Registration-Based Encryption in the Plain Model

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Central Authority

Master Secret Key (MSK)



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Master Public Key (MPK)

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Security must hold even when every registered user is corrupted!

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Valid since attacking unregistered user without honest user is legitimate!

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Observation: if crs contains only the hash key, then all we need is re-randomizable key!

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The CDH chameleon encryption in [DG17], when interpreted as a hash encryption, has re-randomizable key.

Key generation: uniformly sample $\alpha_{i,j} \leftarrow Z_p^*$ and set $g_{i,j} = g^{\alpha_{i,j}}$.

$$k \coloneqq \left(g, \begin{pmatrix} g_{1,0} & g_{2,0} & \cdots & g_{n,0} \\ g_{1,1} & g_{2,1} & \cdots & g_{n,1} \end{pmatrix}\right)$$

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Key re-randomization: uniformly sample $\beta_{i,j} \leftarrow Z_p^*$.

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Hard to invert the re-randomization.

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By hardness of DL, Hash(k', ·) is collision resistant for honestly sampled k.

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• Schemes with better concrete efficiency?

Thanks for listening!