Non-Interactive Distributed Point Functions

PKC 2025

Reichman
 University



Elette Boyle Lalita Devadas Sacha Servan-Schreiber

 $P_{t}\left(x
ight)$

$$P_t\left(x
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eq t \end{cases}$$

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Alice

 K_A



















Alice





Private Information Retrieval and Search [GI'14, BGI'15, DPKY'20]

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Distributed Oblivious RAM [Ds'17 + follow-up work]

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Distributed Oblivious RAM [Ds'17 + follow-up work]

Preprocessing multi-party computation [BCGI'18 + follow-up work]

More efficient secure computation [BGIK'21 + follow-up work]

Can we remove the trusted setup?

Removing the Trusted Setup

The Doerner-shelat Protocol



Removing the Trusted Setup The Doerner-shelat Protocol



Can we remove interaction?

Inspiration

Diffie–Hellman Key Exchange

Diffie–Hellman Key Exchange [DH'76]





Diffie–Hellman Key Exchange ^[DH'76]



Diffie–Hellman Key Exchange ^[DH'76]

$z_A + z_B = f(x, y)$



Diffie–Hellman Key Exchange [DH'76]



Diffie–Hellman Key Exchange [DH'76]



"Diffie-Hellman" for DPF keys?

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*We still allow for a common reference string

Non-Interactive Distributed Point Functions










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Building NIDPFs



Secret-Key Homomorphic Secret Sharing





Some Tricks

NIDPF

Secret-Key Homomorphic Secret Sharing

Adapted from protocols described in [ARS'24, BCMPR'24]





























Using succinct matrix multiplication to realize a NIDPF

Step 1: Reinterpret Indices via CRT





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$$t_A
ightarrow t_A = i_A \cdot \ell + j_A$$



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 $t_A
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ightarrow t_B = i_B \cdot \ell + j_B$



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Step 2: Define matrices representing secret indices



 $t_A
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Step 2: Define matrices representing secret indices



 $t_A \implies t_A = 3 \cdot \ell + 4$ $t_B \Longrightarrow t_B = i_B \cdot \ell + 0$







Step 2: Define matrices representing secret indices













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Some Tricks

NIDPF

Secret-Key Homomorphic Secret Sharing











$\mathsf{ct}\, \leftarrow \mathsf{Encrypt}\,(\mathsf{sk}, x)$





Alice
















Row Shift Matrix





Col Shift Matrix



.

 $\mathsf{ct} \, \leftarrow \mathsf{Encrypt}\left(\mathsf{sk}, \mathbf{B_{row}}\right)$



Row Shift Matrix







$\mathsf{ct} \, \leftarrow \mathsf{Encrypt}\left(\mathsf{sk}, \mathbf{B_{row}}\right)$



Problem 2: cannot evaluate HSS over secret shares

Row Shift Matrix













Row Shift Matrix









Row Shift Matrix









Problem 2: cannot evaluate HSS over secret shares

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Row Shift Matrix





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- Input shares are additively-homomorphic ciphertexts encrypted with key sk
- Memory shares of x are additive secret shares of the tuple $(x, sk \cdot x)$

There exists a Mult algorithm that computes additive shares of the product between an input share and a memory share.



Row Shift Matrix























$$\mathsf{pk}_A := \mathsf{pk}_A^{\mathsf{matmul}}$$



























$$\mathsf{pk}_A := \mathsf{pk}_A^{\mathsf{matmul}}$$













$$\mathsf{pk}_A := \mathsf{pk}_A^{\mathsf{matmul}}$$

















ÍЗ













sk

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Results

NIDPF with domain size N

	Assumptions	Communication	Comments
Spooky [DHRW'16]	LWE OR iO+DDH	log(N)	Requires multi-key FHE

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NIDPF with domain size N

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Still only modestly sublinear. Open problem: \sqrt{N} or better

Thank you!

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Non-Interactive Distributed Point Functions

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