Stateless and Two-Round Threshold Schnorr Signatures

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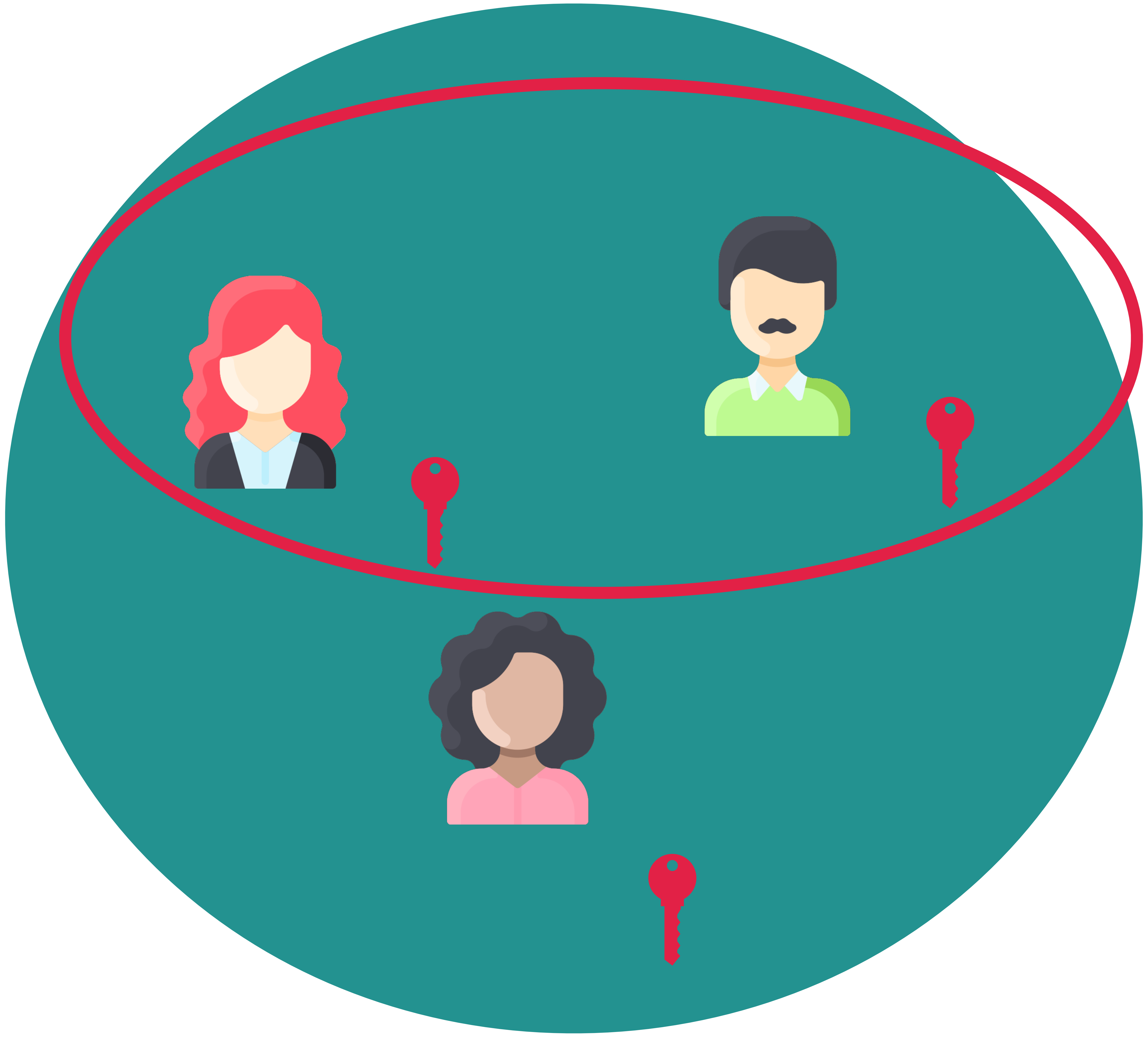
What are Threshold Signatures?



- Ideally t-out-of-n
- Key generation via trusted dealer or DKG
- Secure up to (t-1) corruptions

(2,3) Example

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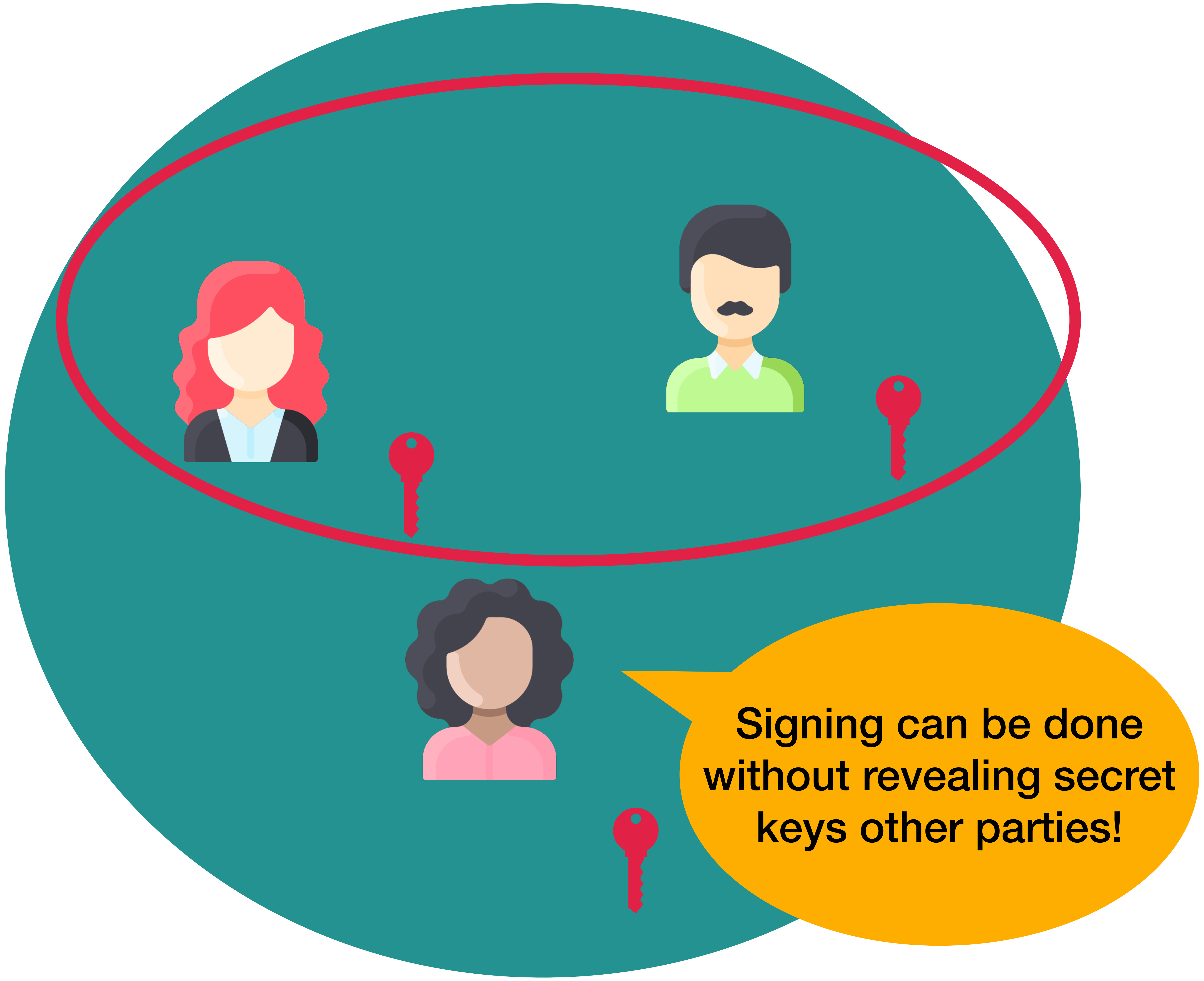

Public Key

- Ideally t-out-of-n
- Key generation via trusted dealer or DKG
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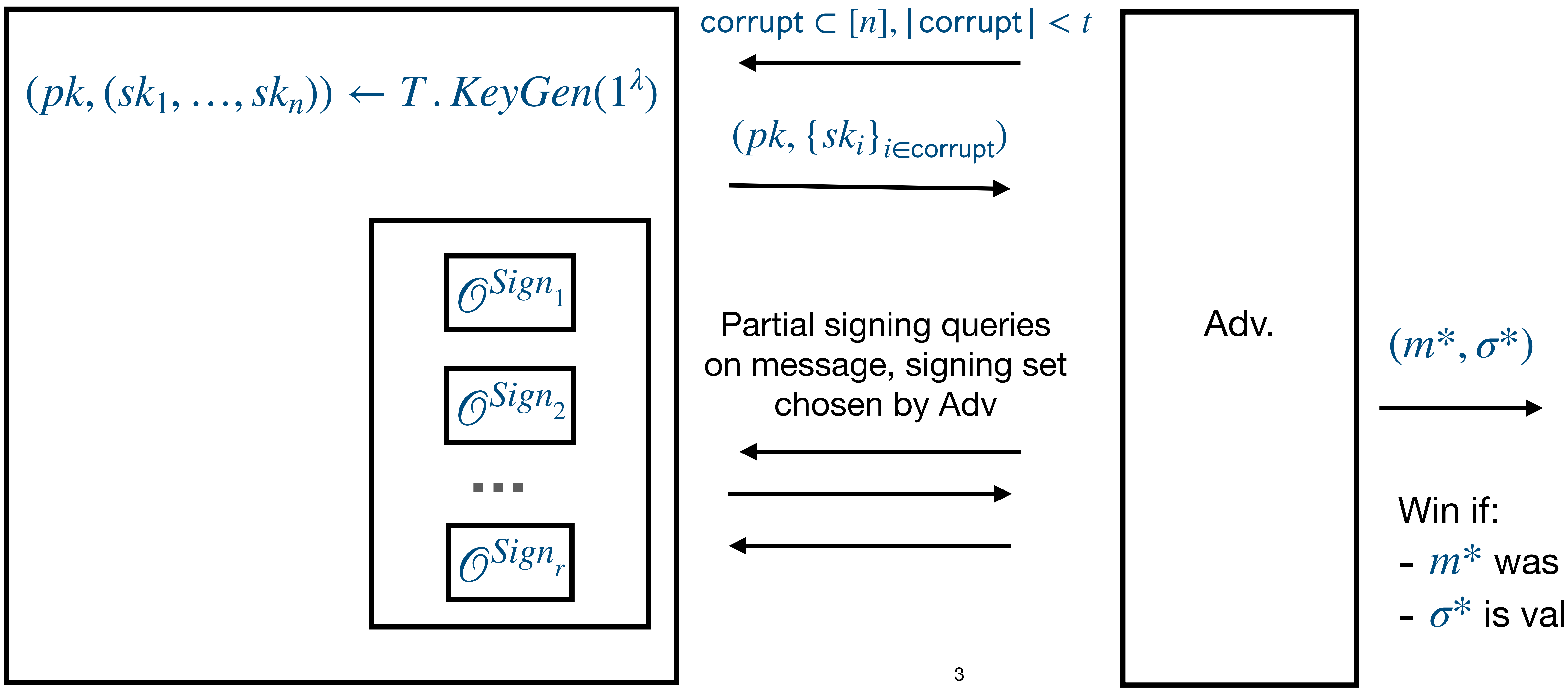
(2,3) Example

Unforgeability

A threshold signature scheme T is secure if no PPT adversary can win the following game with non-negligible advantage:



Win if:
- m^* was never queried to $\mathcal{O}^{Sign}$
- σ^* is valid under (pk^*, m^*)

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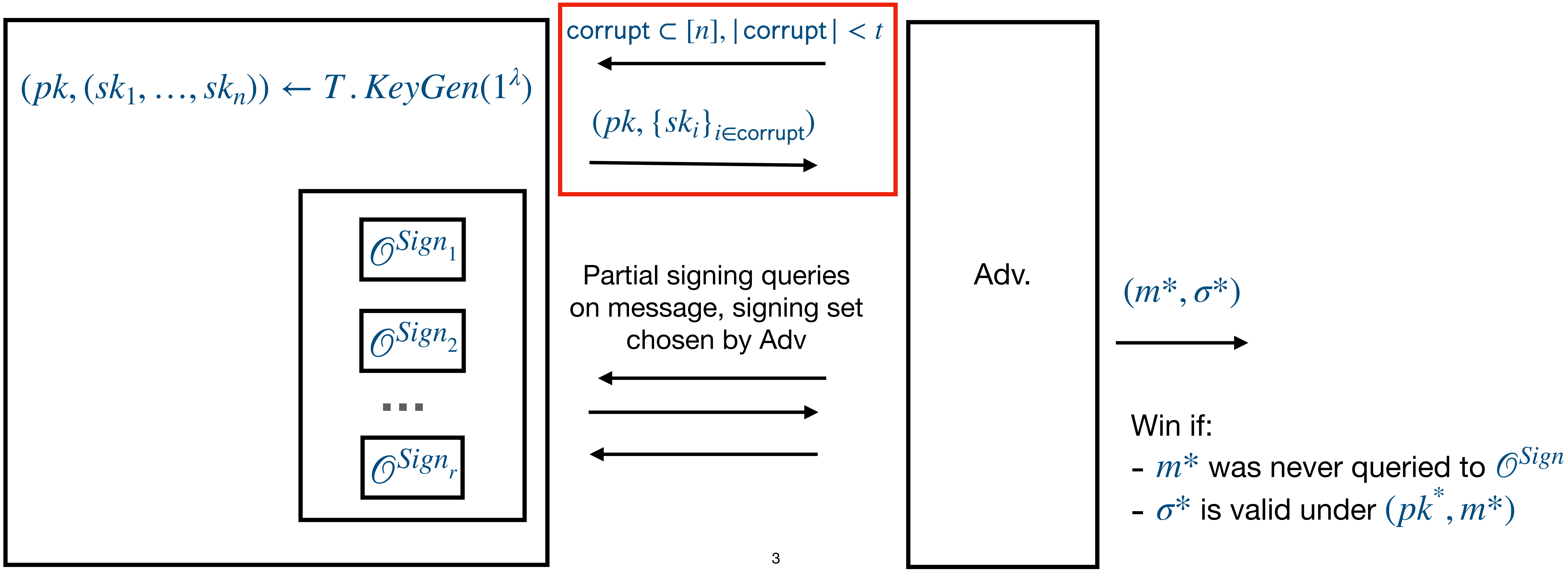

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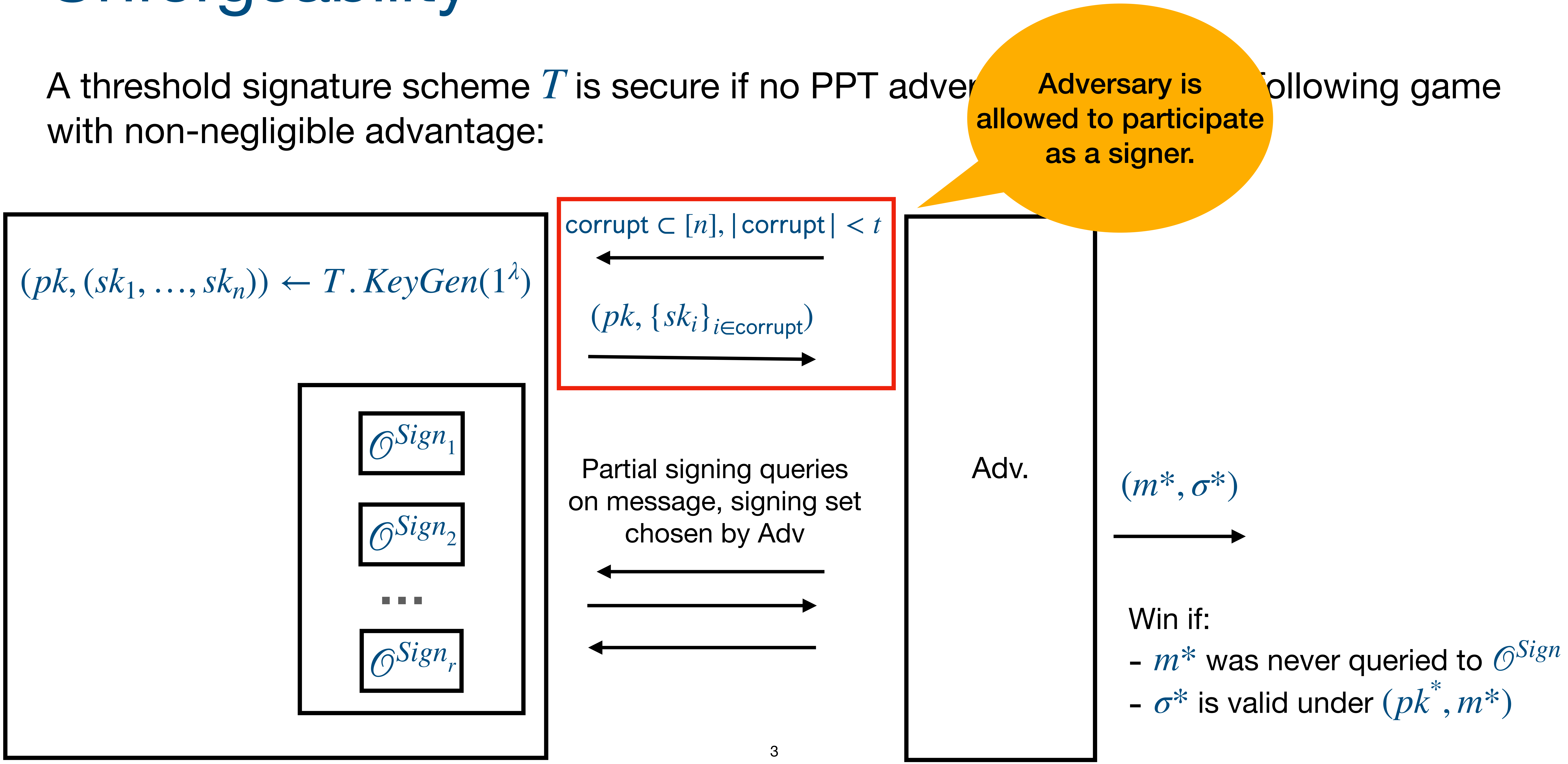

Multi-Party Schnorr Signatures

How to share r ?

How to share sk ?

$$z \leftarrow r + c \cdot sk$$

$$sig = (R, z)$$

	Scheme	Assumptions	Signing Rounds
Multi-sigs (n-of-n)	MuSig [MPSW18, BDN18] SimpleMuSig [BDN18, C**K**M21]	DL+ROM	3
	MuSig2 [NRS21] DWMS [AB21] SpeedyMuSig [C**K**M21]	OMDL+ROM	2
Threshold (t-of-n)	Lindell22 Sparkle [C**K**M23]	Schnorr DL+ROM	3
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Concurrently Secure ✓

Randomized (Stateful) ✗

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Honest minority: up to (t-1) corrupt; at least one honest (t total).

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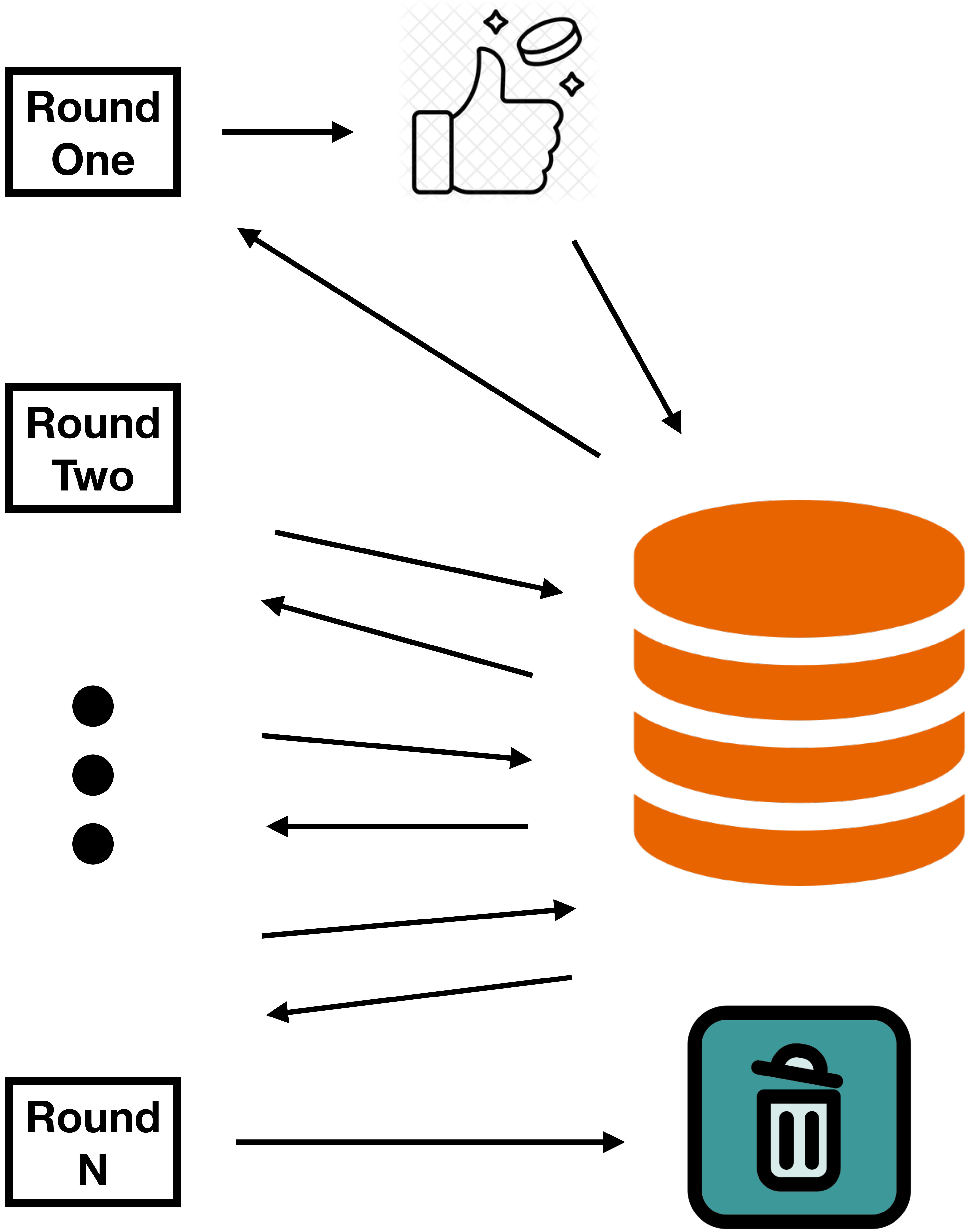

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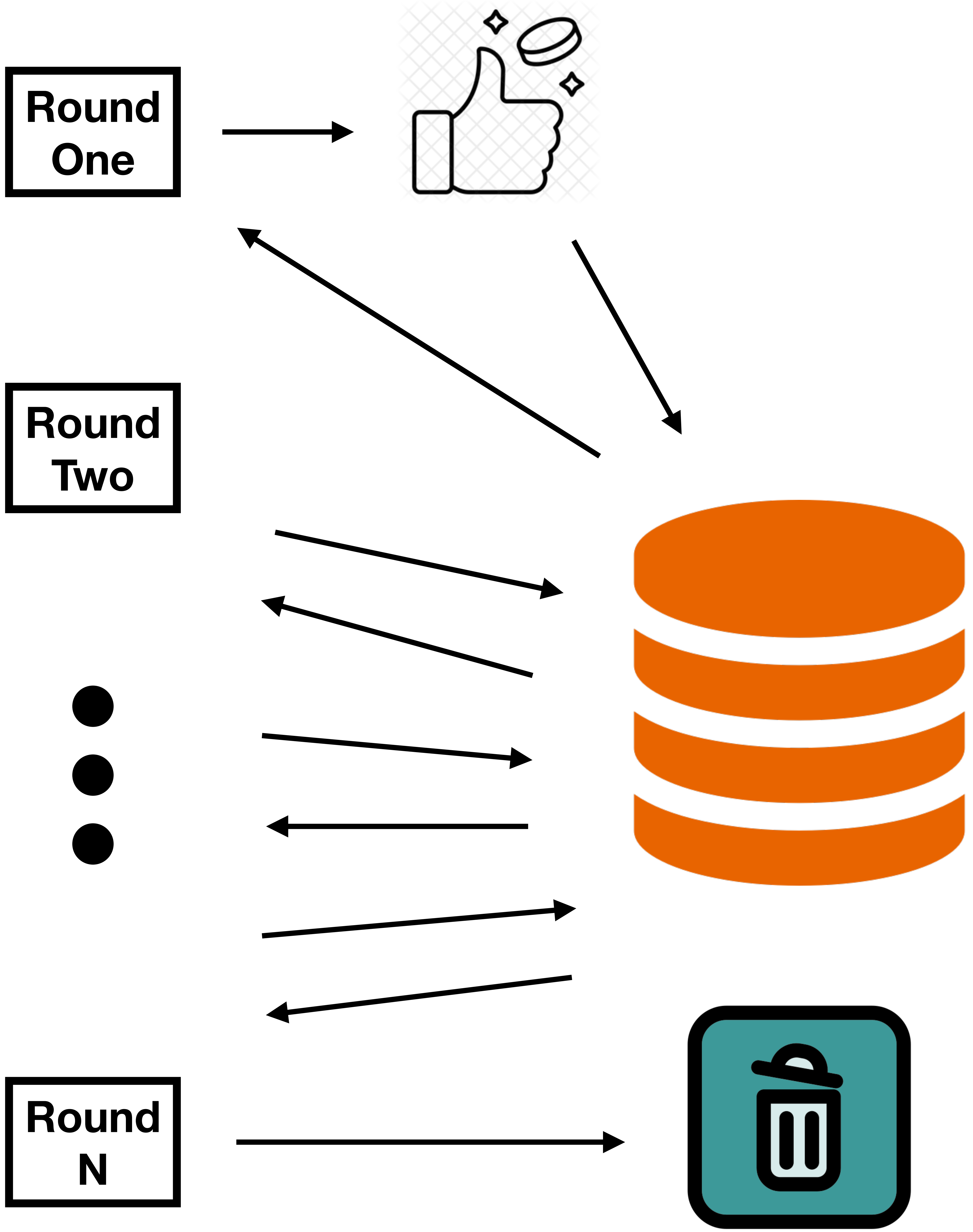

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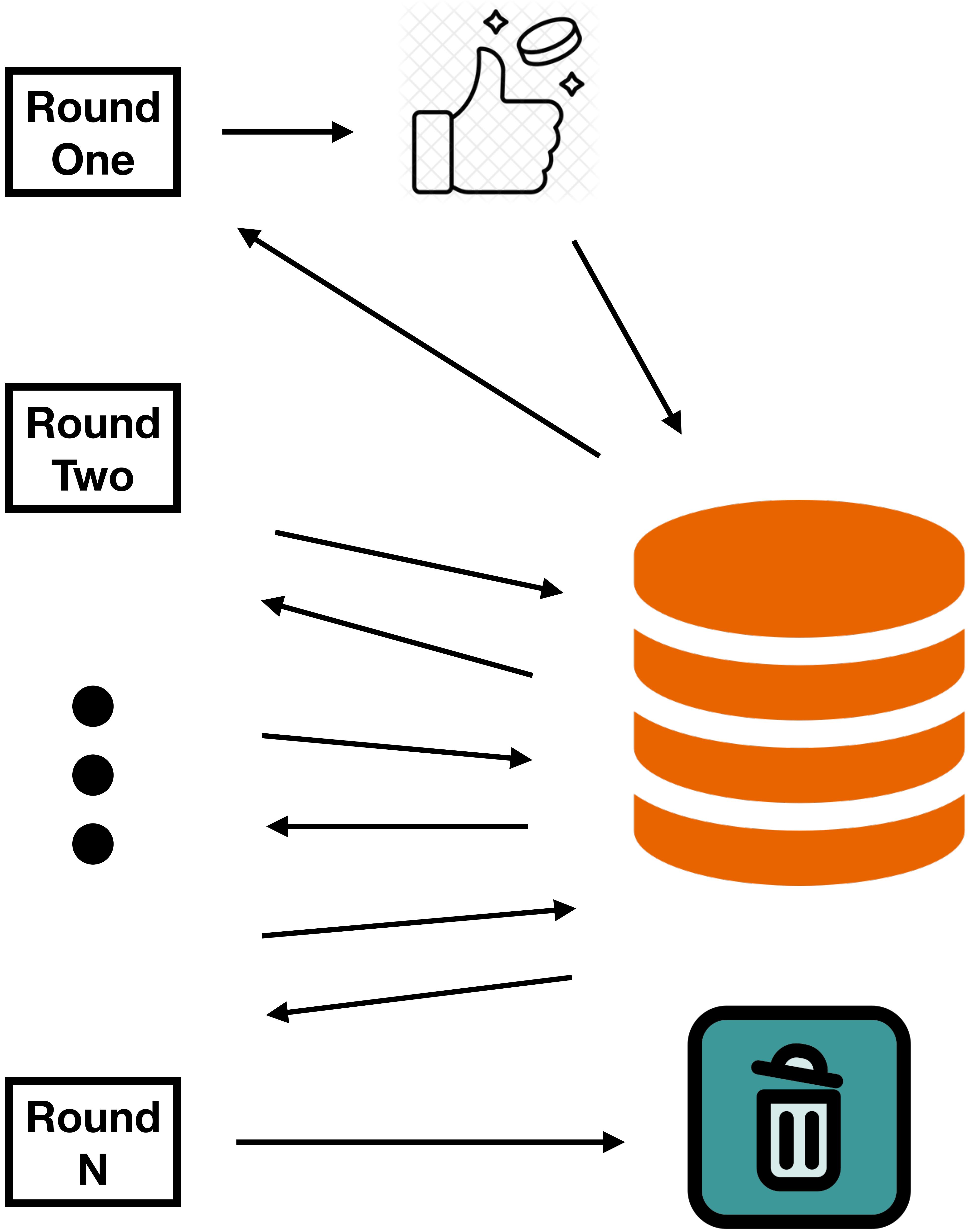

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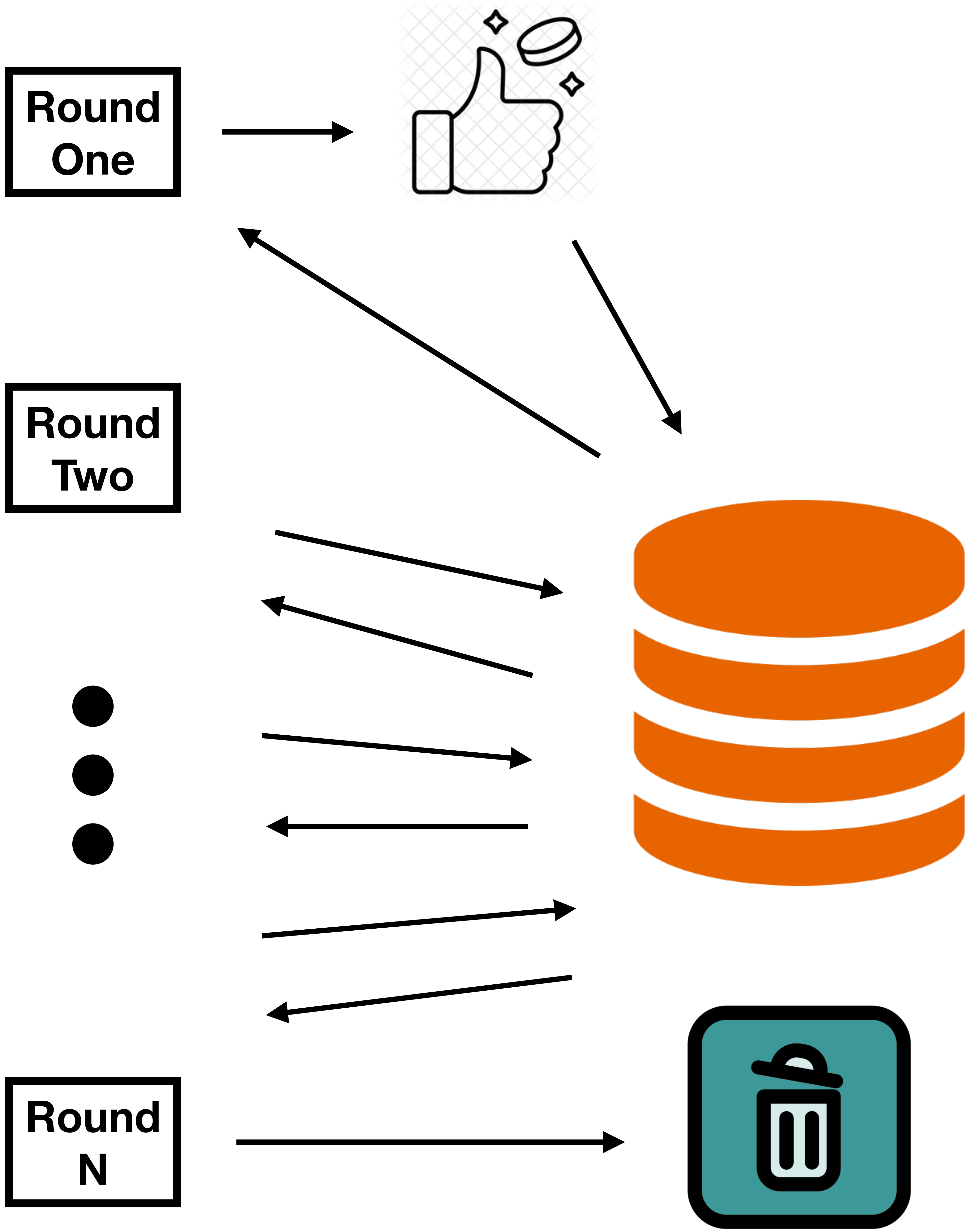

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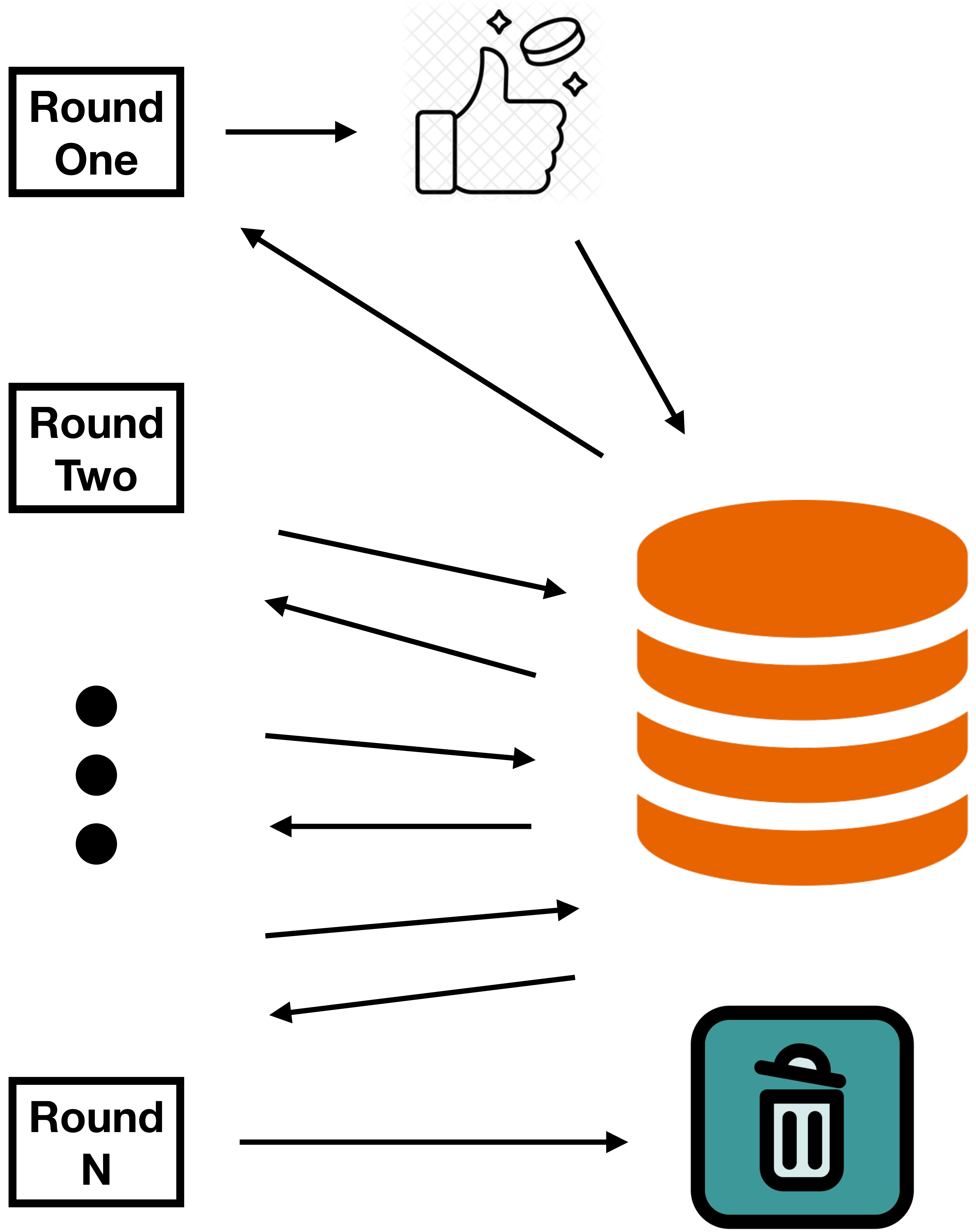
- Determinism is a means to achieve statelessness



(Single-Party) Schnorr Signatures

To generate a key pair:

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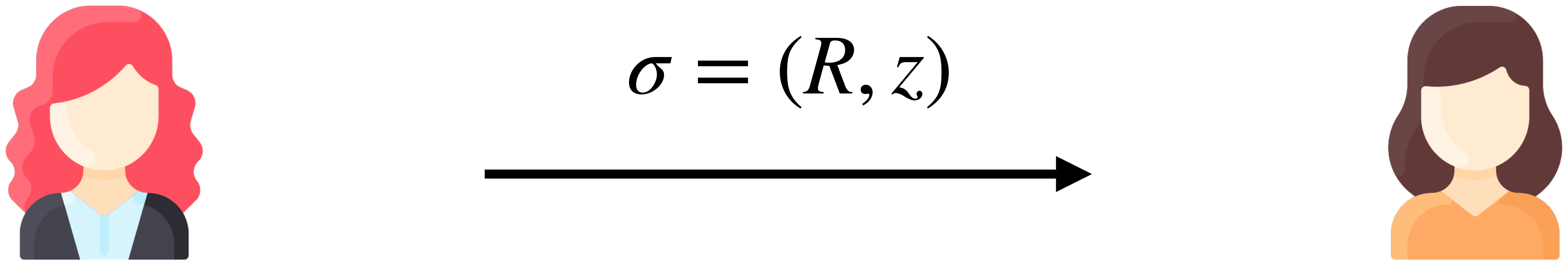

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To verify (PK, σ, m):

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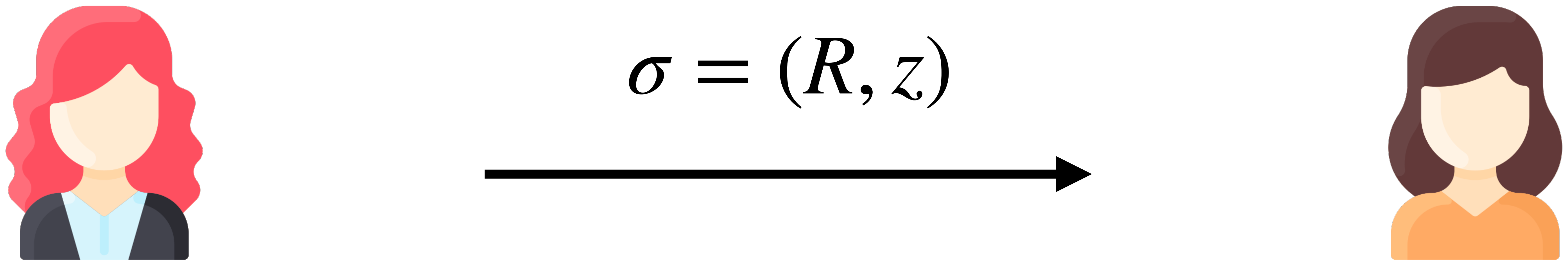
$$R \cdot PK^c \stackrel{?}{=} g^z$$

output accept/reject

(Single-Party) EdDSA Signatures



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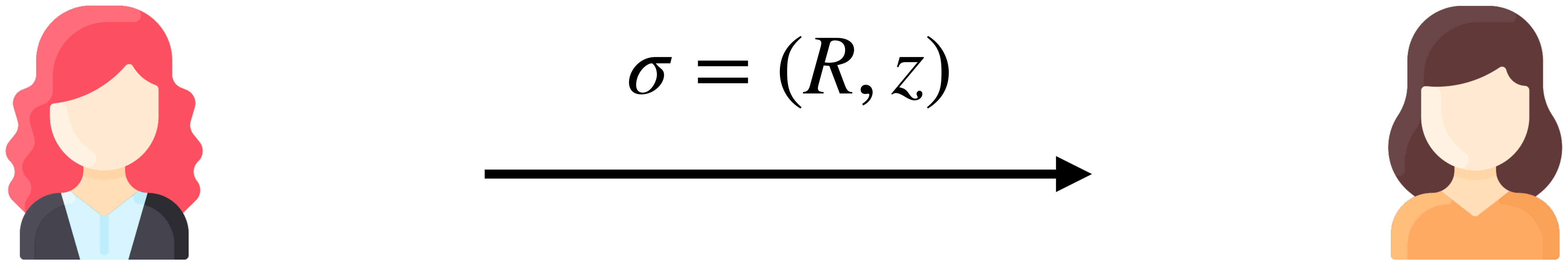

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Prevents issues from bad randomness.

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Summary: EdDSA-style determinism is **not** publicly verifiable; Adversary can pick its nonce randomly without detection

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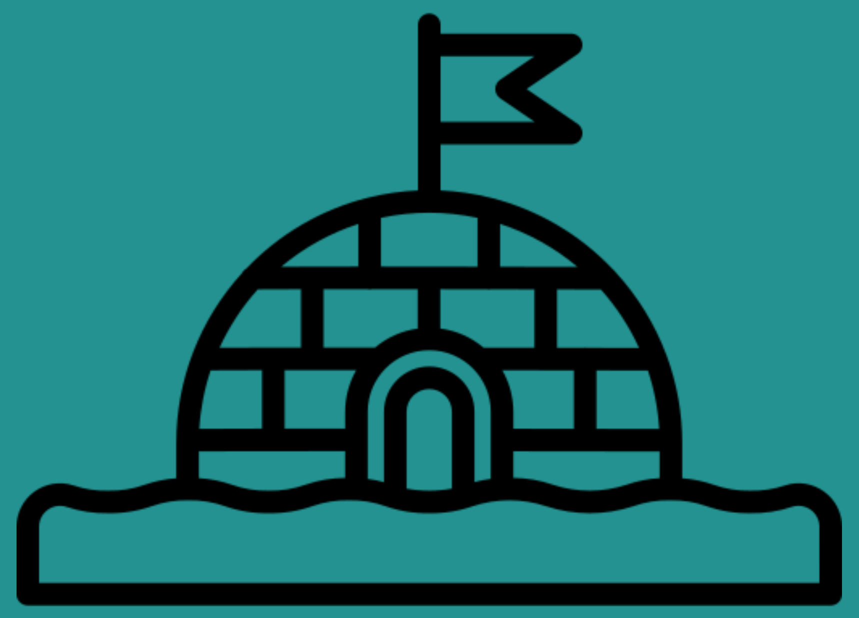
Goal of this work: To design a practical (efficient, simple) deterministic threshold signature.

Arctic: A Two-Round Stateless Threshold Schnorr Signature



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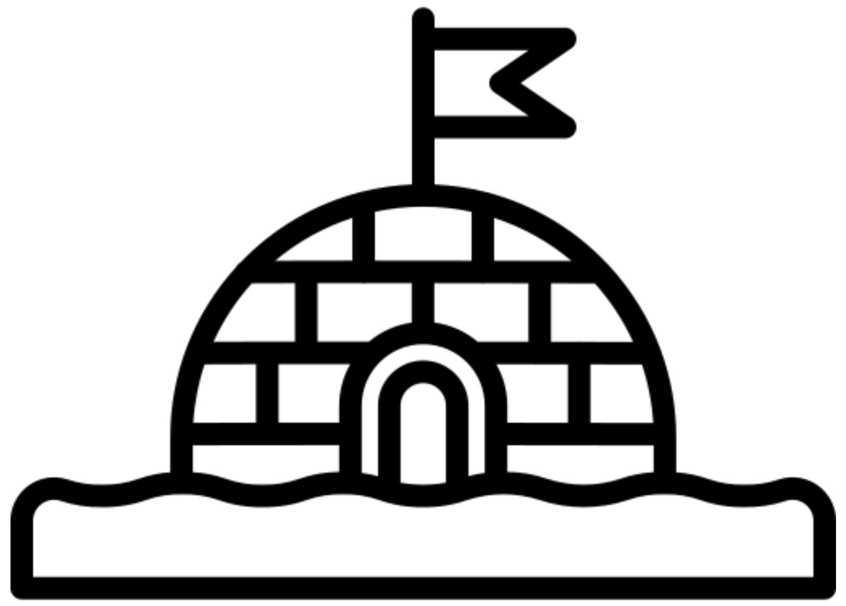

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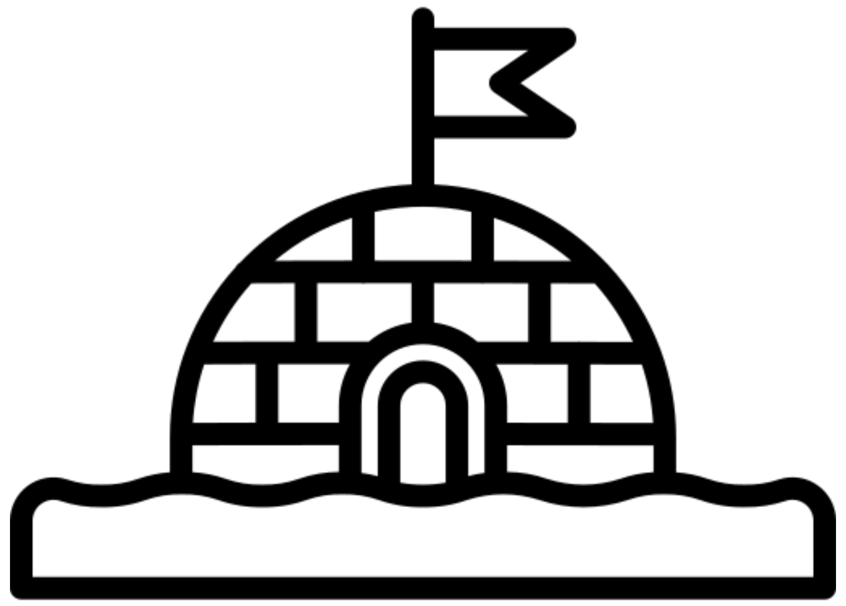

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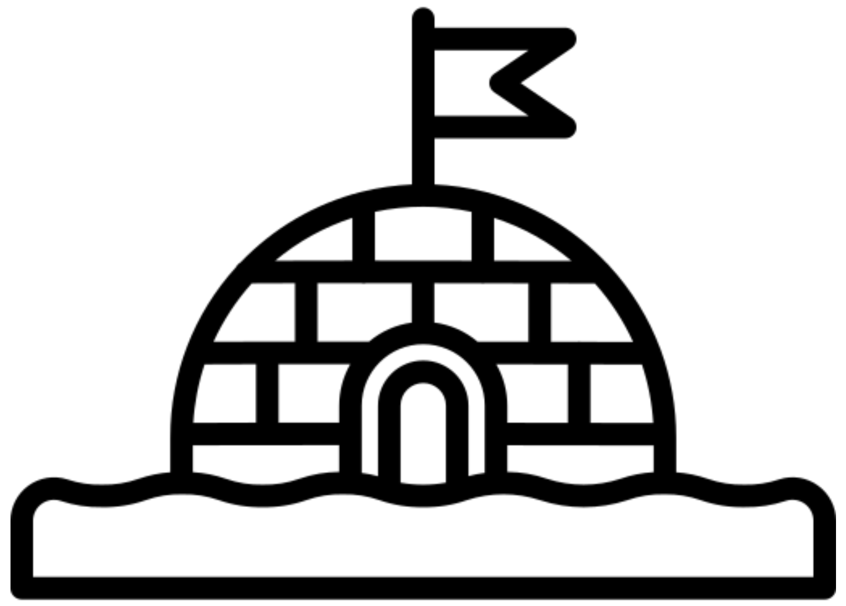

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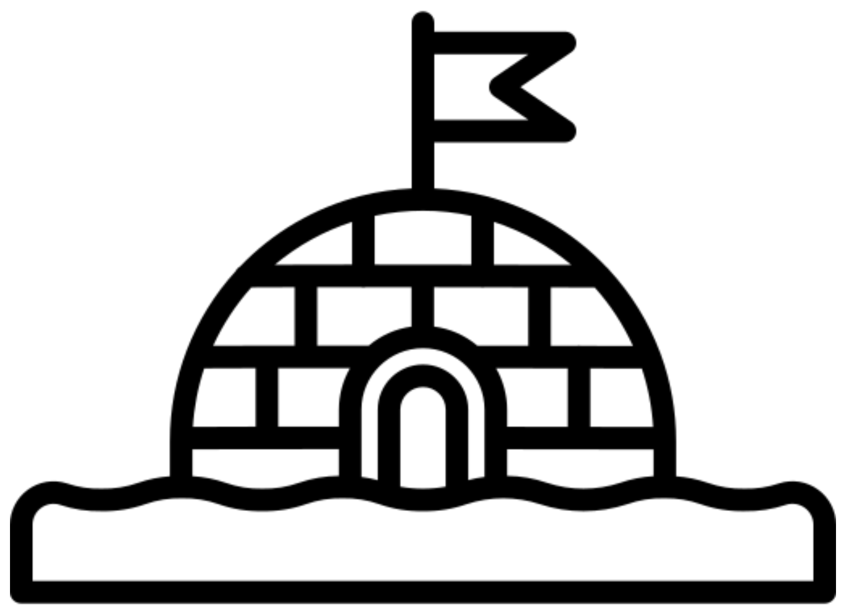
  - Efficient for moderately-sized groups (i.e., less than 25). ✓



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$\underline{\text{KeyGen}(1^\lambda)}$

1. Derive
   $sk^s \leftarrow \mathbb{Z}_q; PK \leftarrow g^{sk^s}$
2. Shamir secret share sk into
   $(sk_1^s, \ldots, sk_n^s)$
3. Generate VPSS keys
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4. Send key shares (sk_i^v, sk_i^s)
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   Output $\perp$

$(r_i, R_i) \leftarrow \text{VPSS.Gen}(sk_i^v, m, C)$

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Combine$(R, \{z_i\}_{i\in C})$

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Correctness: $r = \sum_{i \in C} r_i \lambda_i$ and $sk^s = \sum_{i \in C} sk_i^s \lambda_i$

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- Verification ensures each party followed the protocol honestly.

Replicated Secret Sharing: Example

$a_1 = (2,3,4)$

$a_2 = (1,3,4)$

$a_3 = (1,2,4)$

$a_4 = (1,2,3)$

corruption threshold t=2
minimum signers=3
total signers n=4

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Intuition: sk^v is information-theoretically hidden;
each (t-1) corrupt parties lack exactly one ϕ_i.

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Verifiable Pseudorandom Secret Sharing in Arctic

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To derive Arctic nonces:

$$r_k \leftarrow \sum_{i=1}^{\binom{n-1}{t-1}} H(\phi_i, m) \cdot L_{a_i}(k), \text{ for each } \phi_i \in sk_i^v$$

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Interpolate to
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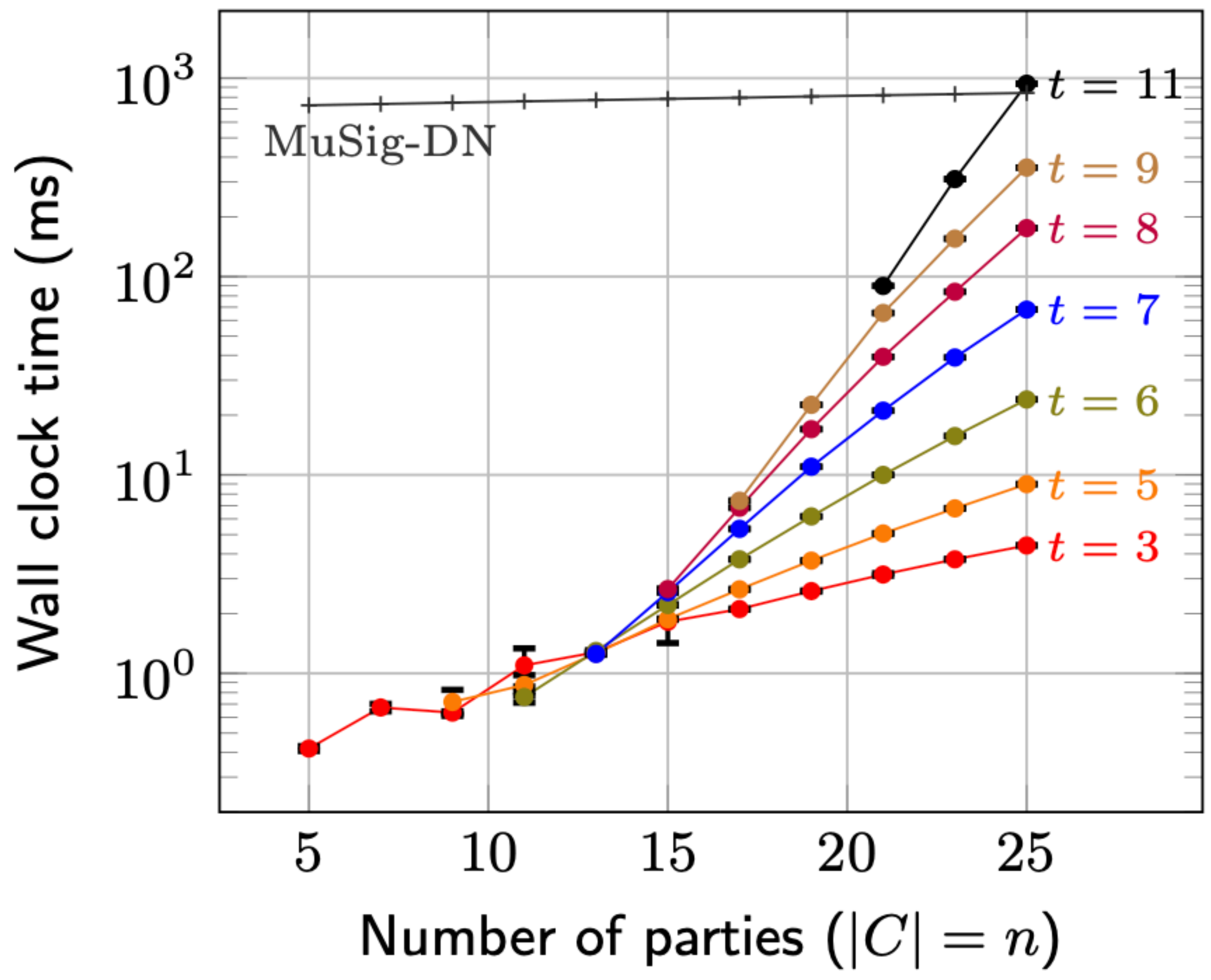
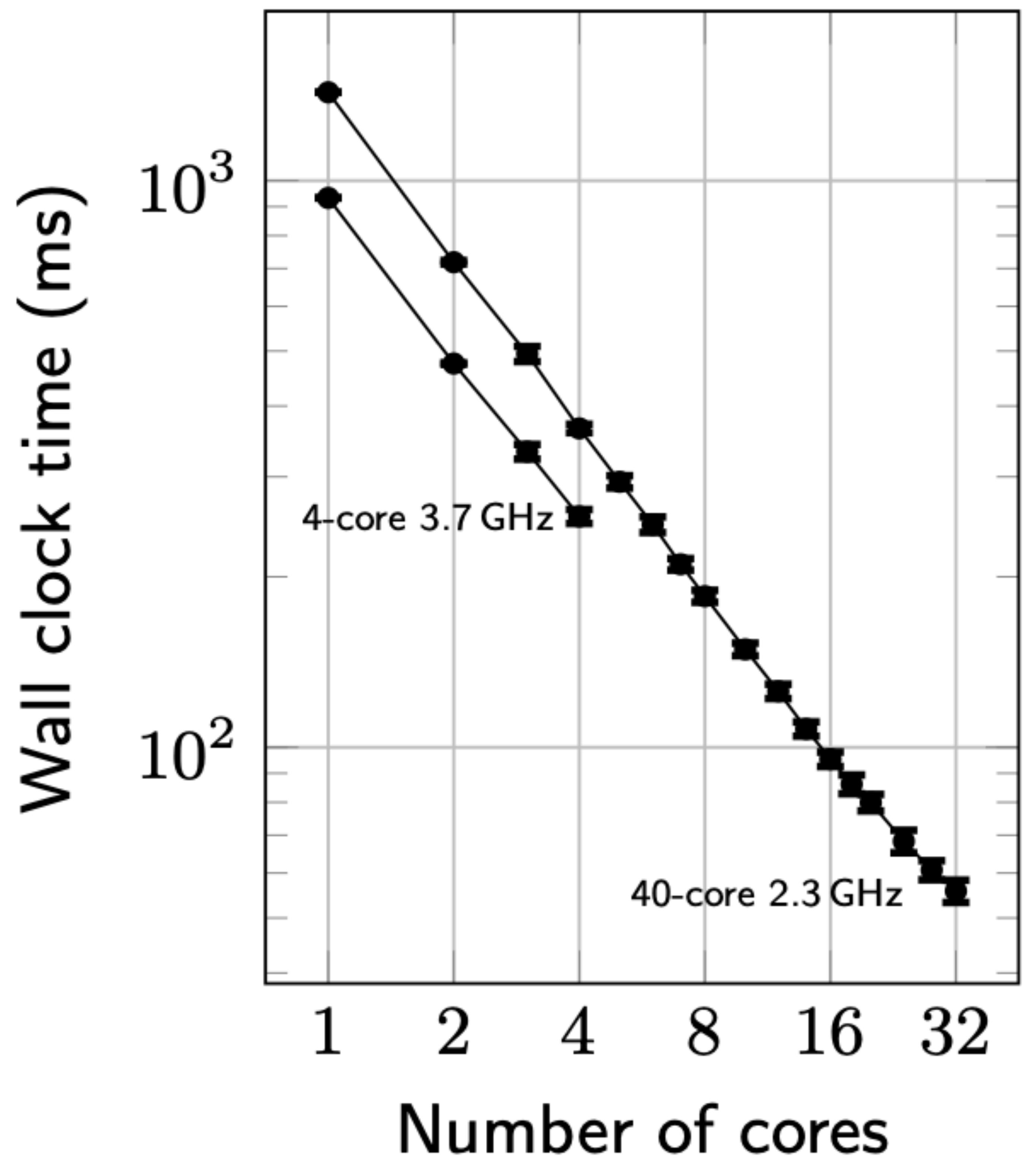
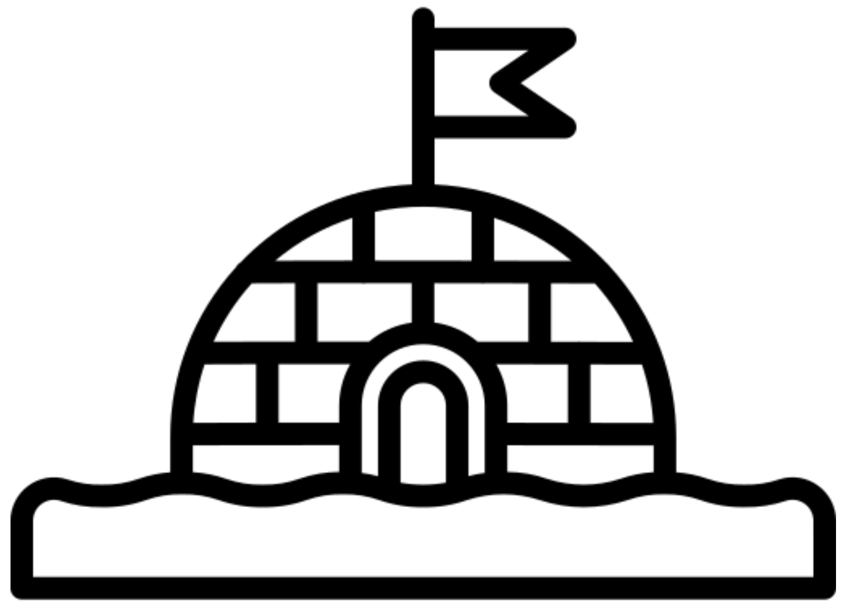
Performance of Arctic





MuSig-DN uses Bulletproofs to prove
a party generated their nonce honestly

Takeaways



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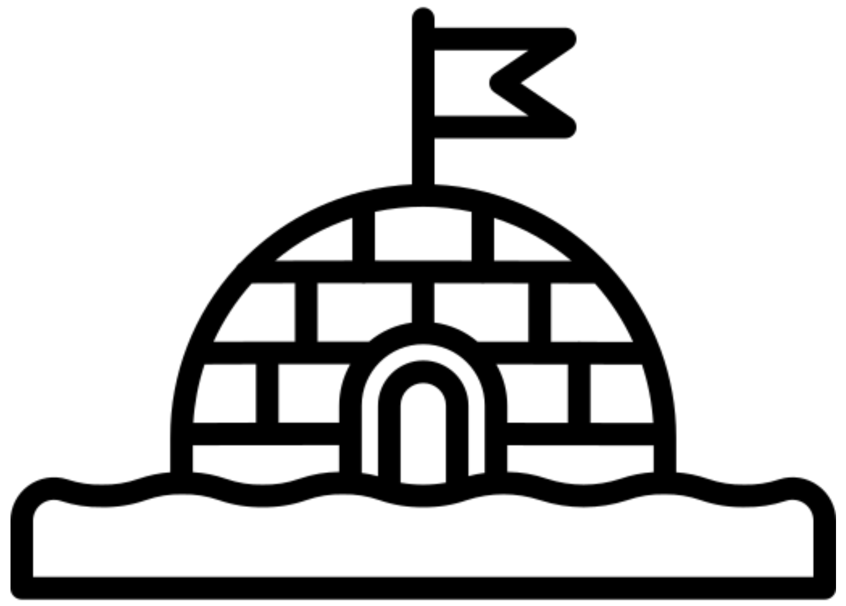

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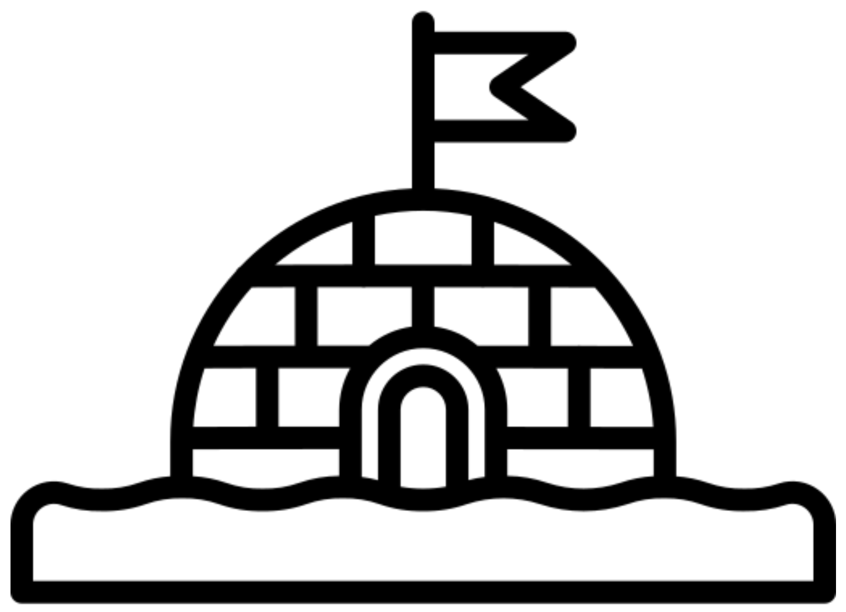

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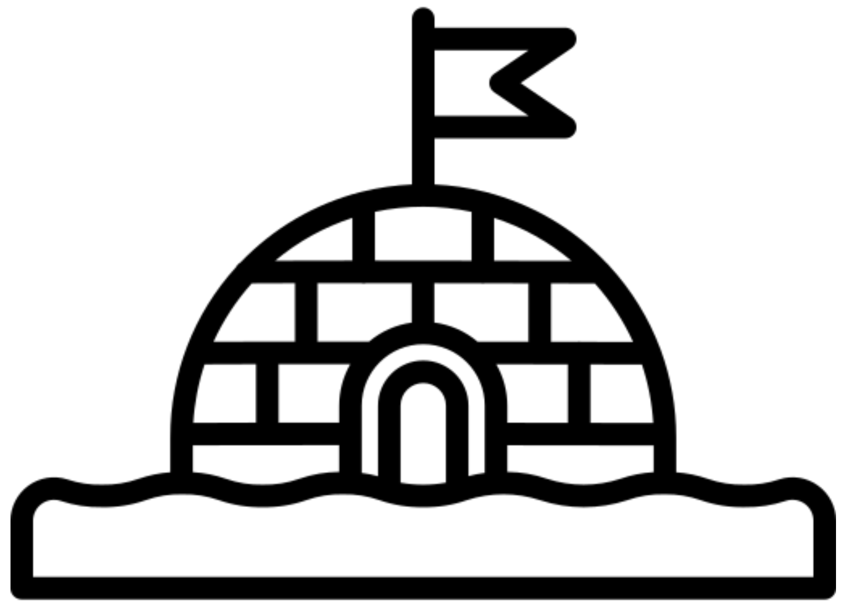

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Publicly verifiable when performed over commitments $(R_i)_{i \in C}$