Radboud University

Probing Secure Composability Without Fresh Randomness: Theory and Application to Ascon

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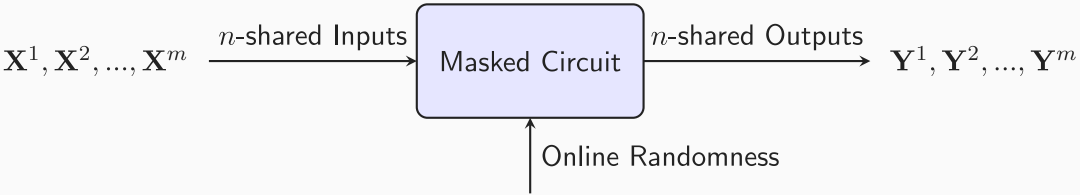

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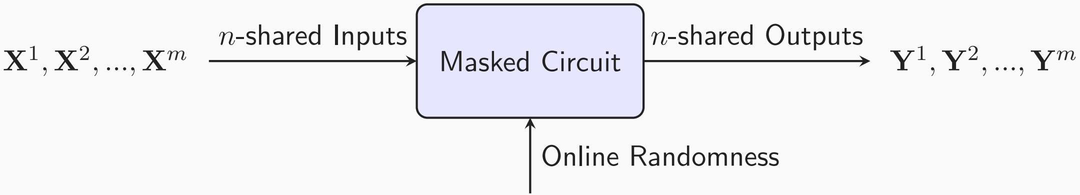

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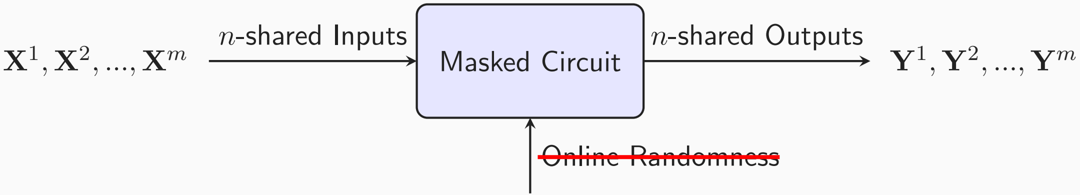
- Resource-constrained devices (IoT, embedded) struggle with randomness generation.
- **Question:** Can we achieve probing security without online randomness?



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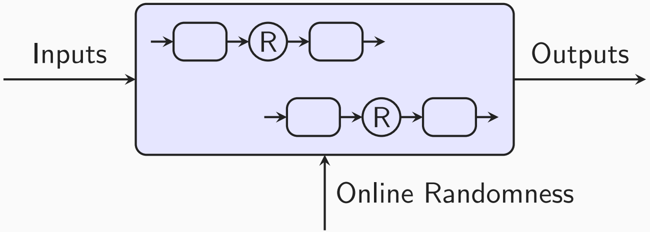

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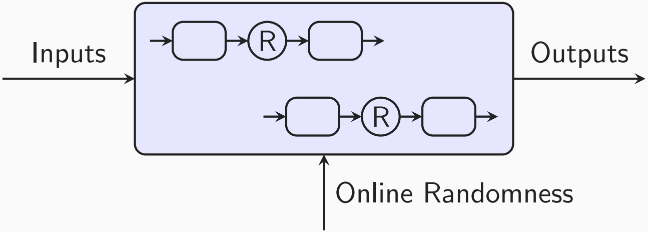

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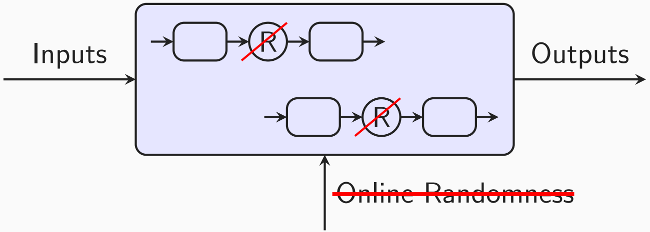

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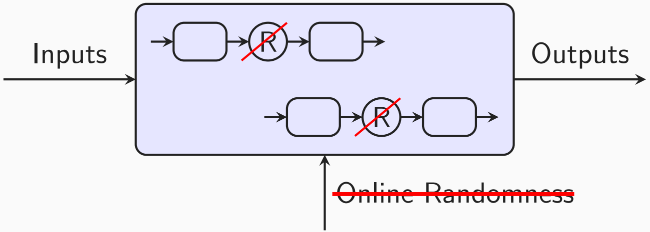
- Our goal: Prove & implement composability without fresh randomness.



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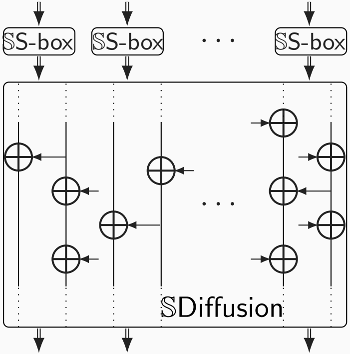

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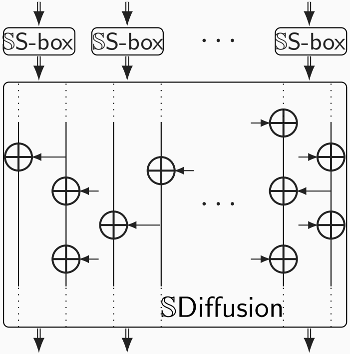

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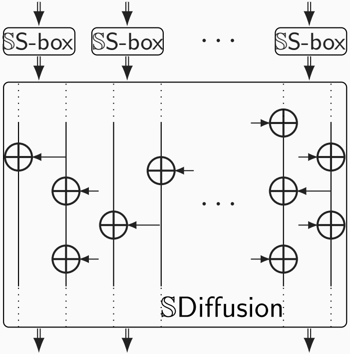

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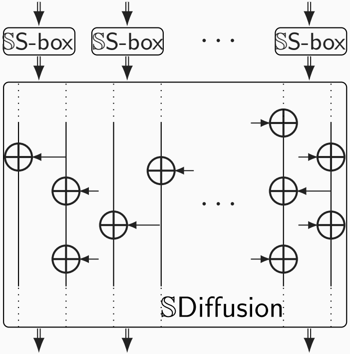
- Security order derives from gadget security $d_{\mathbb{S}\text{S-box}}$ and $\mathcal{B}_{l\text{-box}}$.



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- Practice: Implementation on Ascon (NIST LWC standard).

Composition Theorem (Theorem 1)

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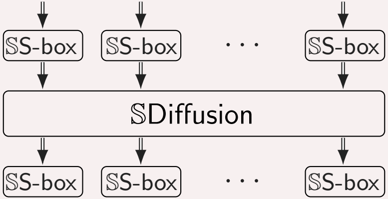

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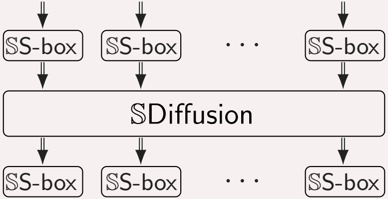

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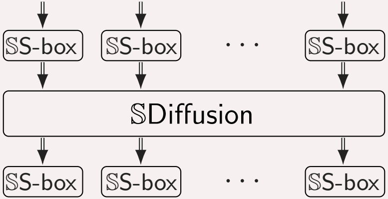

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$$\min\{d_{\mathbb{S}\text{S-box}},\ (\mathcal{B}_{l\text{-box}} - 1)\} \leq d_{\text{comb}} \leq d_{\mathbb{S}\text{S-box}}.$$

Box Linear Branch Number

- Consider a simple linear mapping on $\mathbf{S} \in \{0,1\}^{15}$:

$$\mathbf{S}' \leftarrow \mathbf{S} \oplus (\mathbf{S} \lll 1) \oplus (\mathbf{S} \lll 3),$$

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Definition: Box Linear Branch Number

Let $\mathbf{M}$ be a $b \times b$ binary matrix representing a linear diffusion layer, and let S denote its b-bit input. The box linear branch number $\mathcal{B}_{l\text{-box}}$ is defined as:

$$\mathcal{B}_{l\text{-box}} = \min_{S \neq 0^b} \left[\#\text{active S-boxes}(S) + \#\text{active S-boxes}(S\mathbf{M}^\top) \right],$$

where an S-box is considered active if any of its input bits is non-zero.

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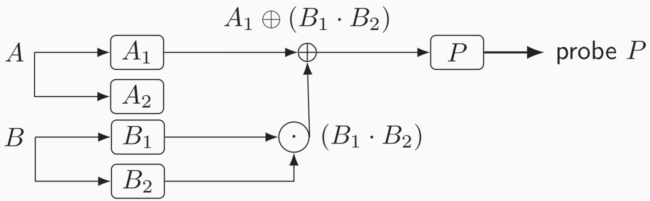

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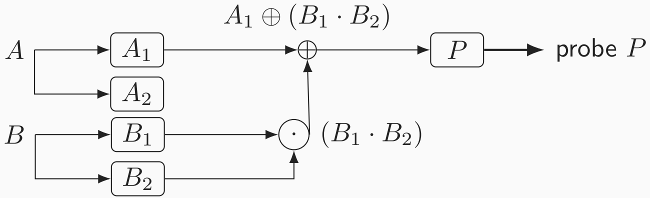

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- However, direct verification shows that P is in fact independent of the secrets A and B.
- $\Rightarrow$ Simulation rules are efficient, but not always *general*.

Key Requirement

The input shares must be mutually independent.

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Takeaway

TI composes via *refresh gadgets* (costing online randomness), whereas our approach composes via the cipher's *diffusion layer* (no online randomness).

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- We implement first- and second-order deterministic masked Ascon.
- Recent work extends this approach to *third-order* masked Ascon [JSB25].

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- This highlights how input shares inherently contribute to probing security.

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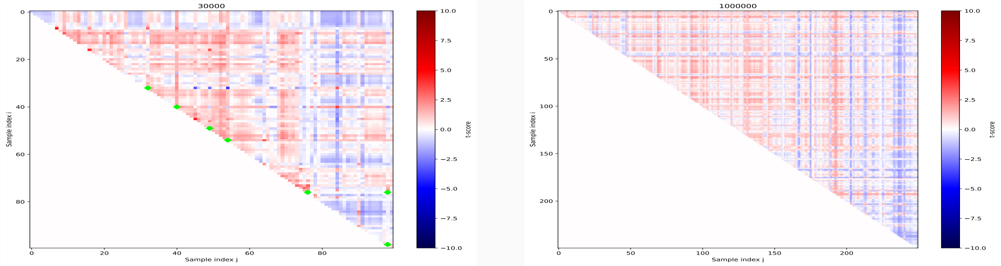

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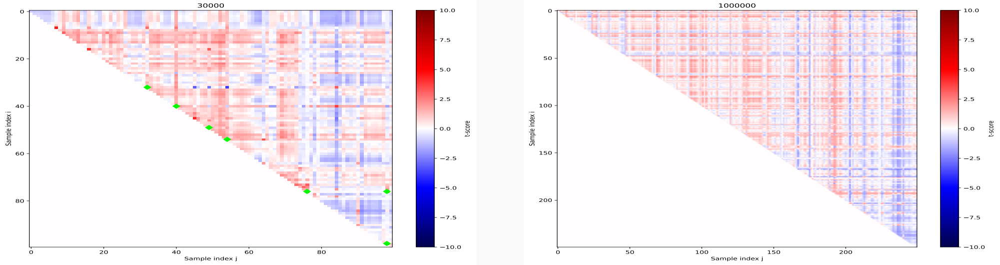

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Thank you for your attention!

Code: `https://github.com/vahid-jahandideh/deterministic-masking-ascon`