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A New Side-Channel Attack on RSA Prime Generation

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Outline

- Introduction and Motivation
- The Attack
  - Basic attack
  - Refinements
- Efficiency (empirical results)
- Experimental results
- Countermeasures
- Final remarks

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Facts (I)

- Side-channel attacks on RSA implementations have a long tradition.
- Nearly all of these attacks aim at the exponentiation with the private key. Only a few papers consider the key generation process (e.g., Clavier & Coron, 2006).

Facts (II)

- If a smart card generates an RSA key outside the personalisation environment the key generation process may be vulnerable by side-channel attacks.

Side-channel attacks on RSA key generation

- Compared to side-channel attacks within the exponentiation phase the prospects for the attacker seem to be worse since
  - the key is generated only once
  - the generation process does not use any (known or chosen) external input
- The type of the weaknesses and their exploitation may be different from side-channel attacks within the exponentiation phase.

Motivation

- We present a side-channel attack on the RSA key generation process on a straight-forward implementation (proposed e.g. by Brandt et al. (1991), cf. also RSAREF toolkit)
- The goal of our paper is two-fold, namely
  - to demonstrate the fundamental vulnerability of the RSA key generation process against side-channel attacks.
  - to encourage the community to study the key generation process with regard to side-channel attacks

Definition

- $T = \{r_2 := 3, 5, 7, \ldots, r_N\}$
  /* trial base, consists of the first odd N-1 primes */

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Prime generation algorithm (I)

1. Generate an odd (pseudo-) random number $v \in \{2^{k-1}+1, \ldots, 2^k\}$

2.

   a) $i := 2;$

   b) while ($i \leq N$) do { /*trial divisions*/

```
        if (r_i divides v) then {
                v := v+2;
                GOTO Step 2a; }
        i++;
}
```

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Prime generation algorithm (II)

c) m := 1;

d) while (m ≤ t) do {

/* t = max # of primality tests */

apply the Miller-Rabin primality test to v;

if the primality test fails then {

v := v+2;

GOTO Step 2a; }

m++;

}

3. p:= v (resp., q := v)

Assumptions

- Power analysis allows the attacker
  - to identify for each prime candidate v after which trial division the while-loop has terminated
  - whether a Miller-Rabin test has been applied.

NOTE: If all trial divisions need approximately the same run-time it suffices to identify the beginning of the while-loop 2b) or the incrementation step v := v+2.

Remark

- We further assume that
  - the RNG is strong
  - the trial division itself and the Miller-Rabin tests are perfectly protected against side-channel attacks

- Otherwise, even stronger attacks may exist.

Basic attack (I)

- Notation: $v_0 = v$, $v_1 = v_0 + 2, \ldots, v_m = v_0 + 2m := p$
- Basic observation:
  - For v_j loop 2b) terminates after trial division by r
  - $\Rightarrow v_j \equiv 0 \pmod r$
  - $\Rightarrow p = v_m = v_j + 2(m-j) \equiv 2(m-j) \pmod r$

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Basic attack (II)

- Generation of p:
  $S_p := \{2\} \cup \{ r \in T \mid$ for at least v_j the algorithm terminated after the division by r $\}$

- The CRT gives

$$a_p \equiv p \pmod{s_p} \qquad \text{with } s_p := \prod_{r \in S_p} r \,,$$

and finally

$$a_q \equiv q \equiv a_p^{-1} n \pmod{s_p}$$

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Basic attack (III)

- Analogously (observing the generation of q)

$$b_q \equiv q \pmod{s_q} \qquad \text{for } s_q := \prod_{r \in S_q} r$$

and

$$b_p \equiv p \equiv b_q^{-1}\, n \pmod{s_q}$$

- Finally, from (a_p, b_p) and (a_q, b_q) the attacker computes

$$c_p \equiv p \pmod{s},\ c_q \equiv q \pmod{s} \quad \text{with } s := \mathrm{lcm}(s_p, s_q)$$

Basic attack (IV)

- $p = sx_p + c_p$, $q = sy_q + c_q$ for unknown integers x_p, y_q

- The pair (x_p, y_q) is a zero of the irreducible bivariate integer polynomial $f: Z \times Z \rightarrow Z$,
  $f(x,y) := sxy + c_p y + c_q x - t$ with $t := (n - c_p c_q) / s$

- If $\log_2(s) > k/2$ the LLL algorithm finds the pair (x_p, y_p) in time polynomial in k (k = bit length of p and q).

Empirical results

- Simulations of the attack with Magma (≅ perfect measurements)
- k = 512; LLL requires at least $\log_2(s) > 256$
- Trial bases: $T_1 = \{3,5,\ldots,251\}$, /* odd primes < 2^8 */
  $T_2 = \{3,5,\ldots,281\}$, $T_3 = \{3,5,\ldots,349\}$

Success Probabilities (basic attack)

	T_1	T_2	T_3
$\mathrm{Prob}(\log_2(s) > 256)$	0.118	0.188	0.283
$\mathrm{Prob}(\log_2(s) > 277)$	0.055	0.120	0.208

Remark

- If $\log_2(s) < k/2$ the LLL-algorithm will not find the zero (x_p, y_p).
- One may guess the remainder p (mod r_i') for some further primes $r_1', .., r_m'$ so that $s' := s \cdot r_1' \cdot \cdot \cdot r_m'$ is sufficiently large.
- <u>Drawback:</u> In the worst case the LLL algorithm has to be applied to all $r_1' \cdot \cdot \cdot r_m'$ admissible candidates (c_p', c_q') for (p (mod s'), q (mod s'))

Refinements of the attack

By exploiting further side-channel information from

- the trial divisions
- the extended Euclidean algorithm (computation of d (mod (p-1)) and d (mod (q-1))

many candidates for c_p' can be excluded.

Remark: For k=512 this provides about 10-15 bits additional information.

Experimental results (I)

- Sample implementation on an ATMEL ATmega microcontroller

```
rnd2r( );               /* generates a random number */
testdiv512 (v,3);       /* trial division by 3 */
testdiv512 (v,5);
testdiv512 (v,7);
incrnd (v);             /* increments v by 2 */
testdiv512 (v,3);
incrnd (v);
testdiv512 (v,3);
testdiv512 (v,5);
```

Experimental results (II)

- Notation: $x_1, x_2, \ldots, x_N$: power consumption during the particular clock cycles
- Goal: Find a characteristic sample that identifies a trial division or an incrementation step

$$x_1, x_2, \ldots, x_{t-1}, \boxed{x_t, x_{t+1}, \ldots, x_{t+M-1}}, x_{t+M}, \ldots, x_N$$

$$y_1, y_2, \ldots, y_M \quad \text{characteristic sample}$$

Experimental results (III)

- The *similarity function*

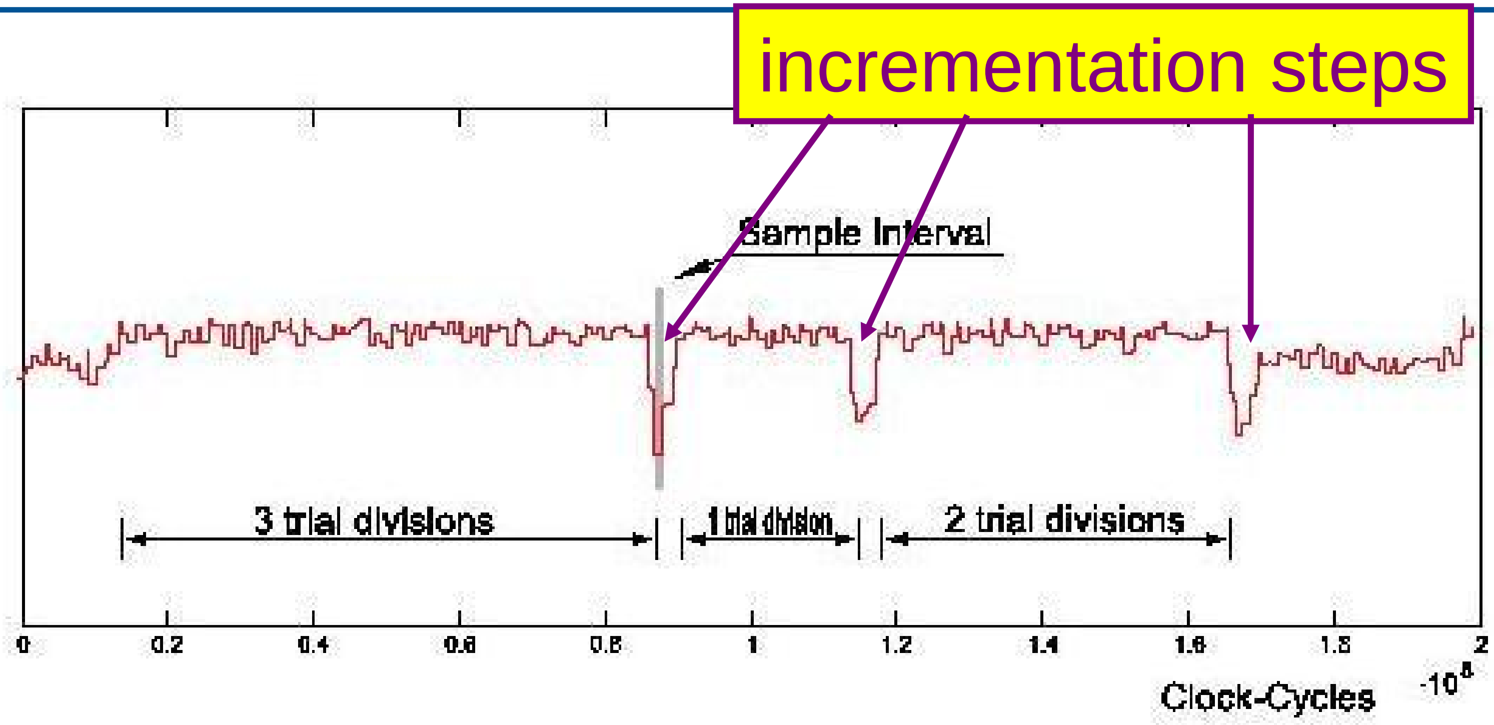
$$a_j = \frac{1}{M}\sum_{i=1}^{M} | x_{i+j} - y_i | \qquad \text{for } j \in \{1,\ldots, N\text{-}M\}$$

compares $(y_1, \ldots, y_M)$ with the power consumption subsequence $(x_{j+1}, \ldots, x_{j+M})$ for all shift parameters j.

To compensate random local effects we finally applied

$b_j := \min \{a_j,\ldots,a_{j+F-1}\}$ /*minimum over a 'window'*/

Experimental results (IV)



- low peaks: large similarity, high peaks: large dissimilarity
- sample sequence within the first incrementation step
  low peaks = positions of incrementation steps

Possible countermeasure

- regular refreshment of the prime candidates v_j by updating some bytes (e.g., XORing 8 bytes of every 10^{th} candidate v_j with random bytes)

Final remarks

- We have demonstrated the power of a side-channel attack on a straight-forward prime generation algorithm.
- Simulations yielded success probabilities of 10 – 15 %, and practical experiments verified that the above-mentioned assumptions are indeed realistic.
- Moreover, this paper shall motivate the community to devote more attention to the key generation step.

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