

Randomness beacons in theory and practice



Joseph Bonneau

Real World Crypto
March 28, 2025



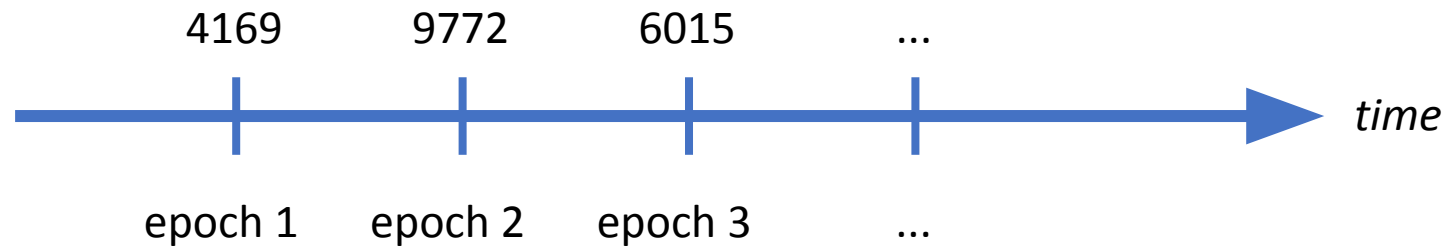
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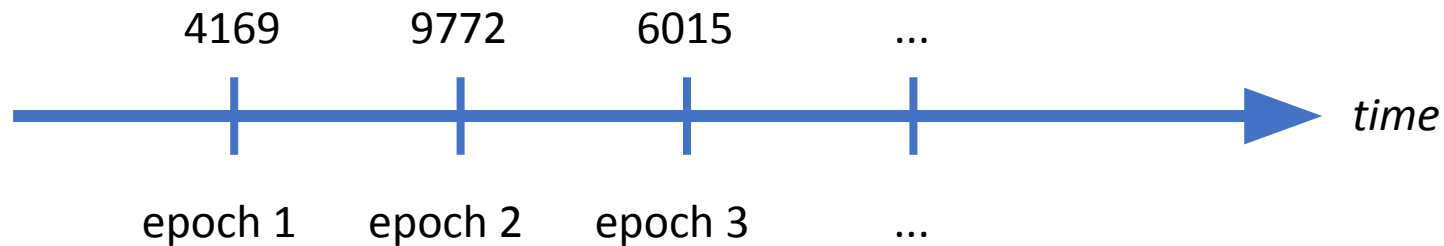
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The *randomness beacon* abstraction [Rabin83]



The *randomness beacon* abstraction [Rabin83]



Goals (high level):

- Statistically uniform randomness
- Public consensus on values
- Regular service, high bandwidth
- Attackers can't:
 - Predict
 - Manipulate
 - Block

Beacons can power verifiable lotteries



GROUP B	GROUP C	GROUP D
PARIS SAINT-GERMAIN	FC BAYERN MÜNCHEN	CR FLAMENGO
ATLÉTICO DE MADRID	AUCKLAND CITY FC	ESPÉRANCE SPORTIVE DE TUNISIE
BOTAFOGO	CA BOCA JUNIORS	CHELSEA FC
SEATTLE SOUNDERS FC	SL BENFICA	CLUB LEÓN
GROUP F	GROUP G	GROUP H
FLUMINENSE FC	MANCHESTER CITY	REAL MADRID C. F.
BORUSSIA DORTMUND	WYDAD AC	AL HILAL
ULSAN HD	AL AIN FC	CF PACHUCA
MAMELODI SUNDOWNS FC	JUVENTUS FC	FC SALZBURG



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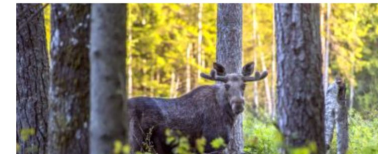
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[Home](#) > [Hunting in NH](#) > [Moose Hunting in New Hampshire](#) > [Moose Hunt Lottery](#)

Moose Hunt Lottery

Important information on the NH Moose Hunt Lottery

The window to apply for a moose hunting permit is mid-January to midnight on the last Friday in May.



Lottery Drawing/Unit Assignment:
Permittee candidates are selected through a computer-generated random number draw.

Each applicant selected in the lottery drawing is **assigned to hunt** within a

Quick Links

[Online Lottery Application](#)

[Mail-in Lottery App](#)

[Lottery Overview and Odds](#)

The New York Times

New Federal Judiciary Rule Will Limit 'Forum Shopping' by Plaintiffs

For years, litigants have tried to cherry-pick the judges in sweeping cases on abortion and immigration. Random judge selection is about to make that harder.

By [Mattathias Schwartz](#)

March 12, 2024

Many use cases beyond lotteries

- Games
- Sampling ballots for election audits
- Selecting parameters for cryptographic protocols
- Leader election in BFT consensus & blockchains
- Randomized transaction ordering
- Challenges for non-interactive cryptographic proofs

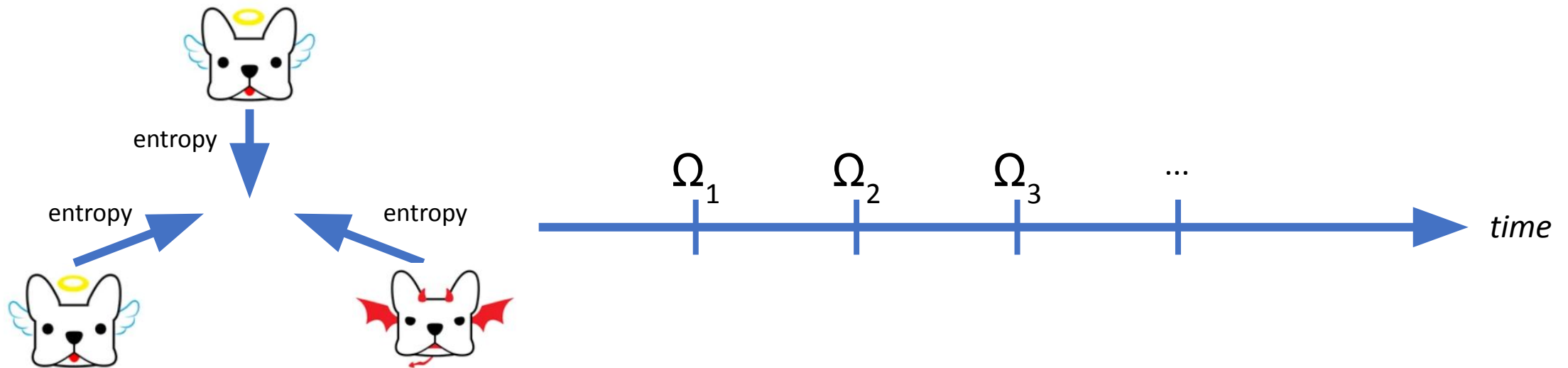
Goal: Many applications driven by a public randomness beacon

State of the art has barely changed for millenia!



This talk: *distributed* randomness beacons

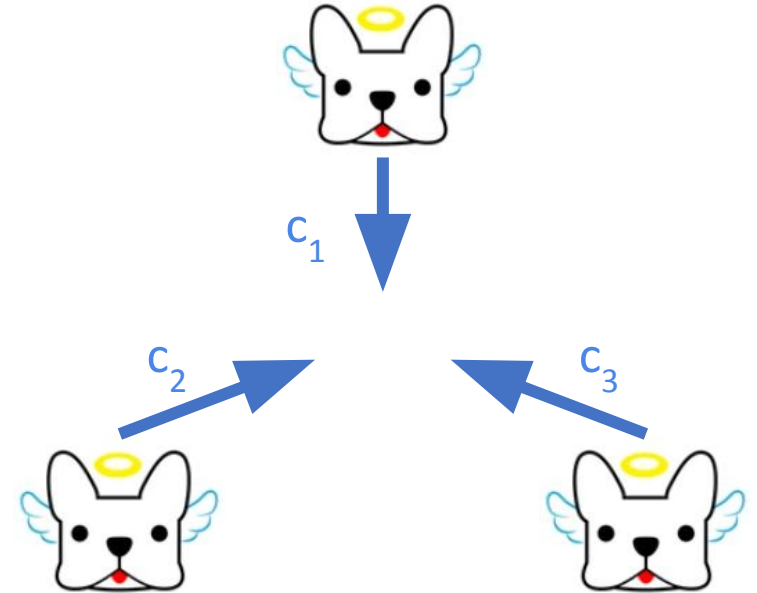
- Multiparty protocol with n participants, produce output Ω_i in epoch i
- Up to t out of n nodes are controlled by the adversary



Classic: Commit-Reveal

1. Commit

- Publish a cryptographic commitment
 $c_i = \text{Commit}(e_i)$ to a random value e_i



Classic: Commit-Reveal

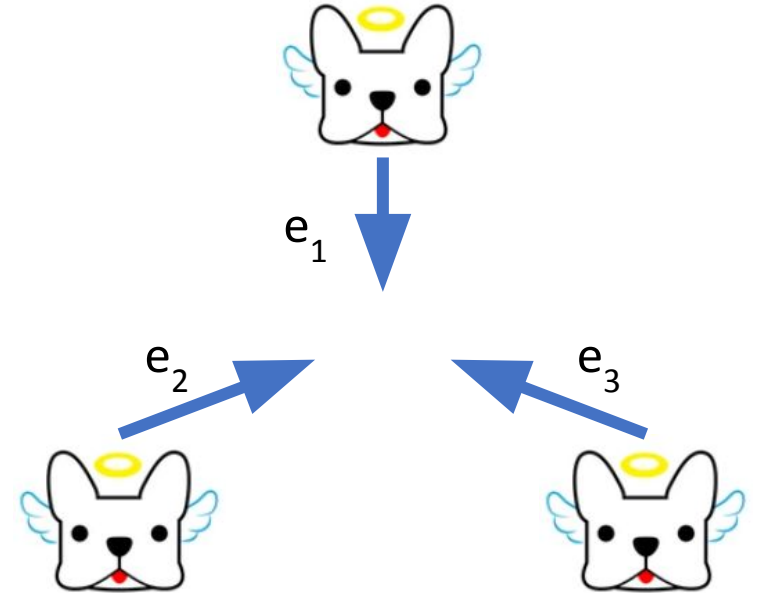
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2. Reveal

- Participants reveal their e_i values

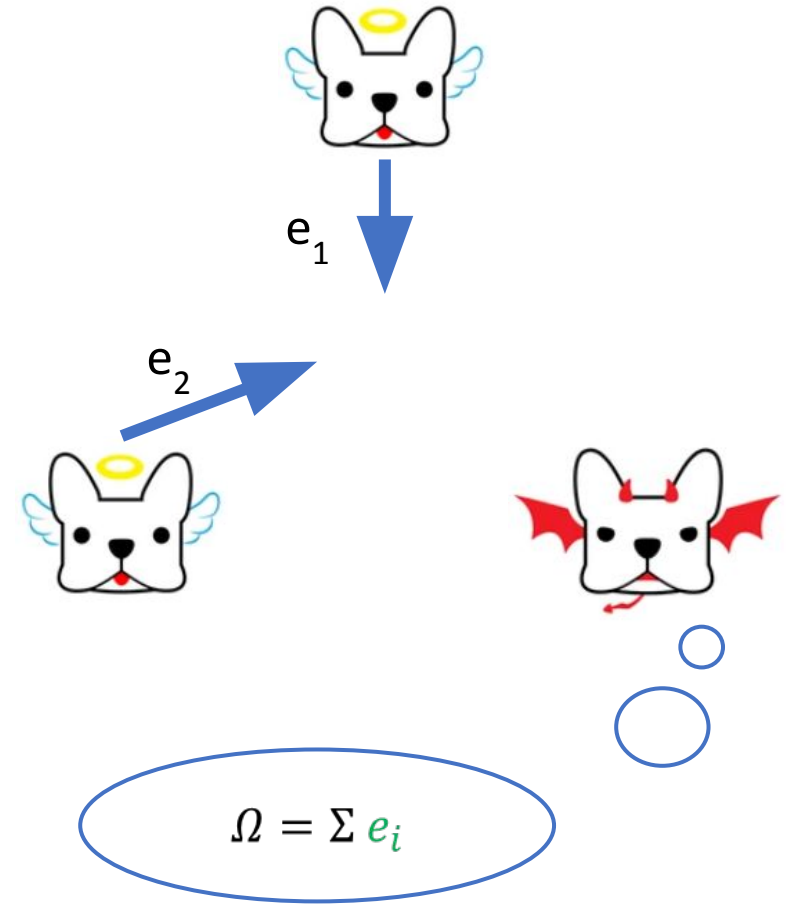
Beacon output: $\Omega = \sum e_i$



Classic: Commit-Reveal

1. Commit
 - Publish a cryptographic commitment $c_i = \text{Commit}(e_i)$ to a random value e_i
2. Reveal
 - Participants reveal their e_i values

Main problem: last-revealer attack



Classic: Commit-Reveal

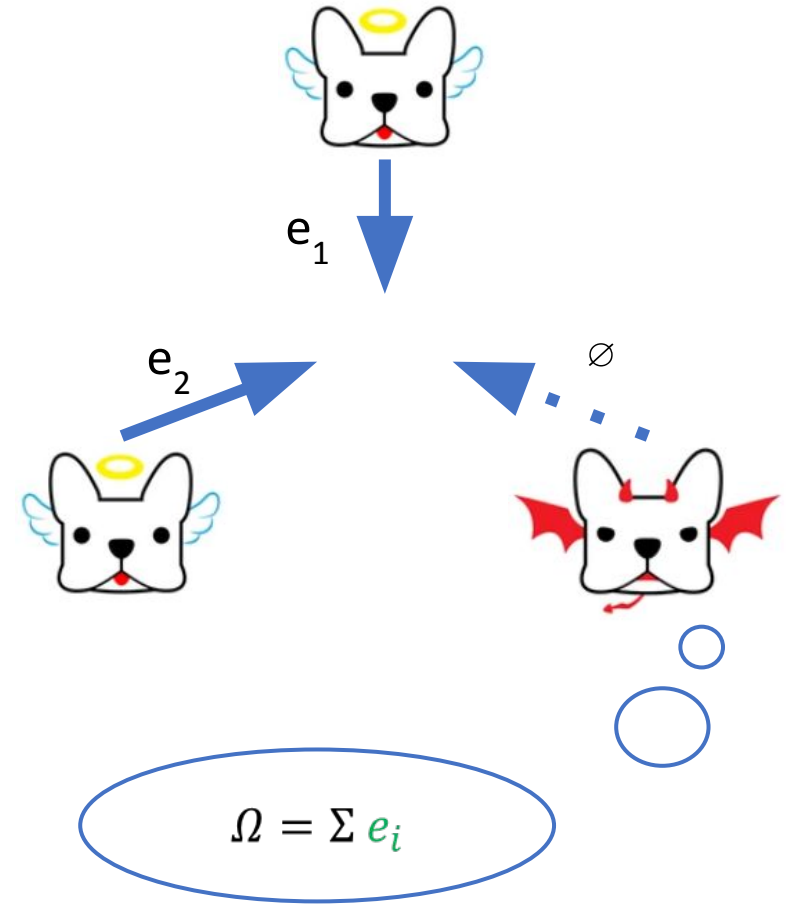
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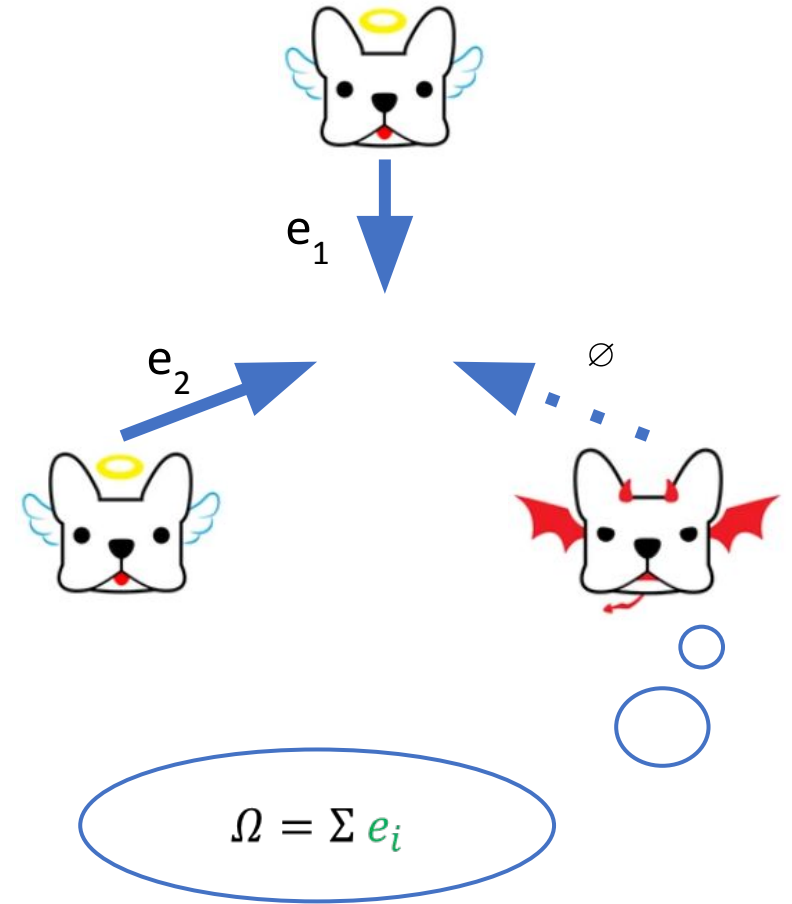
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Classic: Commit-Reveal

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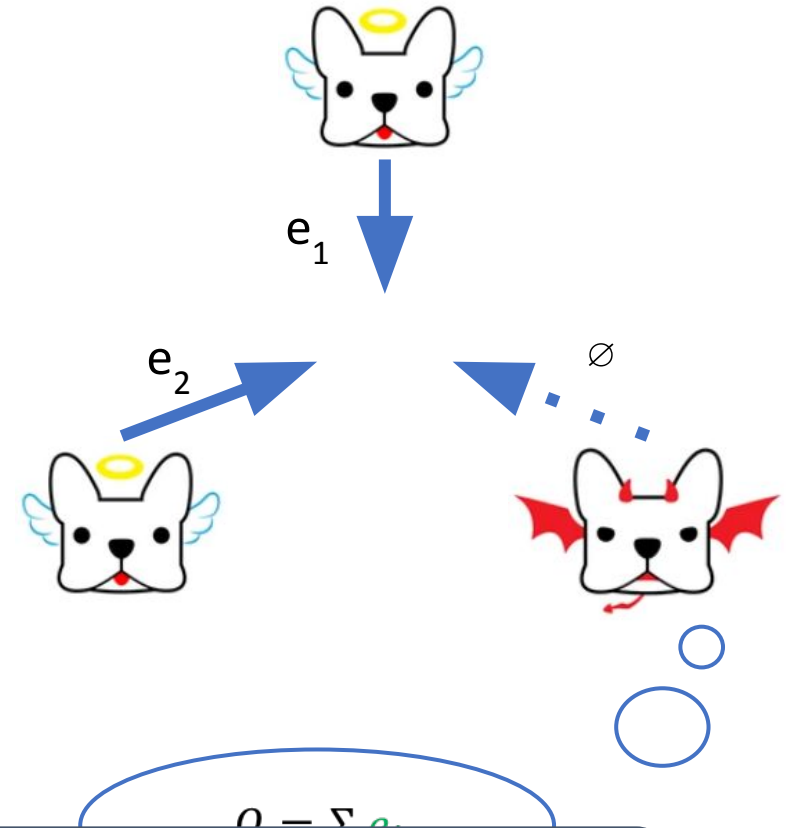
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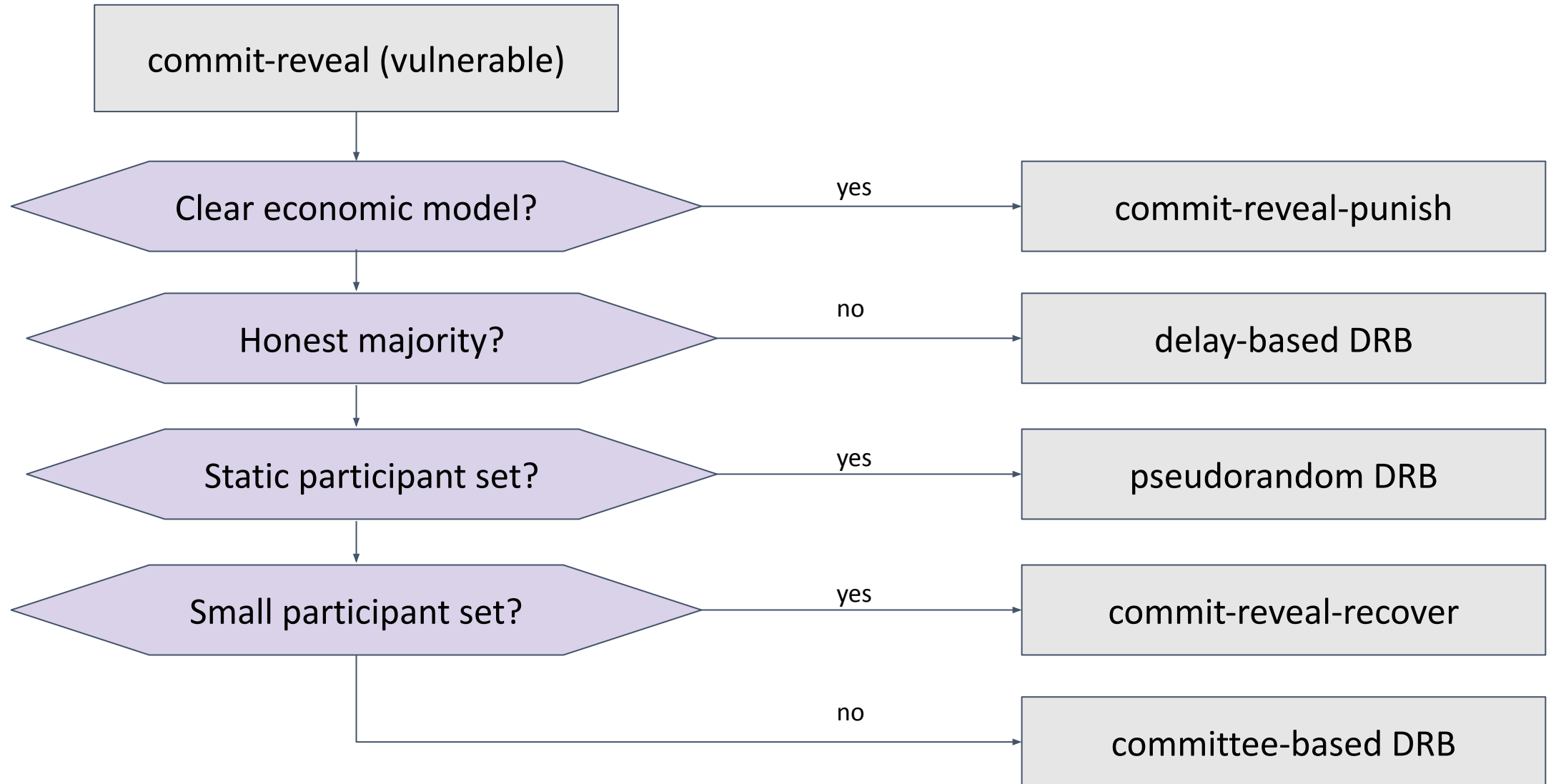
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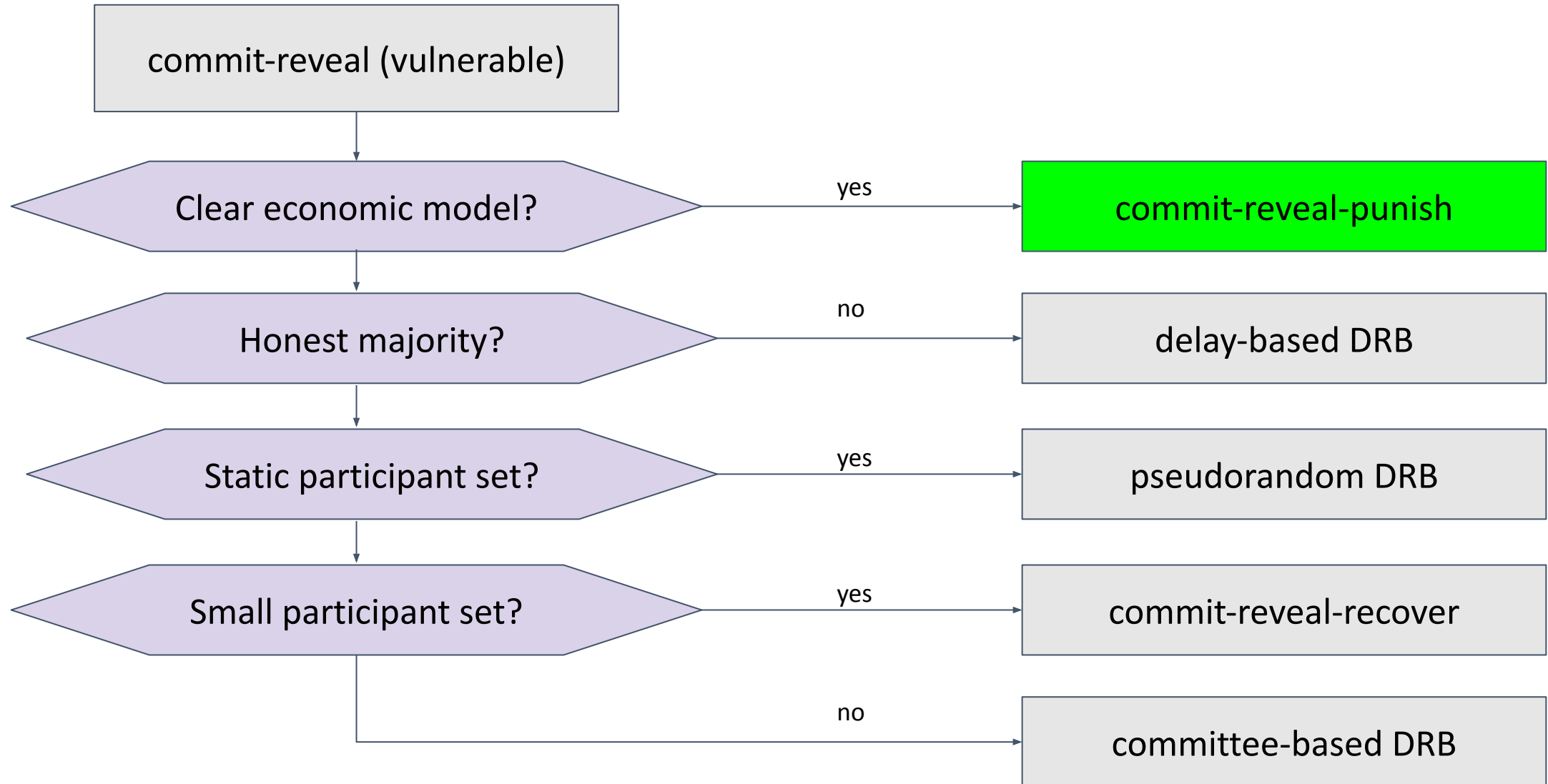


All DRBs are essentially **patches to the last revealer attack**

DRB design flow chart

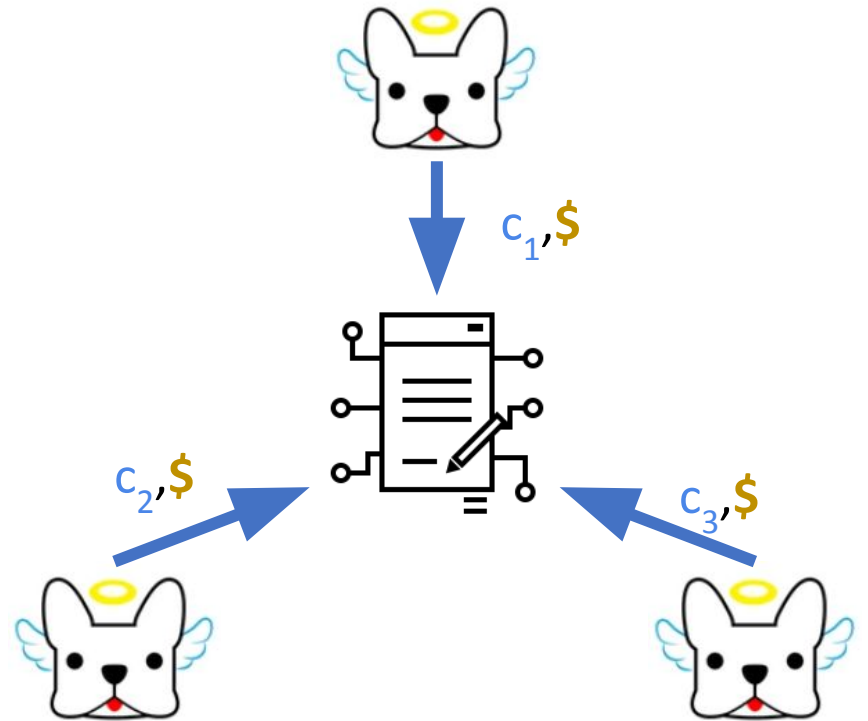


DRB design flow chart



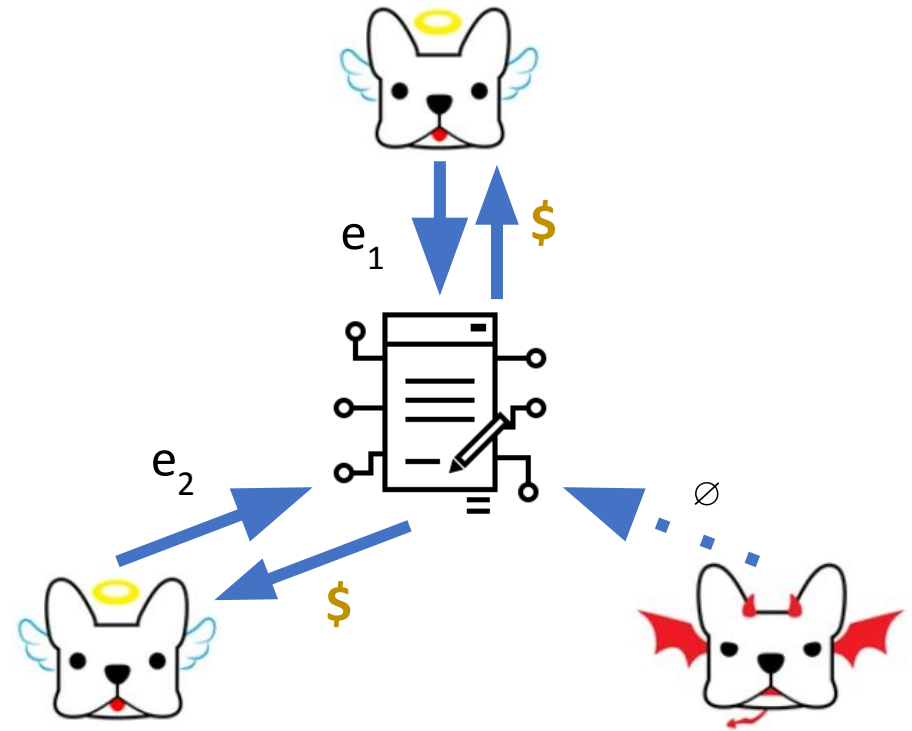
Commit-reveal-punish

1. Commit/deposit



Commit-reveal-punish

1. **Commit/deposit**
2. **Reveal + refund**
 - a. Participants who don't reveal lose funds
 - b. Restart if any participant aborts



Commit-reveal-punish

- **Advantages**

- efficient
 - $O(n)$ communication, compute
- easily implemented

- **Cons**

- Requires capital lockup
- Benign faults must be punished
- Hard to bound attacker utility if beacons have multiple purposes

Commit-reveal-punish: RANDAO

- Deployed in Ethereum since 2020
 - Used for committee selection
- Also available to smart contracts
 - `block.prevrandao`
- Proposer can reveal VRF or withhold
 - Withholding precludes block reward
- Withholding is profitable! [AW24]

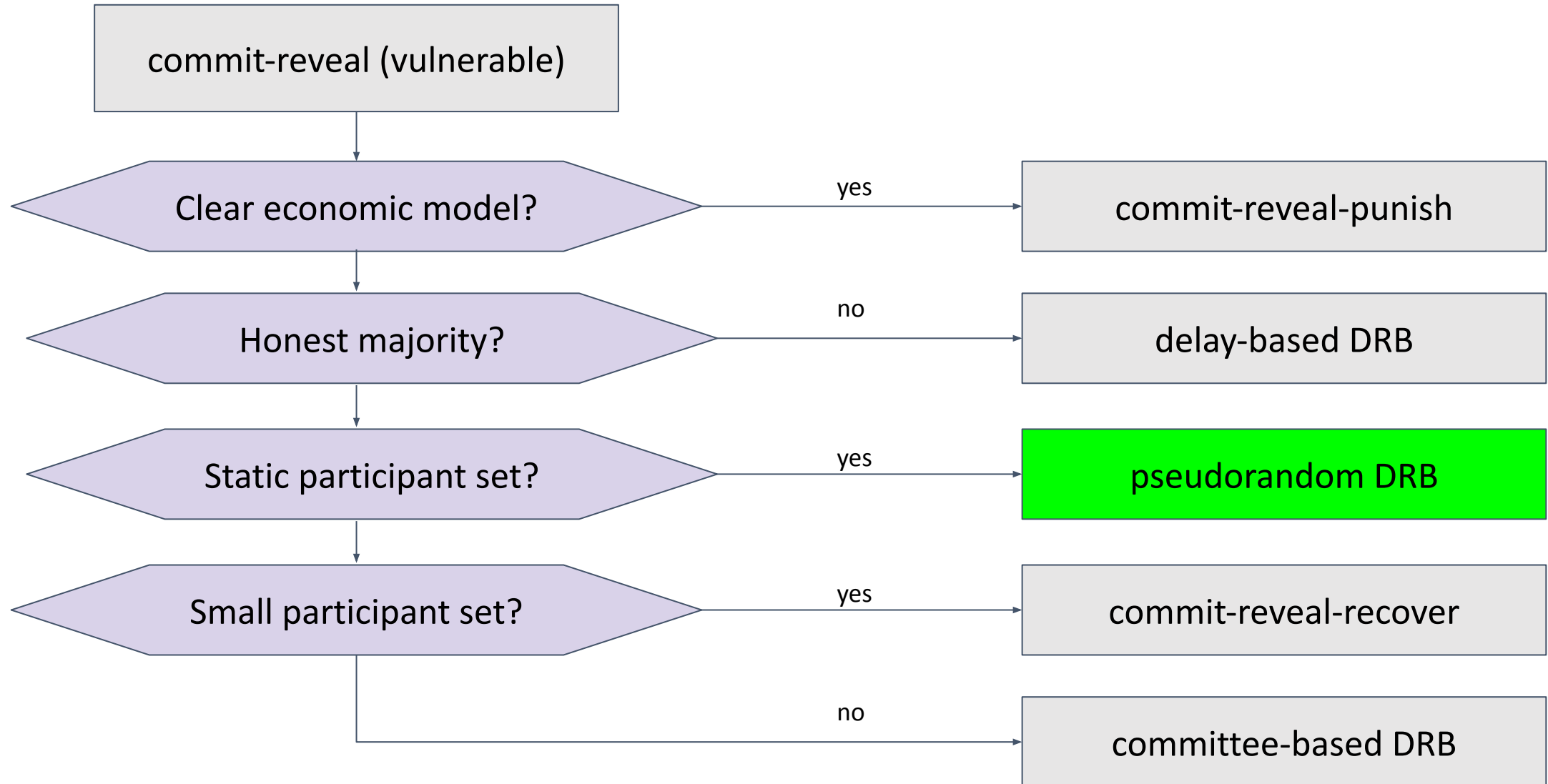
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- Withholding is profitable! [AW24]

α	optimal reward
1%	1.00107%
5%	5.04834%
10%	10.18807%
15%	15.39960%
20%	20.67770%
25%	26.02472%
30%	31.45164%
35%	36.97348%
40%	42.62435%
45%	48.49184%

Optimal RANDAO Manipulation in Ethereum.
Kaya Alpturer, S. Matthew Weinberg. AFT 2024.

DRB design flow chart

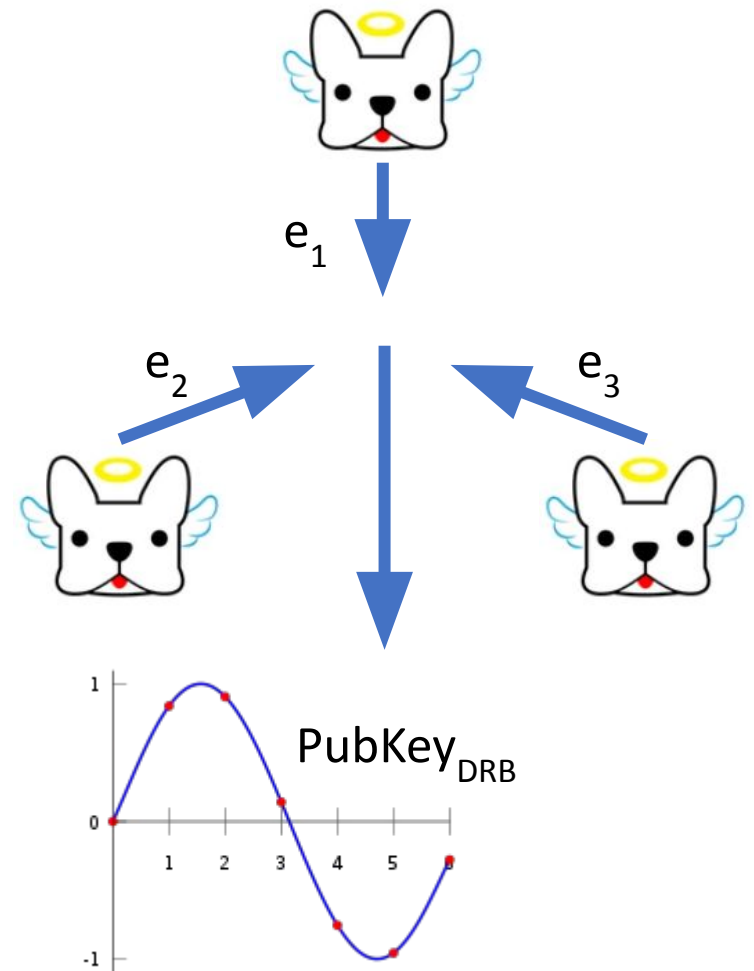


Pseudorandom DRBs

1. Setup

- Output is t -out-of- n secret-shared VRF key
- Can be distributed setup (DKG) or centralized

Setup itself must be a DRB!



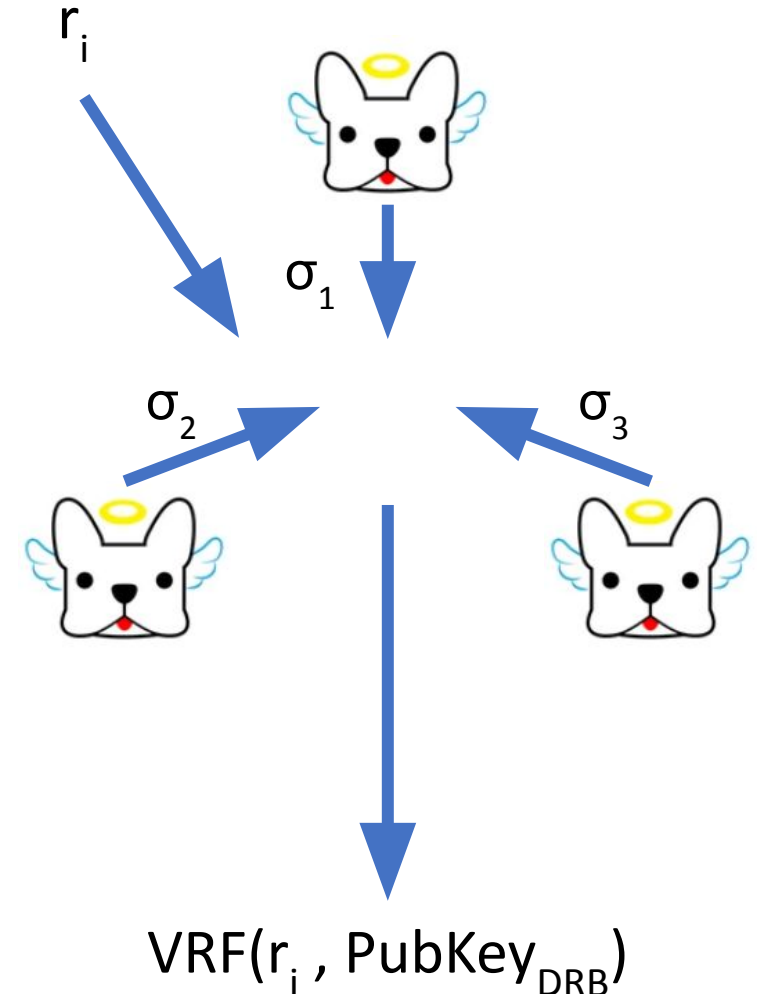
Pseudorandom DRBs

1. Setup

- a. Output is t -out-of- n secret-shared VRF key
- b. Can be distributed setup (DKG) or centralized

2. Output

- b. Compute VRF on round input r_i
- c. Can be static (epoch number) or prior Ω
- d. Collect partial VRF evaluations
- e. Combine to output distributed VRF



Pseudorandom DRBs

- **Advantages**

- Efficient
 - $O(n)$ communication, compute
- Dishonest majority cannot manipulate
 - Can only predict or stall

- **Challenges**

- If t needed to compute, $n-t$ can block liveness
- Malicious coalition can predict infinitely far into the future
- No recovery from compromise
 - Prudent to periodically re-key

Pseudorandom DRB variants

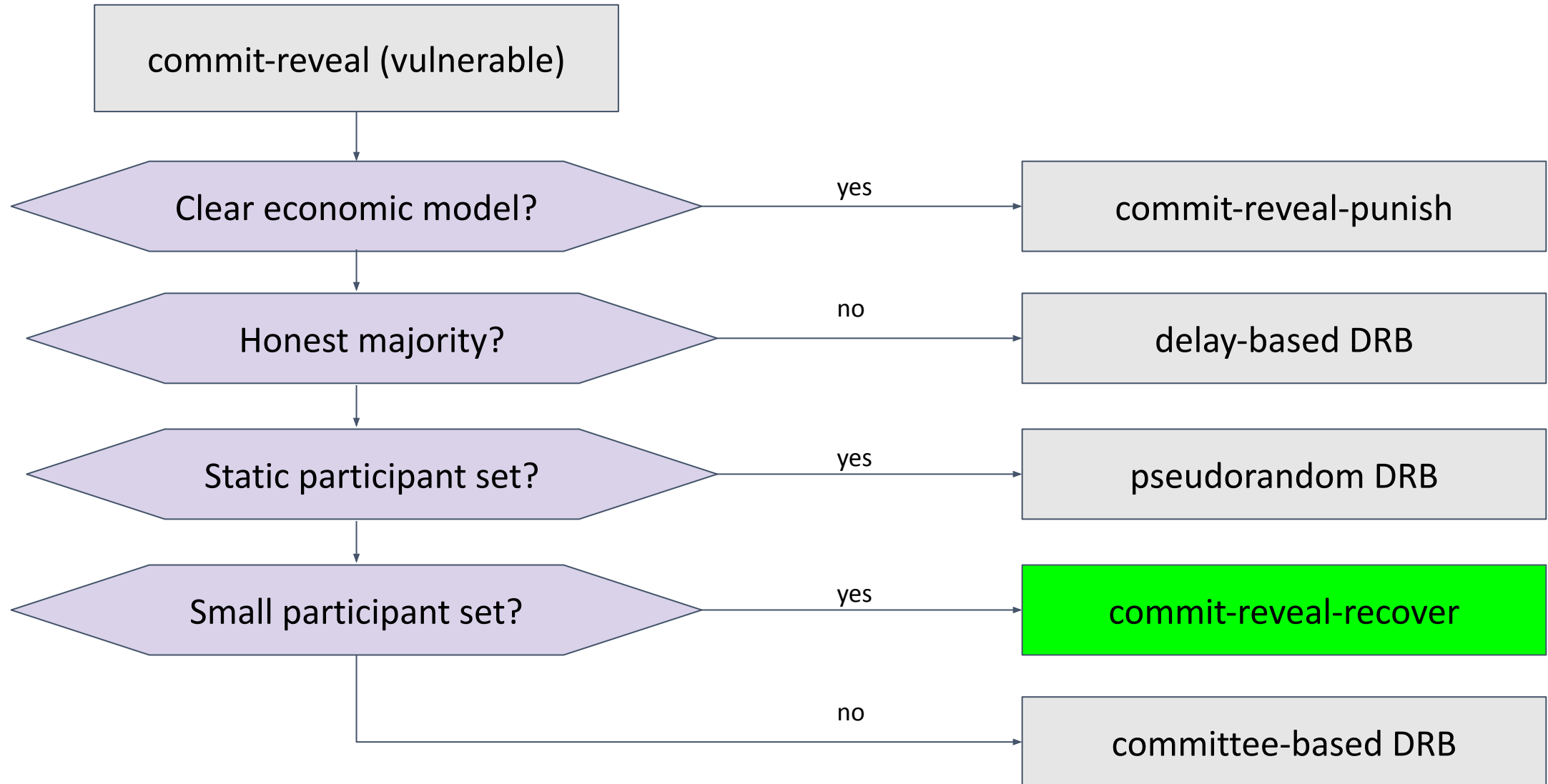
- drand/DFINITY
 - Threshold BLS signatures
- STROBE [BCKKLNNRS 21]
 - Threshold RSA decryption, with *history generation*
- RandHerd [SJKGGKFF 17]
 - Threshold Schnorr, sharded into groups
- DDH-DRB, GLOW-DRB [GLOW 20]
 - Threshold DDH-VRF (like BLS, but NIZK instead of pairings)

drand: a production pseudorandom DRB

- Launched 2019
- Threshold BLS signatures
 - BLS12-381 curve
- Currently 20 nodes
 - $t=11$ required to sign
- 256 bits every 30 seconds
- Nodes run by academics + industry



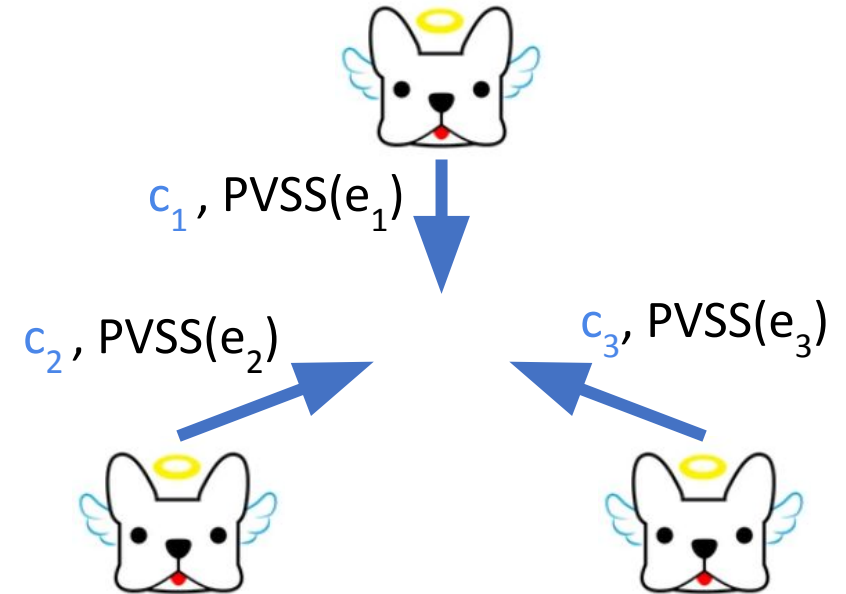
DRB design flow chart



Commit-reveal-recover

1. Commit

- a. Publish $c_i = \text{Commit}(e_i)$ as in classic CR
- b. Secret-share e_i with all other nodes
 - i. PVSS: Publicly verifiable secret sharing



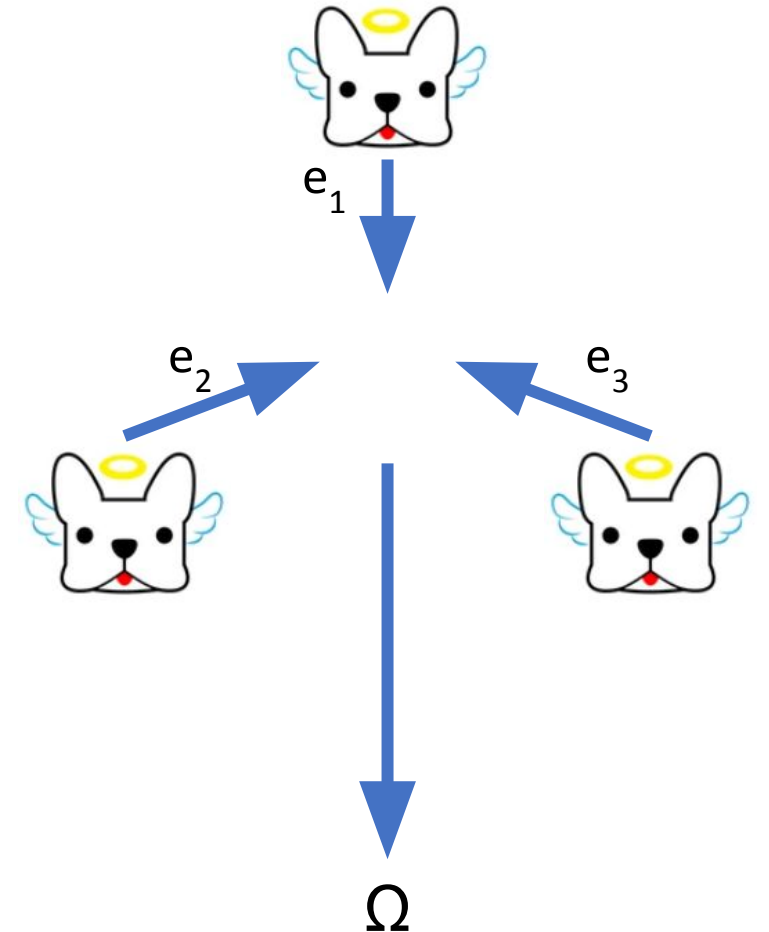
Commit-reveal-recover

1. Commit

- Publish $c_i = \text{Commit}(e_i)$ as in classic CR
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 - PVSS: Publicly verifiable secret sharing

2. Reveal

- Publish e_i as in classic CR



Commit-reveal-recover

1. Commit

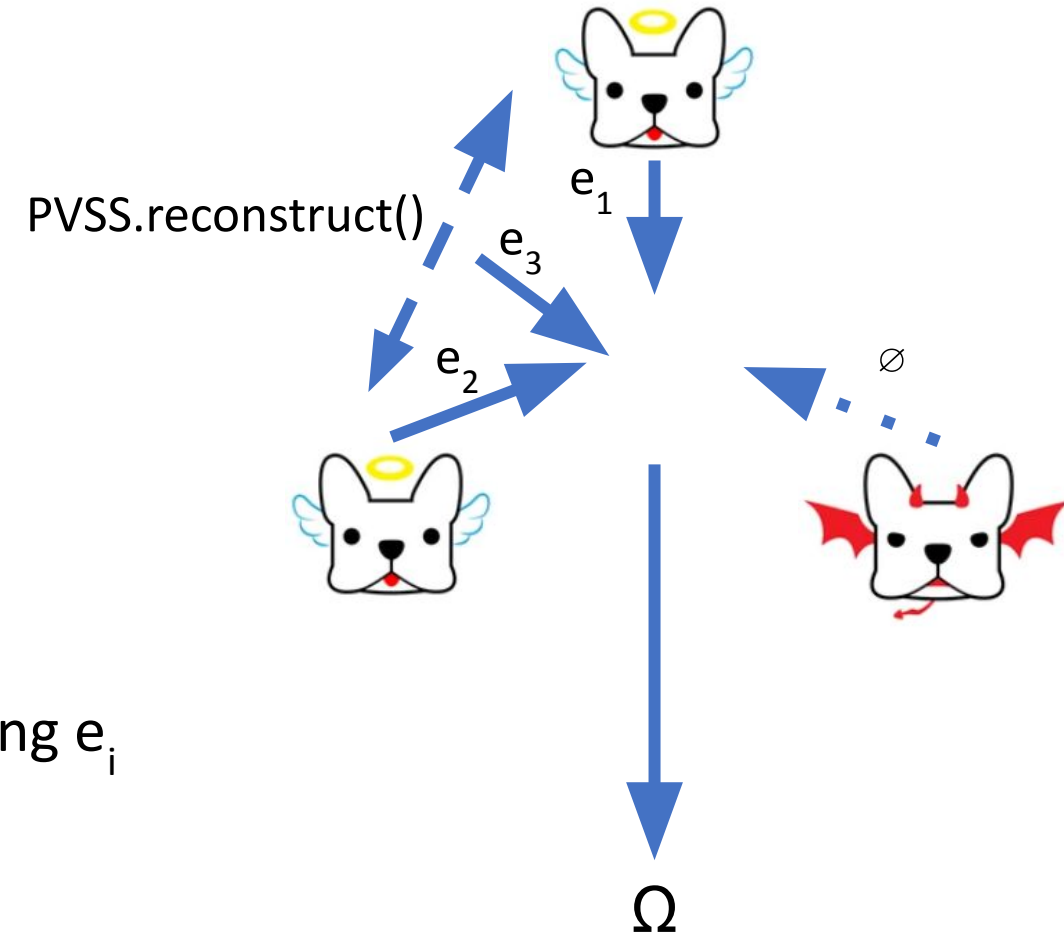
- Publish $c_i = \text{Commit}(e_i)$ as in classic CR
- Secret-share e_i with all other nodes
 - PVSS: Publicly verifiable secret sharing

2. Reveal

- Publish e_i as in classic CR

3. Recover

- Honest participants reconstruct any missing e_i



Commit-reveal-recover variants

- Better PVSS
 - SCRAPE [CD 17]
- Amortized PVSS
 - HERB [CSO 19]
 - Albatross [CD 20]
- Remove optimistic case (share-reconstruct-aggregate)
 - RandShare [SJKGGKFF 17]
 - SecRand [GSX 20]

Commit-reveal-recover

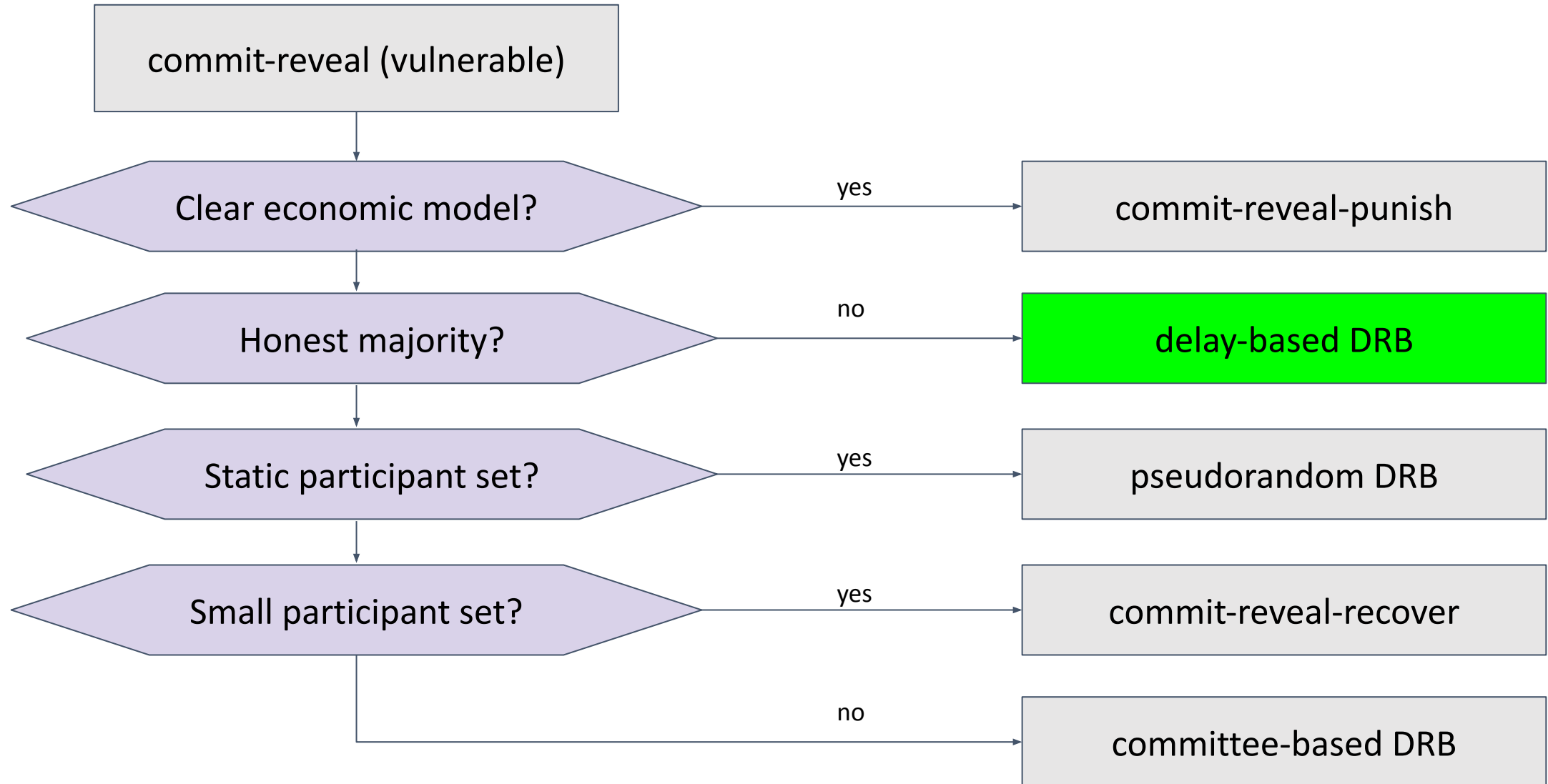
- **Advantages**

- Flexible participation
- Per-round entropy

- **Challenges**

- Relatively inefficient
 - $O(n^2)$ communication
- Complex protocols
- If t needed to compute, $n-t$ can block liveness
- Reconstruction causes extra overhead

DRB design flow chart



Delay functions are *slow* (sequential) but *tractable*

Fast

Encryption
Decryption
Signing
Verification
Hashing

Delay functions:

take a *specified* number of sequential steps

VDFs
Timed commitments
Time-lock encryption
Delay encryption
...

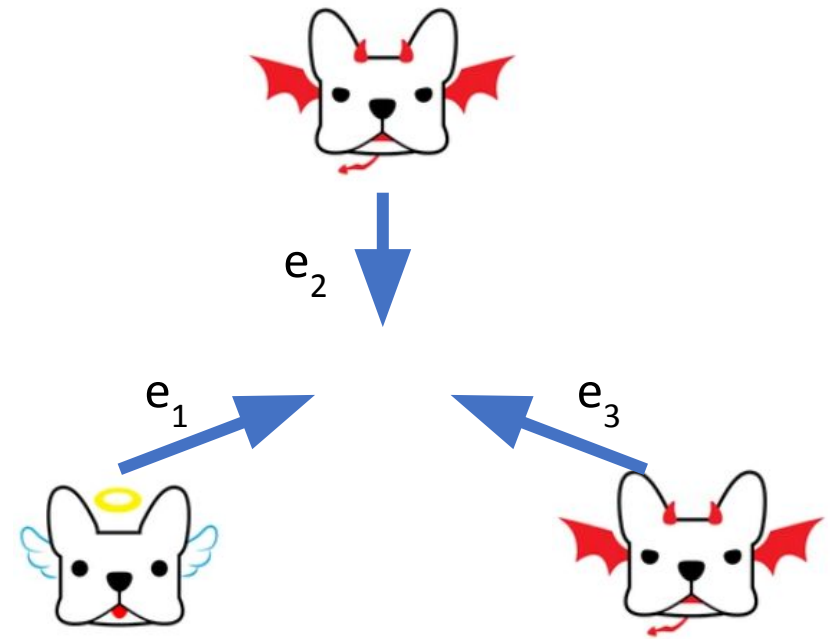
Intractable

Key search
Discrete log
Factoring
Collision-
finding

Reveal-delay (Unicorn) [LW15]

1. Reveal

- a. Raw entropy, no commitments needed!



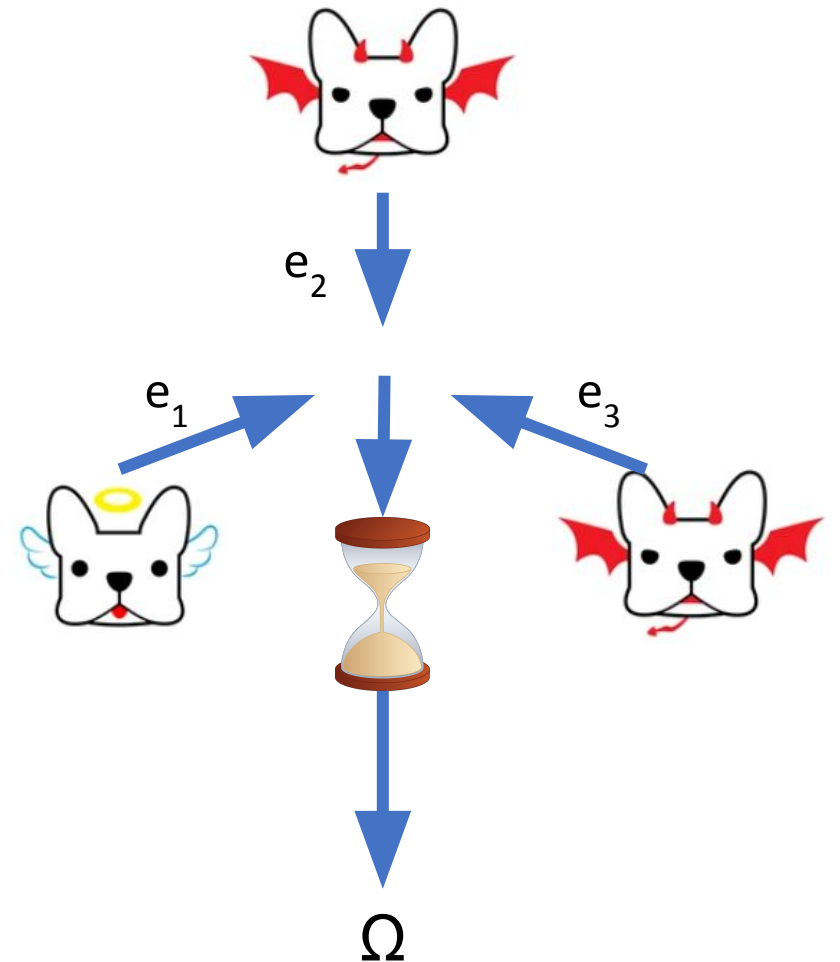
Reveal-delay (Unicorn) [LW15]

1. Reveal

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2. Delay + combine

- b. Modern approach: use a VDF
 - i. Slow (sequential) to compute
 - ii. Efficiently verifiable



Reveal-delay (Unicorn) [LW15]

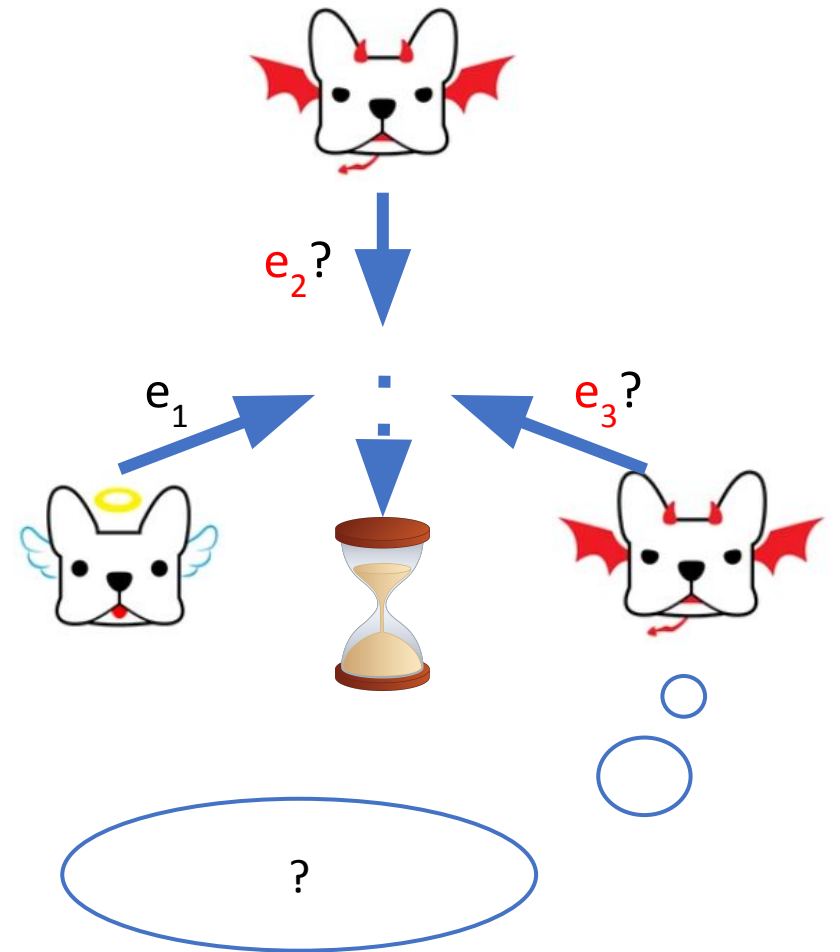
1. Reveal

- a. Raw entropy, no commitments needed!

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- b. Modern approach: use a VDF
 - i. Slow (sequential) to compute
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Last revealer(s) can't compute VDF fast enough to bias



Delay-based DRB variants

- **Frequent output:** RandRunner [SJSHW20]
 - Deliver output more often than delay parameter via pipelining
- **Efficient optimistic case:** Bicorn [CATB23]
 - Skip delay function if *all* participants are honest
- **Sublinear communication:** Cornucopia [CCB24]
 - Leader gathers contributions and broadcasts succinct commitment, proofs

Delay-based DRBs

- **Advantages**

- Secure under dishonest majority
- Efficient
 - $O(n)$ communication, compute. Can be reduced w/leader
- Flexible participation
- Per-round entropy
- Guaranteed output delivery

Delay-based DRBs in practice: Chia

- Used for consensus
 - Launched 2021
- VDF: Repeated squaring in class group
 - 1024-bit discriminant
 - Wesolowski proofs [W19]



Delay-based DRBs

- **Advantages**

- Secure under dishonest majority
- Efficient
 - $O(n)$ communication, compute. Can be reduced w/leader
- Flexible participation
- Per-round entropy
- Guaranteed output delivery

- **Challenges**


- Delay functions induce latency
- Some party must compute the delay function
 - a public good?
- Relatively new cryptographic assumptions
- Intra-predictability: attacker with faster VDF may learn outcome early

VDF designs use relatively new assumptions



Paper 2024/873

Cryptanalysis of Algebraic Verifiable Delay Functions

Alex Biryukov , University of Luxembourg, Esch-sur-Alzette, Luxembourg

Ben Fisch , Yale University, New Haven, USA

Gottfried Herold , Ethereum Foundation, Bonn, Germany

Dmitry Khovratovich, Ethereum Foundation, Luxembourg, Luxembourg

Gaëtan Leurent , INRIA, Paris, France

María Naya-Plasencia , INRIA, Paris, France

Benjamin Wesolowski, CNRS, ENS Lyon, Lyon, France

Abstract

Verifiable Delay Functions (VDF) are a class of cryptographic primitives aiming to guarantee a minimum computation time, even for an adversary with massive parallel computational power. They are useful in blockchain protocols, and several practical candidates have been proposed based on exponentiation in a large finite field: Sloth++, Veedo, MinRoot. The underlying assumption of these constructions is that computing an exponentiation x^e requires at least $\log_2 e$ sequential multiplications.

Metadata

Available format(s)



Category

Attacks and cryptanalysis

Publication info

A minor revision of an IACR publication in CRYPTO 2024

Keywords

Verifiable Delay Functions

MinRoot

Veedo

Sloth++

cryptanalysis

smoothness

Contact author(s)

Theorem: dishonest majority DRBs require delay!

- **Classic result:** Dishonest majority DRBs impossible in “plain model”
 - *Limits on the security of coin flips when half the processors are faulty.* Richard Cleve. TOC 1986.
- **Practical observation:** Dishonest majority DRBs possible with VDFs
- **New result:** Dishonest majority DRB *require* delay functions!
 - *Good Things Come to Those Who Wait: Dishonest-Majority Coin-Flipping Requires Delay Functions.* Joseph Bonneau, Benedikt Bünz, Miranda Christ, Yuval Efron. Eurocrypt 2025.
 - Simple delay function, not full VDF
 - Not parameterizable, not efficiently verifiable
 - Assumes network synchrony

Open questions (protocol design)

- *Secret Leader Election*
 - DRBs where only winner finds out they have won!
 - Applications in consensus and other protocols
- *Silent setup*
 - Use existing public keys for threshold DRB with no (or limited) setup phase
- *Optimistic protocols*
 - Faster execution if a chosen leader is honest
 - Faster execution if *all* nodes are honest

Open questions (engineering)

- Simpler API for developers
 - Smart contract integration: Aptos Roll, Mysten sui::random
- VDF deployment
 - Security model requires public access to VDF hardware
- Local randomness generation
 - DRB is no better than nodes' RNGs!
- Public trust



Thank you!

For more, please see 3 derailed surveys:

SoK: Decentralized randomness beacon protocols. Raikwar, Mayank, and Danilo Gligoroski. ACISP 2022. <https://arxiv.org/pdf/2205.13333>

SoK: Distributed Randomness Beacons. Kevin Choi, Athira Manoj and Joseph Bonneau. IEEE S&P 2023. <https://eprint.iacr.org/2023/728>

SoK: Public Randomness. Kavousi, Alireza, Zhipeng Wang, and Philipp Jovanovic. EuroS&P 2024. <https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=10629002>

Backup slides

Is a dishonest majority model worthwhile?

- Consensus requires an honest majority...
 - **Counterpoint:** Attacks on DRBs are *invisible*
- Dishonest majority enables *open participation*
 - No security downside to adding more participants!