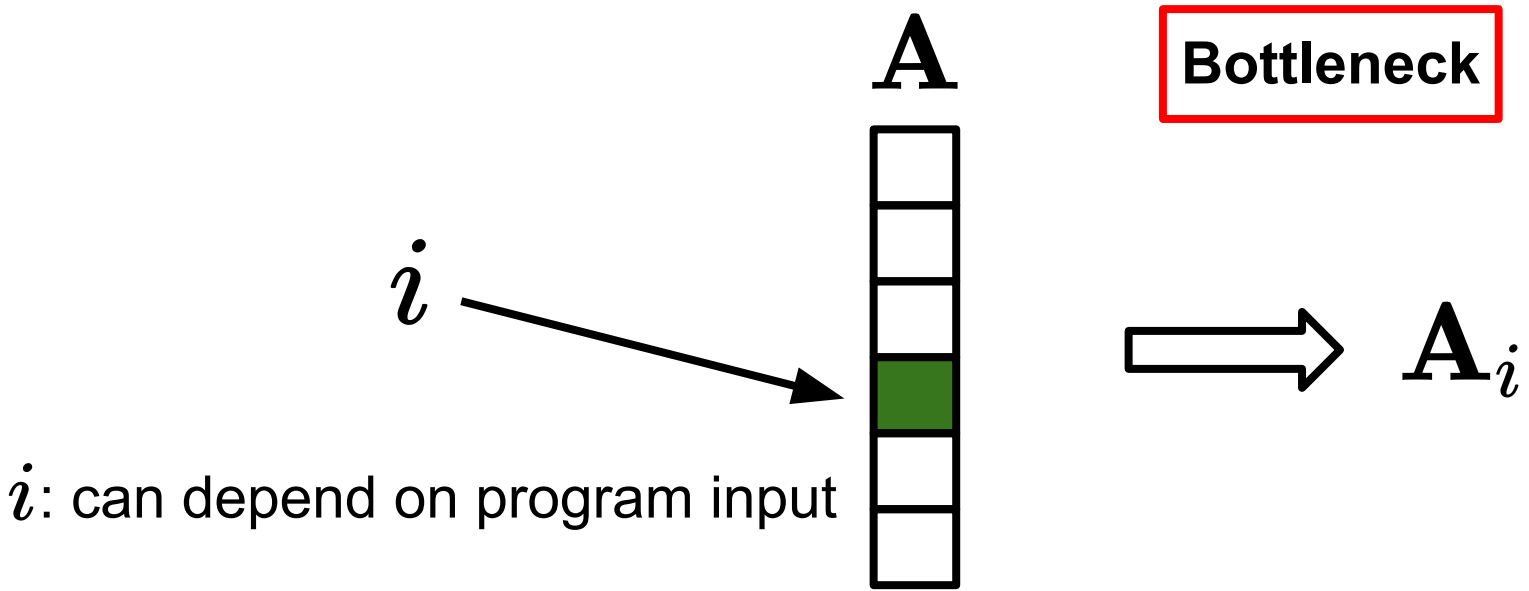


i: can depend on program input

Array Access



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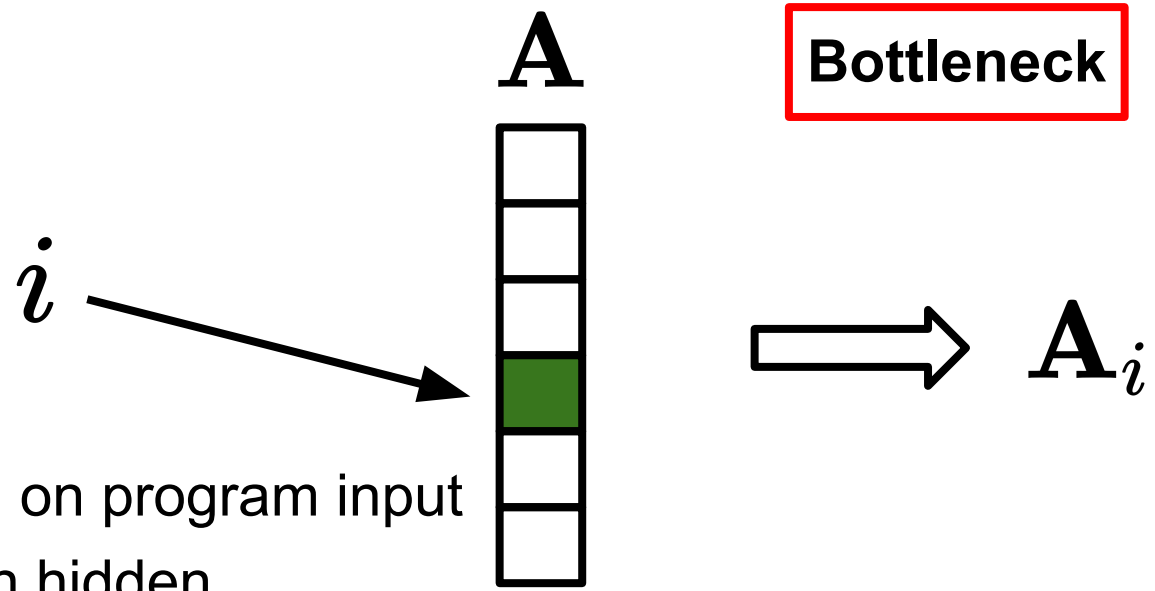
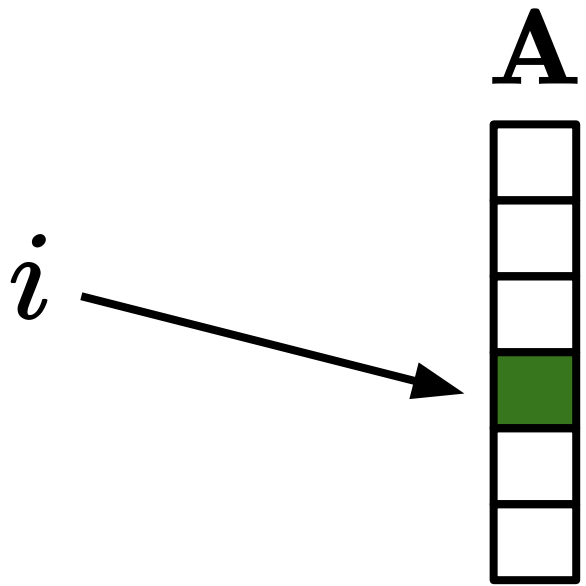


i: can depend on program input
must remain hidden

Linear Scan



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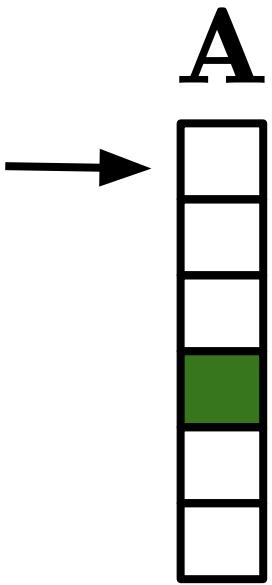

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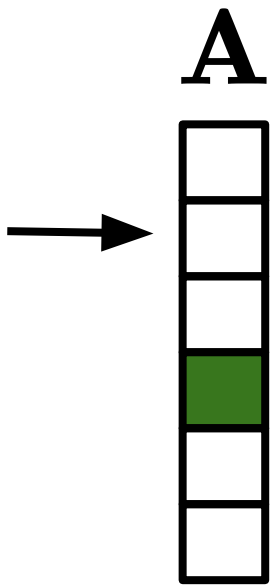

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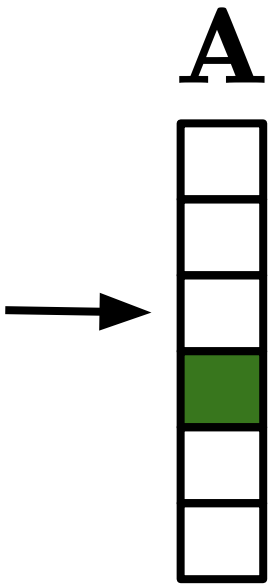

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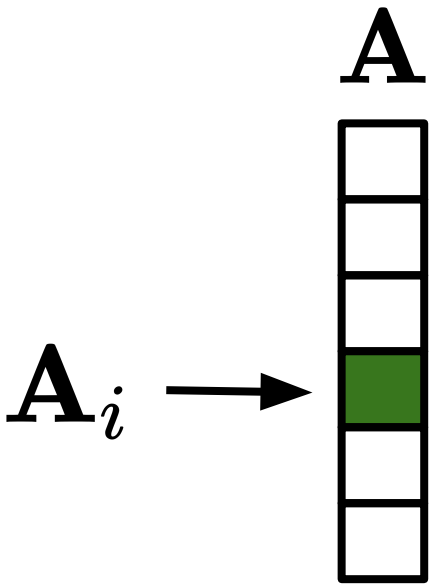

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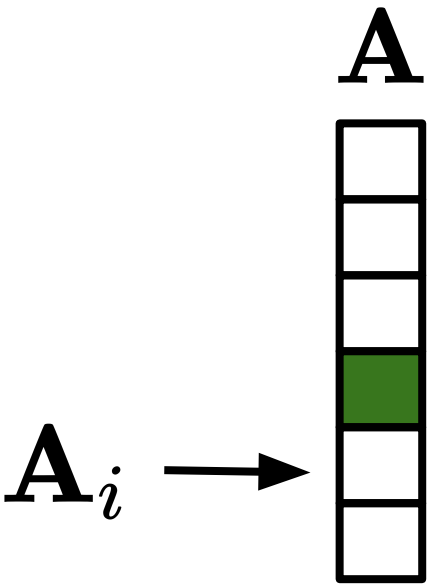

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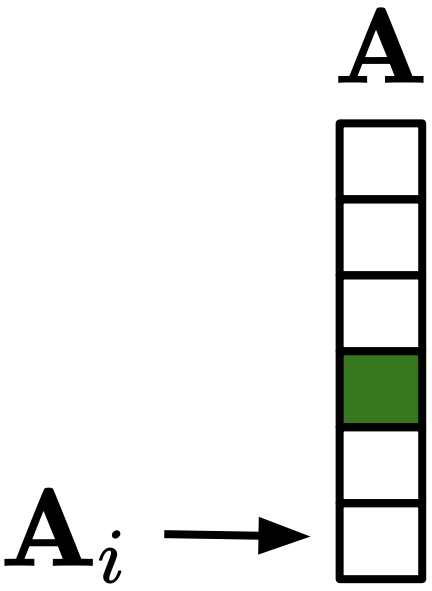

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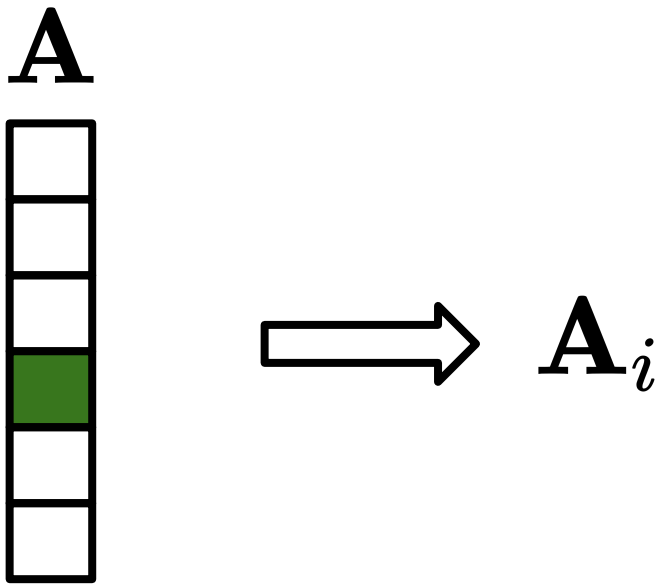
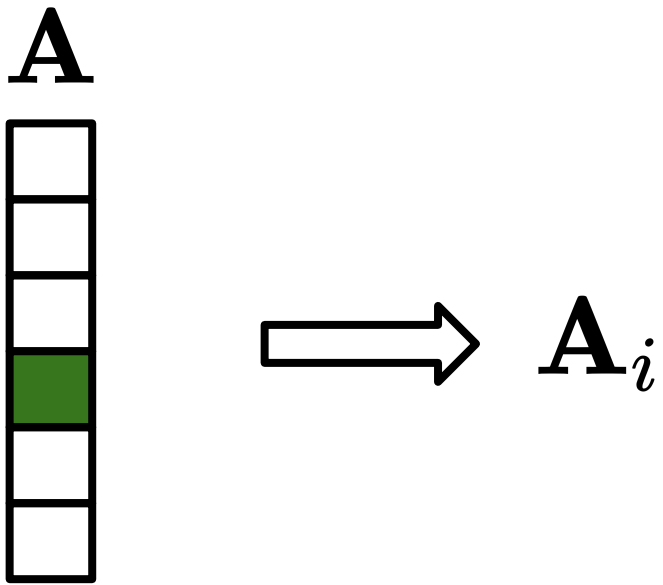


Linear Scan

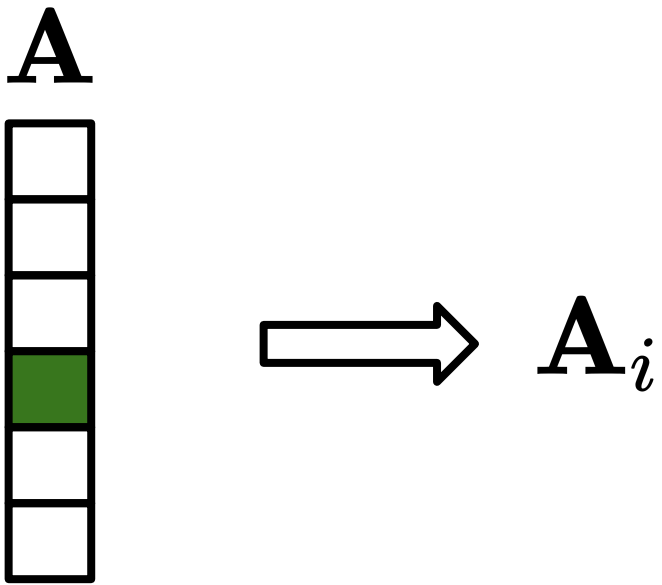
Linear Overhead



Linear Scan

Linear Overhead

Low Constants



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Linear Overhead

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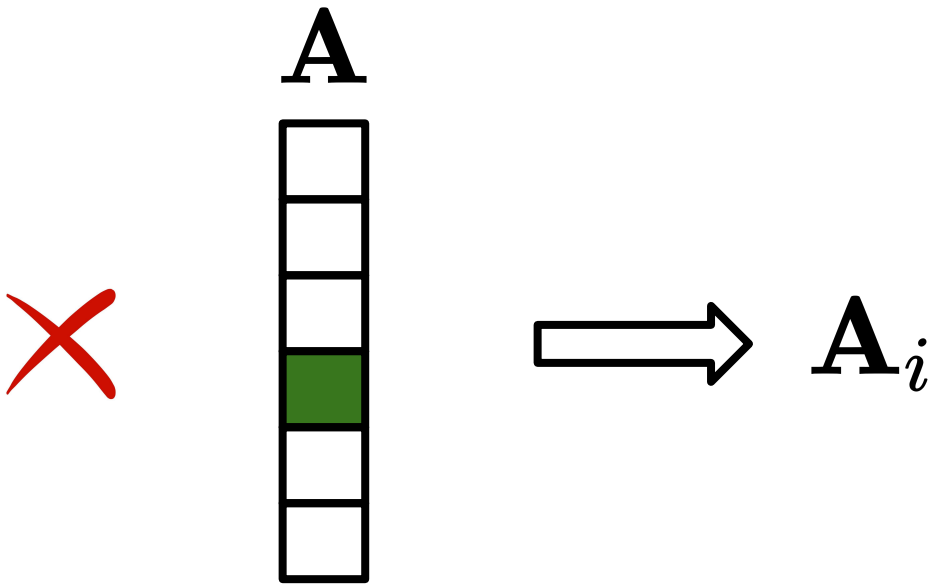
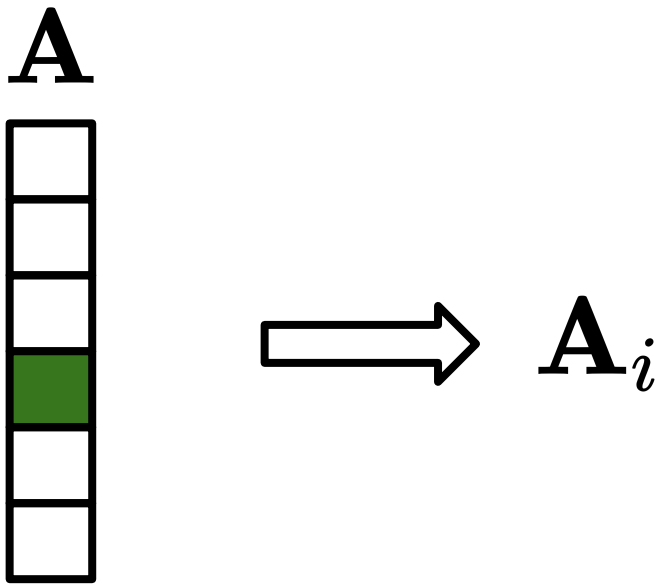


Oblivious RAM

Polylog Overhead



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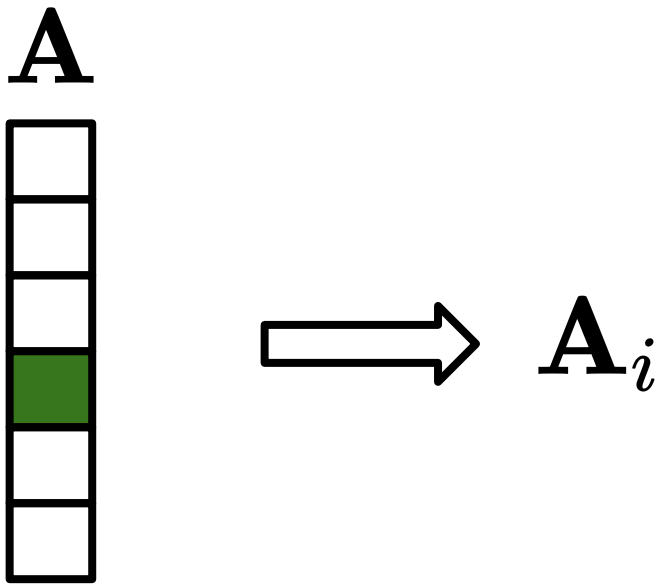

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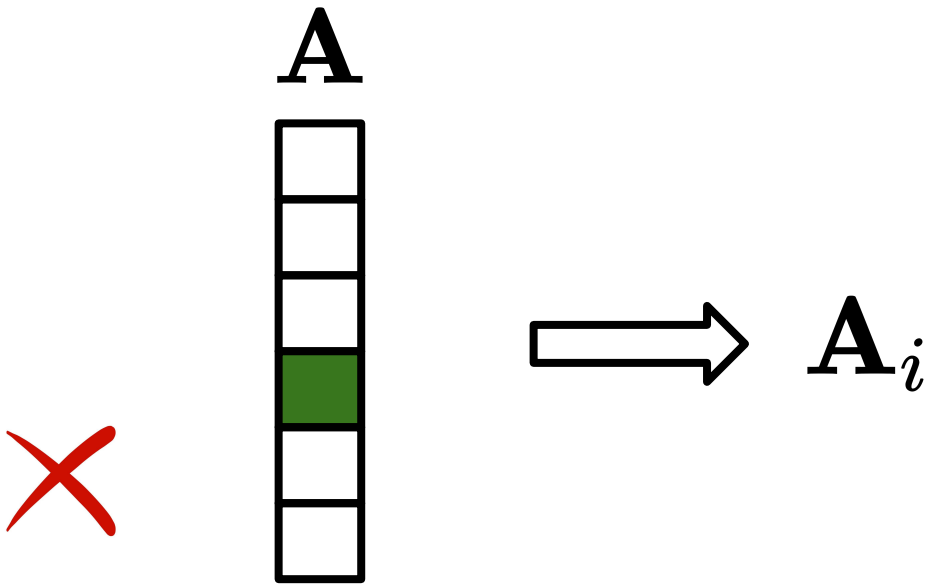


Oblivious RAM

Secret Sharing Schemes

Garbled RAM

Oblivious RAM

Secret Sharing Schemes

- Mostly sequential accesses
- Adds polylog rounds

Garbled RAM

Oblivious RAM

Secret Sharing Schemes

- Mostly sequential accesses
- Adds polylog rounds

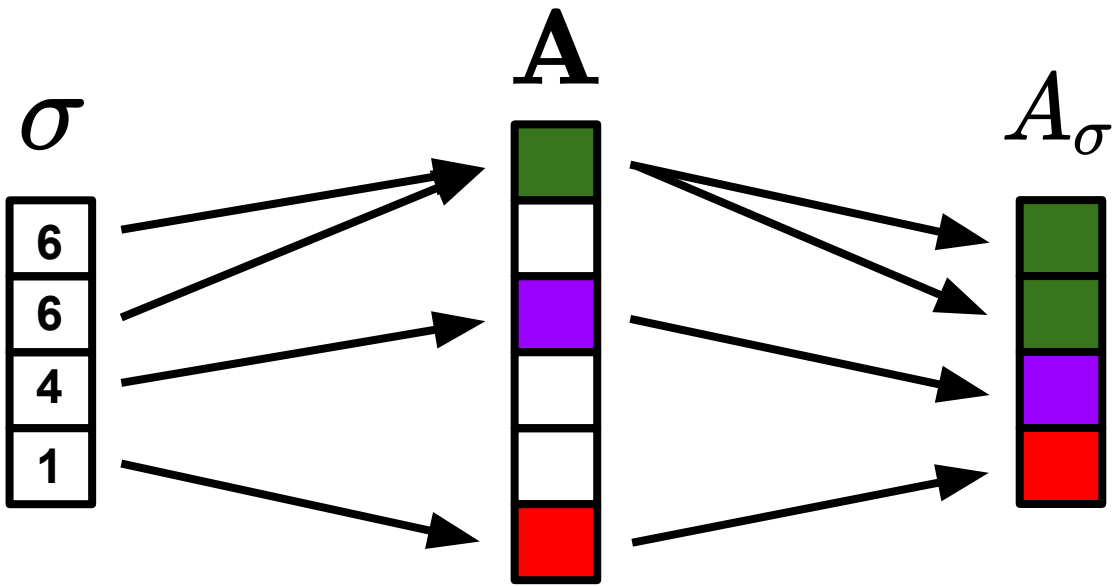
Garbled RAM

- Constant rounds
- κ overhead

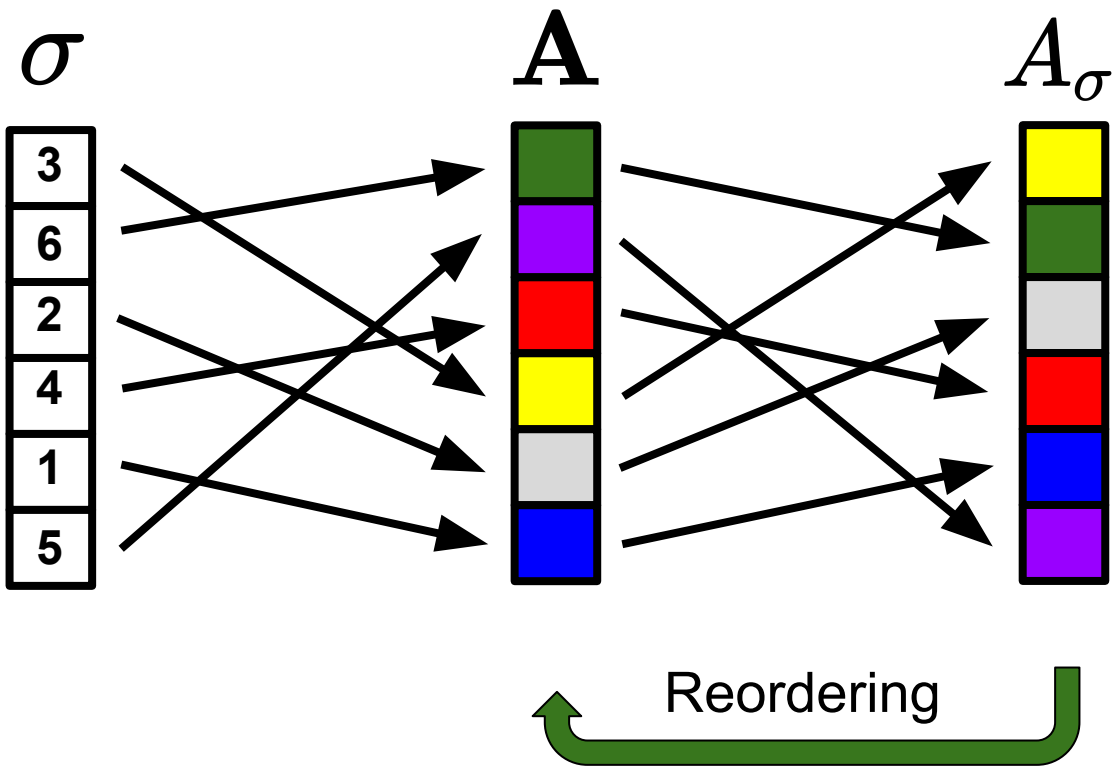
Batched Array Access

Many MPC applications with RAM can be implemented in a **batched** manner

Batched Array Access



σ a **Permutation**



Permutation

A key primitive for batched RAM

We show they allow for more complex batched RAM

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A key primitive for batched RAM

We show they allow for more complex batched RAM

Amortized O(1) overhead per access

Contribution

Suite of 2-party semi-honest protocols:

Contribution

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- 2 novel permutation correlations
- Expressive batched RAM read/write gates
- Toolbox: derandomization protocols, extraction, sort

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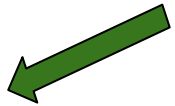
Suite of 2-party semi-honest protocols:

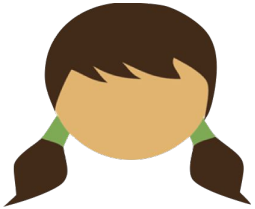


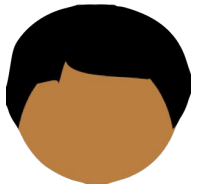
2 novel permutation correlations

Expressive batched RAM read/write gates

Toolbox: derandomization protocols, extraction, sort

Permutation Correlation

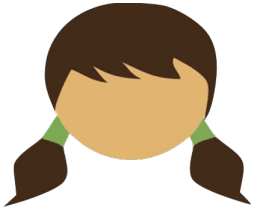


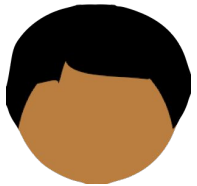


π

$\mathcal{F}_{perm}$

Permutation Correlation





$\mathcal{F}_{perm}$

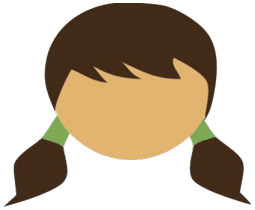
π

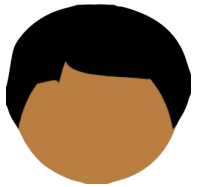
(A, B)

C

$$\pi(A) = B + C$$

Permutation Correlation





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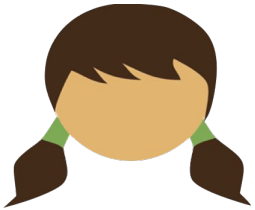
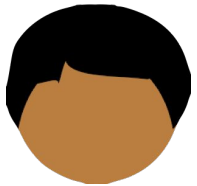
Permutation-equivalent to random OLE in MPC

Permutation Correlation





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Permutation equivalent to random OLE in MPC

‘Share translation’ in [CGP20]

2 Novel Protocols

Weak PRF-based

- O(nl) comm., 3 rounds
- 7.3 seconds and 182MB for $n=2^{20}$, $l=128$

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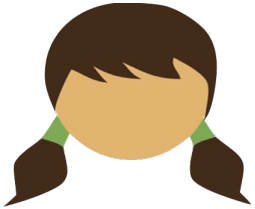
Weak PRF-based

- O(nl) comm., 3 rounds
- 7.3 seconds and 182MB for $n=2^{20}$, $l=128$

PCG-based

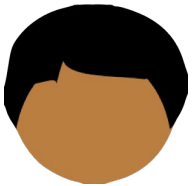
- First sublinear protocol
- O(n log l) comm., O(log (l/κ^2)) rounds
- Higher constants

Weak PRF



k

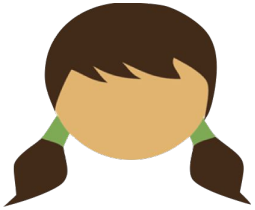
$\mathcal{F}_{wprf}$



x

x is random

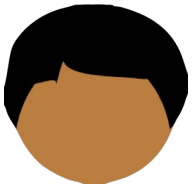
Weak PRF



k

$[F_k(x)]_A$

$\mathcal{F}_{wprf}$

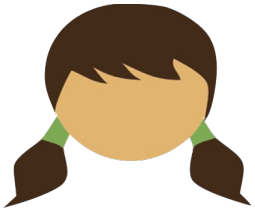


x

$[F_k(x)]_B$

x is random

Weak PRF



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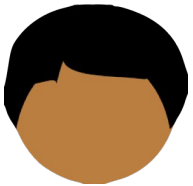

x

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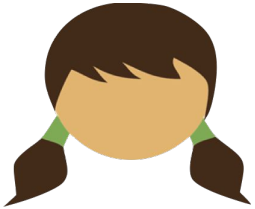
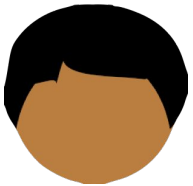
x is random

Alternating Moduli [APRR24], LowMC [ARS+15]

wPRF Permutation Correlation

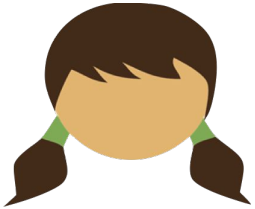


$k \leftarrow \mathcal{K}$



π

wPRF Permutation Correlation



$k \leftarrow \mathcal{K}$

$A_i := F_k(i)$

π

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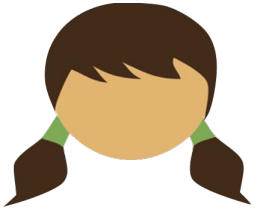

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π

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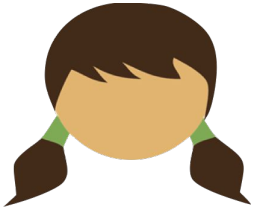
k $\rightleftharpoons$ Π_{wprf} $\rightleftharpoons$ $\pi(i)$

B_i $\rightleftharpoons$ C_i

wPRF Permutation Correlation



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π

$A_i := F_k(i)$

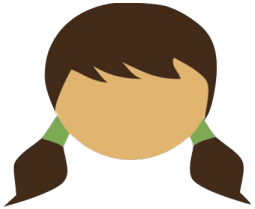
k → ← $\pi(i)$

Π_{wprf}

B_i ← → C_i

$i = 1, \ldots, n$

wPRF Permutation Correlation



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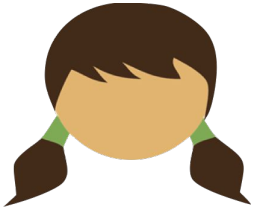

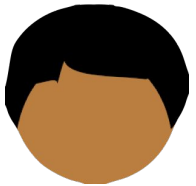

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$A_i := F_k(i)$

k

B_i

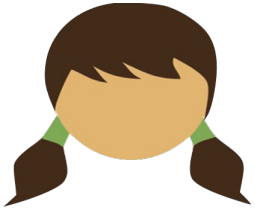
Π_{wprf}

$\pi(i)$

C_i

$$\pi(A) = B + C$$

wPRF Permutation Correlation



$k \leftarrow \mathcal{K}$

π

$A_i := F_k(H(i))$

k

B_i

Π_{wprf}

$H(\pi(i))$

C_i

$$\pi(A) = B + C$$

Ours vs Related Work

Chosen-input, $n=2^{20}$, $l=128$

Ours: 7.3s, 182MB

Chase et al. [CGP20]: 44s, 4GB

Gazelle [JVC18]: 303s, 215MB

Batched Array Access

(1) Batched read and (2) batched write

Multiple reads/write to single index allowed

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O(n log n) time and O(log n) rounds

n accesses with amortized O(1) overhead

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Multiple reads/write to single index allowed

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O(n log n) time and O(log n) rounds

n accesses with amortized O(1) overhead

n accesses to $n=2^{16}$ array ~10s and ~160MB

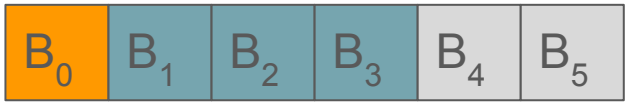
Batched-RAM-Read Tool

Aggregation Tree [BDG+22]:

Input: List of blocks B, List of control bits c

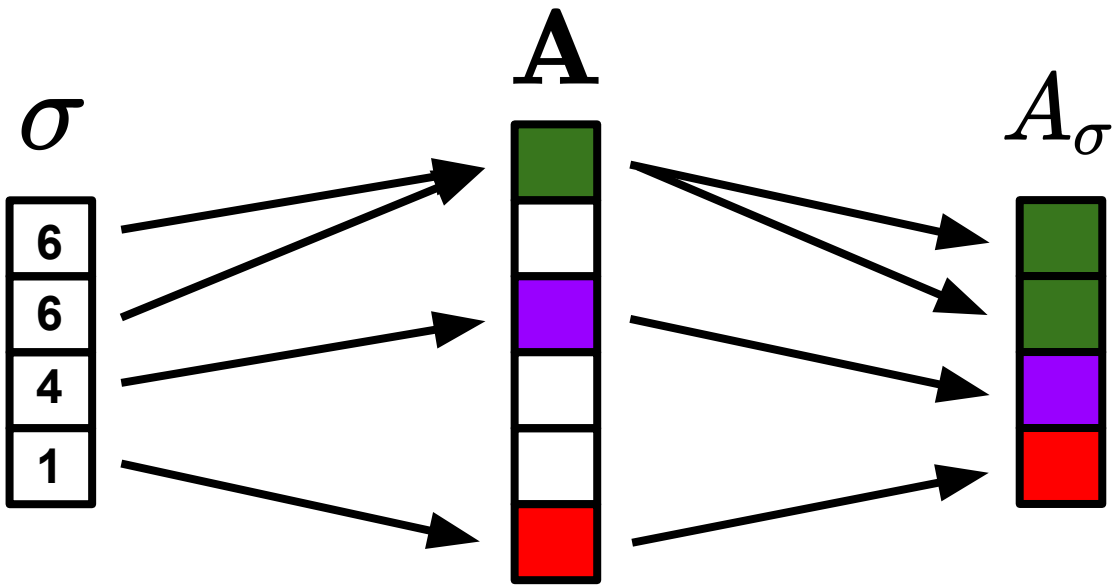
[[B]]: 

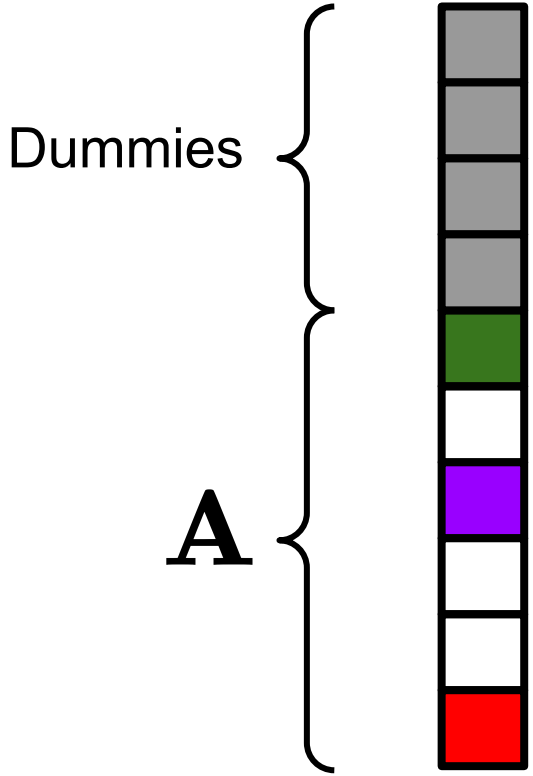
[[c]]: 0 0 1 1 0 1

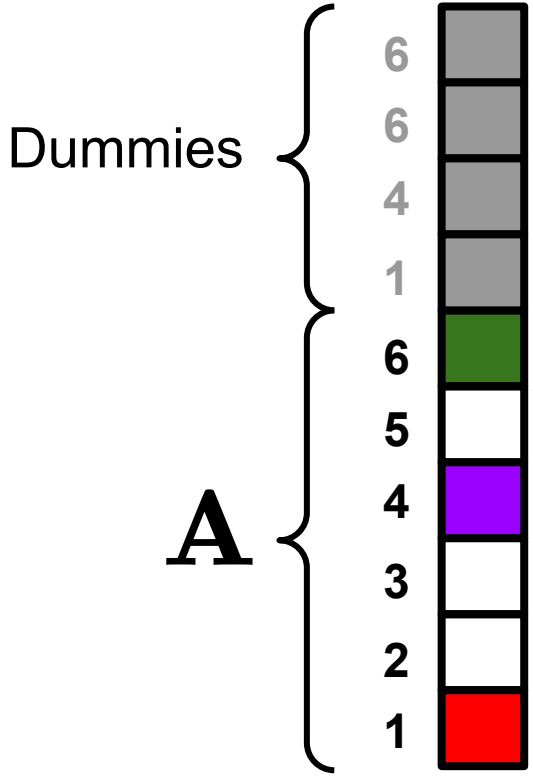
c partitions B into sublists: 

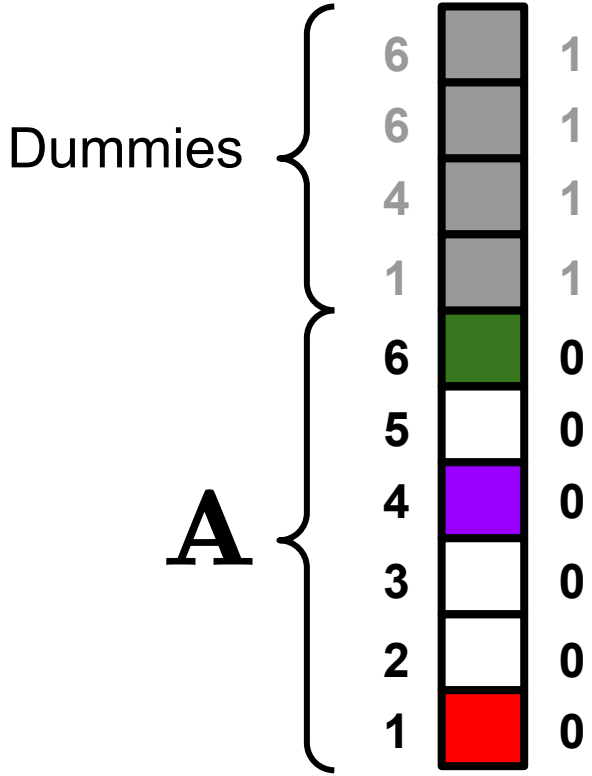
Output: Copy first item of each sublist into rest of sublist

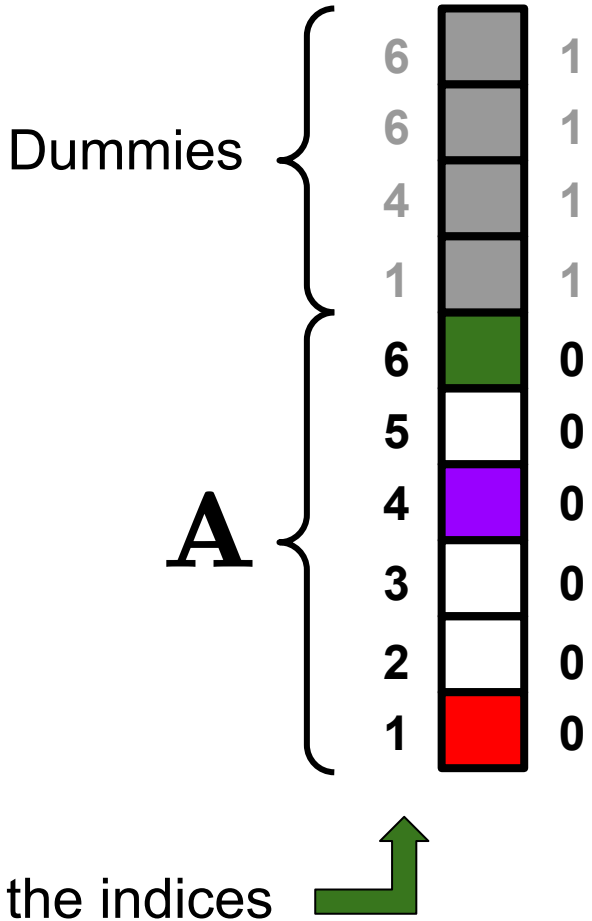
Batched Array Access

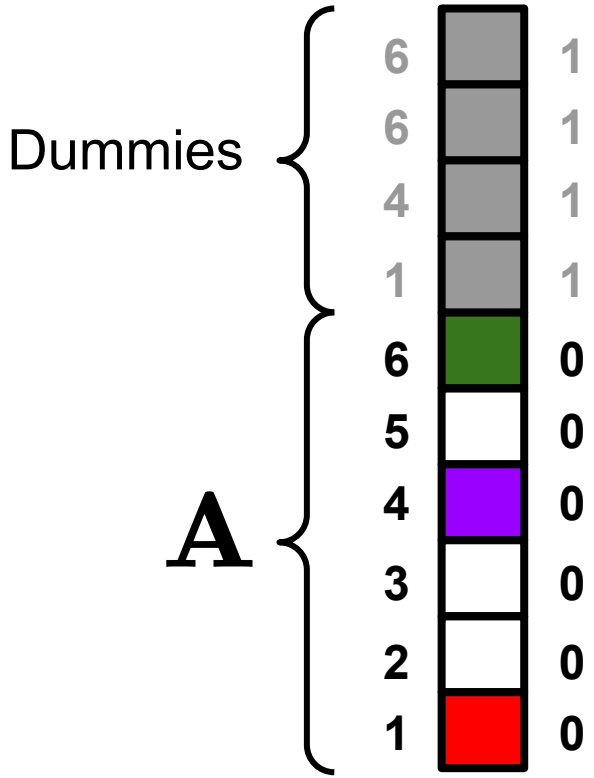



Dummies
A


Dummies
A
6
6
4
1
6
5
4
3
2
1

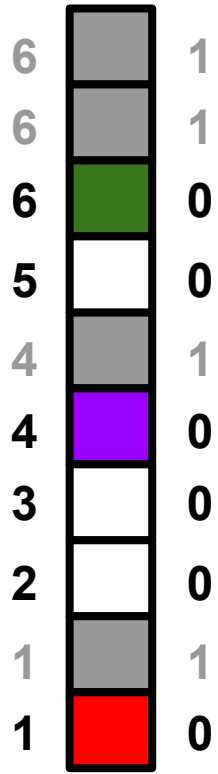

Dummies
A
6
6
4
1
6
5
4
3
2
1
1
1
1
1
0
0
0
0
0
0


Dummies
A
6
6
4
1
6
5
4
3
2
1
1
1
1
1
0
0
0
0
0
0
Stable sort using the indices

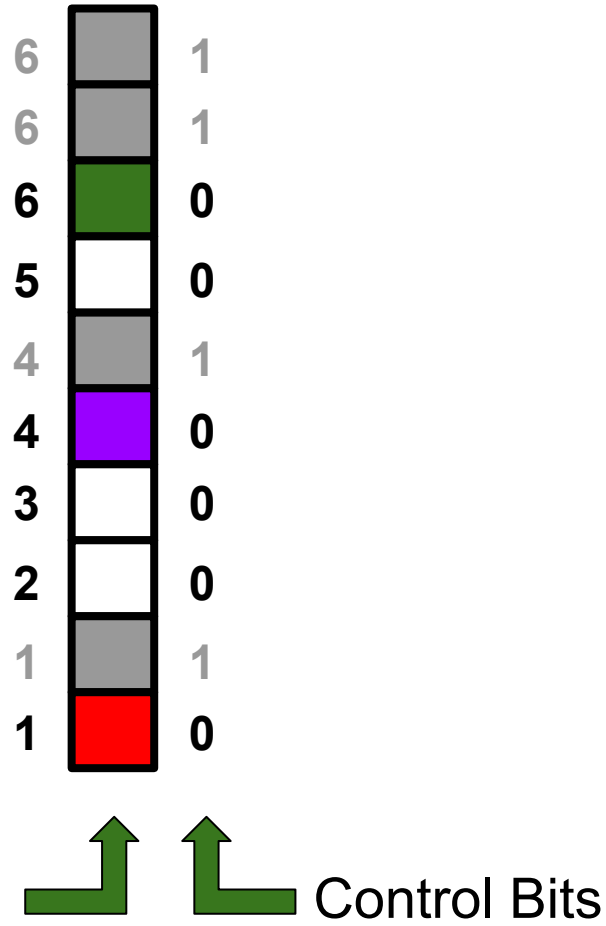


Stable sort using the indices

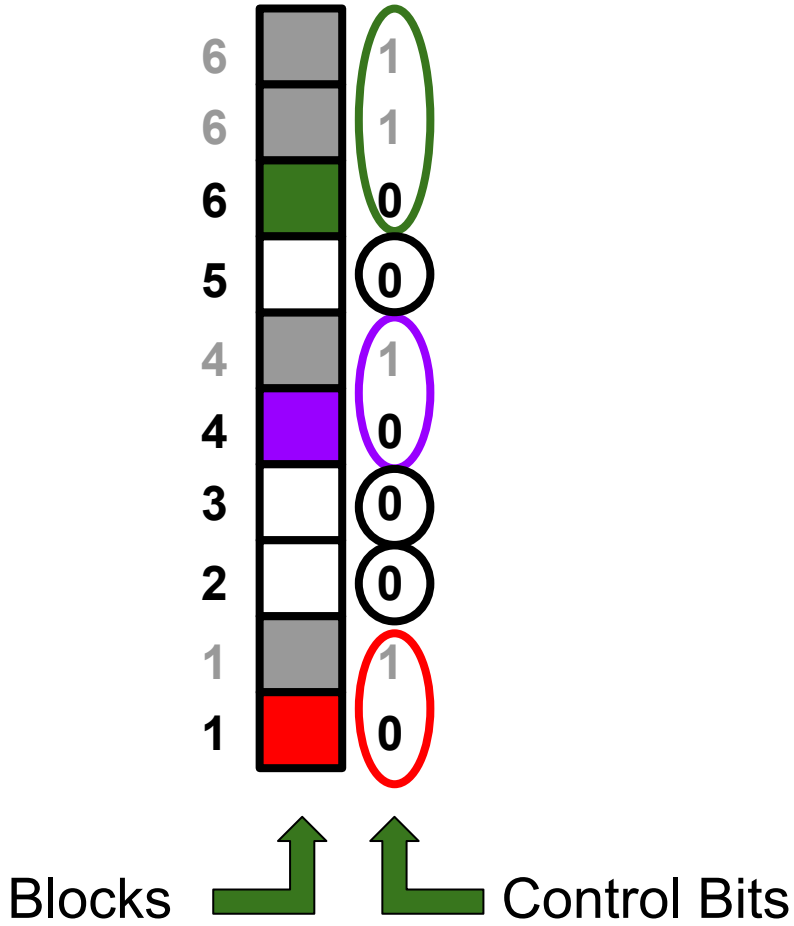
Returns sorting permutation π
Permute with π


6 1
6 1
6 0
5 0
4 1
4 0
3 0
2 0
1 1
1 0

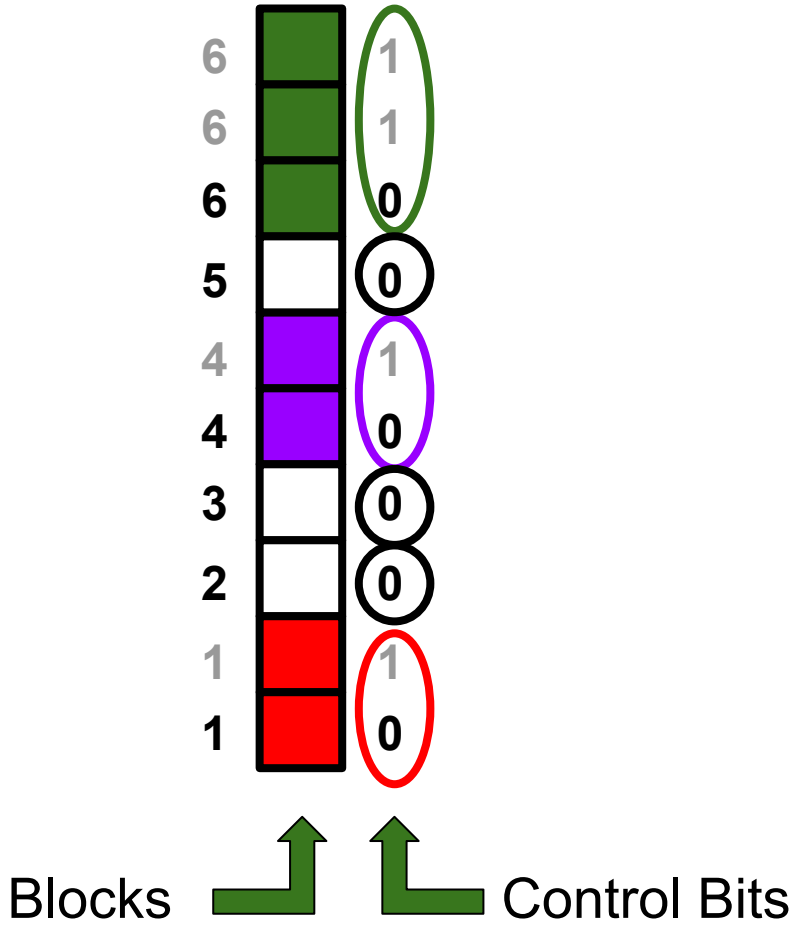
Aggregation Tree

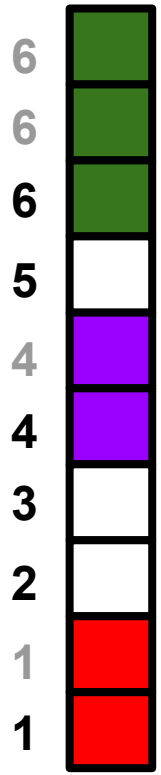


Aggregation Tree

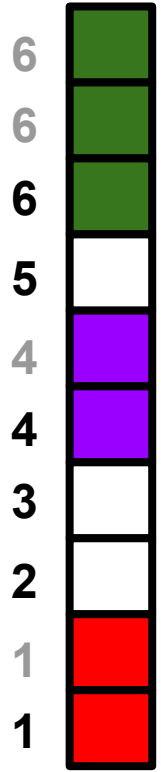


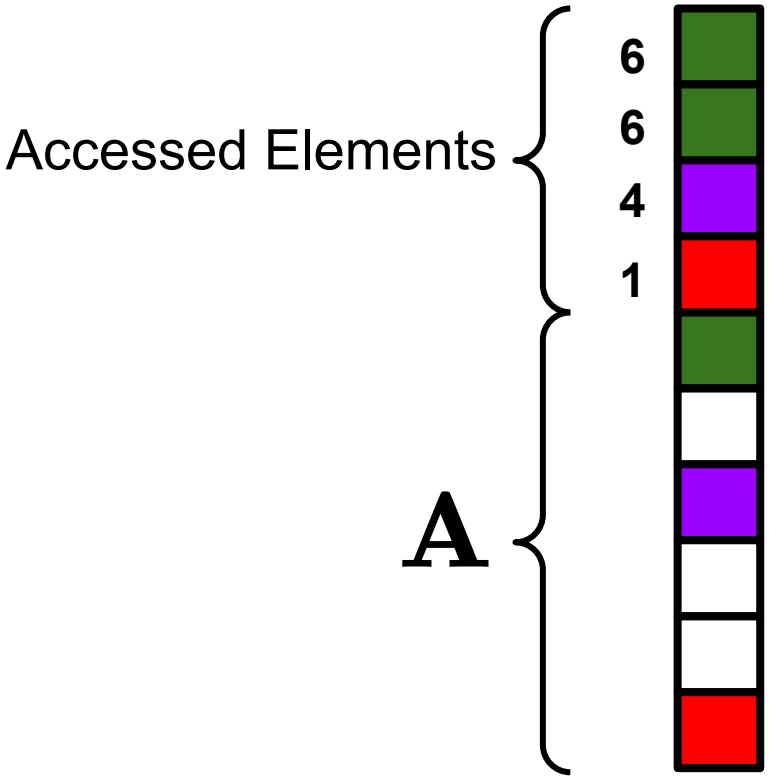
Aggregation Tree




6
6
6
5
4
4
3
2
1
1

Unpermute with π^{-1}




Accessed Elements
6
6
4
1
A

Toolbox

Derandomize:

- Chosen input and permutation
- Input list is secret-shared
- Permutation is secret-shared (as composition)
- Many permutations with 1 correlation
- Inverse permutation

Toolbox

Derandomize:

- Chosen input and permutation
- Input list is secret-shared
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- Inverse permutation

Additive ⇔ composition secret-shared permutation

Sorting and extraction protocols

Efficient Permutation Correlations and Batched Random Access for Two-Party Computation

Suite of 2-party semi-honest protocols:

- 2 novel permutation correlations
- Expressive batched RAM read/write gates
- Toolbox: derandomization protocols, extraction, sort