Side Channel Attacks against Block Ciphers Implementations and Countermeasures

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ANSSI

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Part I

Introduction

Outline

Plan

Smart Card

- A Smart Card is a circuit embedded on a plastic support. It moreover has communication means, storage capacities and computation capacities.
- The physical characteristics of a smart card are standardized.
- The smart card enables the secure storage of sensitive data: a part of its memory is indeed protected in both writing and reading modes.

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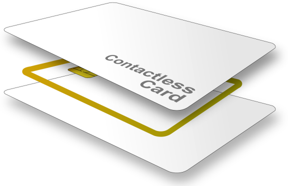

- A plastic Support
- Storage and computation means
  - Micro-controller (ST, Atmel, NXP, Samsung, Infineon, etc.)
- Communication means
  - Connectors
  - Antenna

- Main Goal: embed private data and manipulate them in a secure way.

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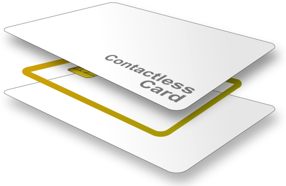

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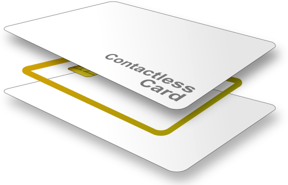

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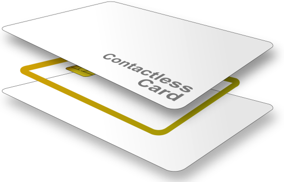

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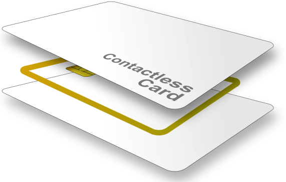

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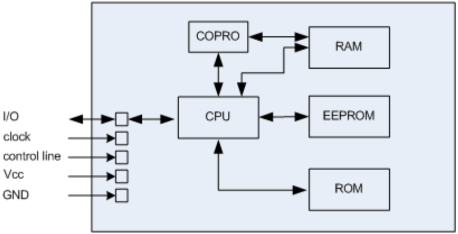

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- RAM is dedicated to the storage of local and volatile variables during the processings.
- EEPROM contains code and some data.
- Co-processor is dedicated to particular cryptographic (*e.g. arithmetic*) *calculations.*

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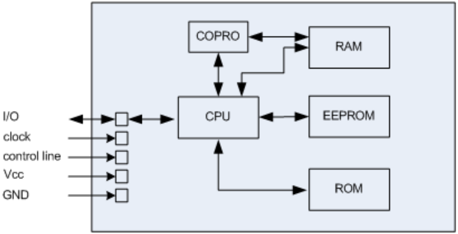

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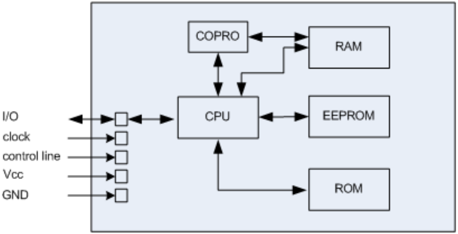

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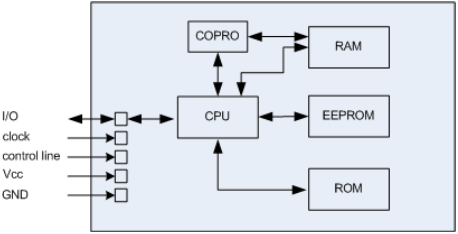

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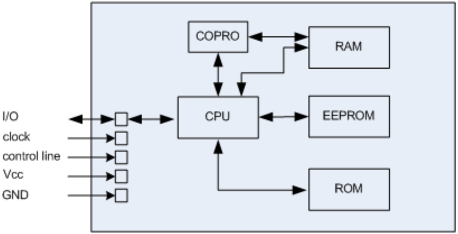

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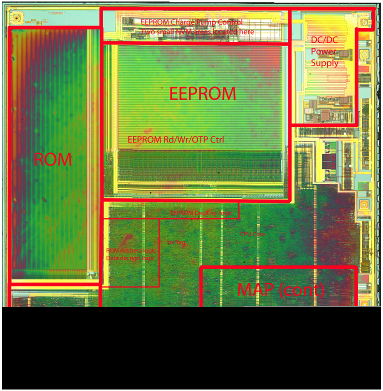

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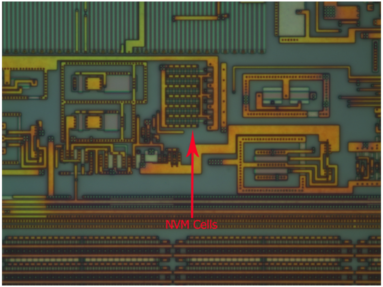

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Smart Card Evolution

- The first smart cards (Bull and Motorola) only had 36 bytes of RAM and 1600 bytes of ROM.
- Today, a smart card has;
  - between 16 and 512 Kbytes of ROM,
  - between 1 and 32 Kbytes of RAM,
  - a processor running at 100 MHz.
- ... it can;
  - embed several mega-bytes of Flash memory,
  - communicate in USB2.0,
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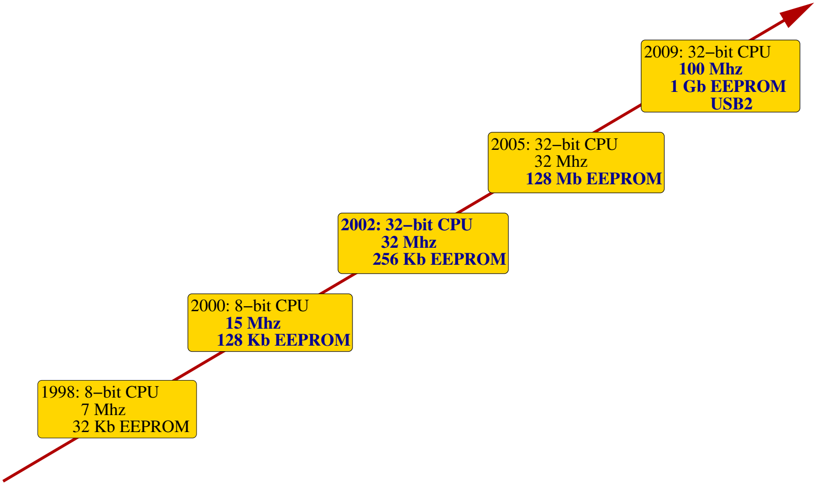

Plan

1. Smart Cards
2. Embedded Cryptogaphy

Which Algorithm for which Security Issue?

- Client (Bank, Operator, Government, etc.) has to deal with security issues: securing transactions, protecting citizen anonymity, limiting access to services, etc.
- In more than 95% of the cases it asks its internal security experts to find a solution.
  - A standard exists: it will certainly be chosen!
  - No satisfying standard exists: a new standardization process is initiated (*e.g. ETSI 3GPP, ISO, ICAO).*
  - *No satisfying standard exists: a solution is designed by the internal experts (proprietary algorithms).*
- Sometimes (in less than 5% of the cases) the Client asks Industrial experts to find a solution.
- As a consequence, the Smart Card Industry essentially implements standards (and even a very few of them!).

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Cryptography in Smart Cards

Smart Cards implement a wide range of cryptographic algorithms:

- Block Ciphers: (Triple-)DES, AES, proprietary algorithms
- Hash functions: SHA family
- Data authentication: CBC-MAC, HMAC
- Symmetric key cryptography: RSA (OAEP, PKCS1-v1.5)
- Signature : RSA (PKCS1-v1.5, PSS), DSA, ECDSA
- Key exchange protocols: Diffie-Hellman, Diffie-Hellman on elliptic curves

Part II

Passive Side Channel Attacks

Outline

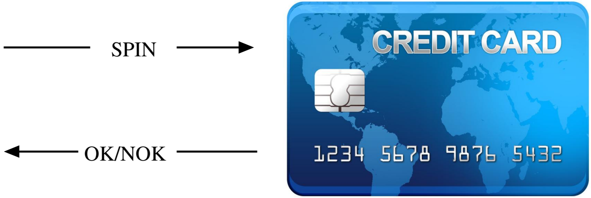
Plan

Exploitation : Simple Attacks (SPA)

SPA refers to attacks where the adversary focus on a single execution of an implementation (with possibility to average the observation for fixed inputs).
In some cases, this gives the adversary information about the manipulated secrets.

- The information leakage must be important.
- The secret must have a simple relationship with the leakage.

Example of SPA; PIN verification



Algo PIN comparison

INPUT(S) : SPIN, PIN
OUTPUT(S) : `ok/nok`

1: **for** $i = 0$ to 3 **do**
2: **if** SPIN[i] $\neq$ PIN[i] **then**
3: **return** `nok`
4: **return** `ok`

The observation of execution timing enables to retrieve PIN with 4×10 tries instead of $10^4 = 10\,000$.

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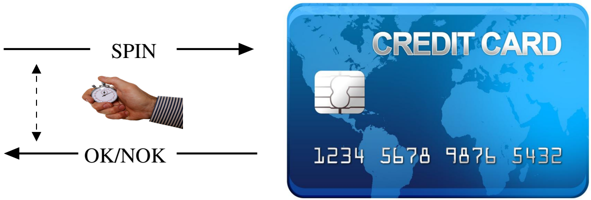

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- Advanced Side Channel Attacks can extract information from observations in contexts where SPA fails.
- They involve statistical tools (simple – difference of means tests – or sophisticated – mutual information processing –).
- They need several (between 10 and more than 10^6) traces such that:
  - the secret is constant,
  - the inputs are different and [optional] known.
  - [optional] some knowledge about the device architecture, the implementation or the noise characteristics.
- They follow a divide-and-conquer approach: the secret is rebuild piece by piece, where each piece is deduced from the behavior of an intermediate result. The size of the piece usually depends on the architecture size (*e.g.* 8, 16 or 32 bits).

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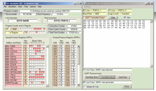

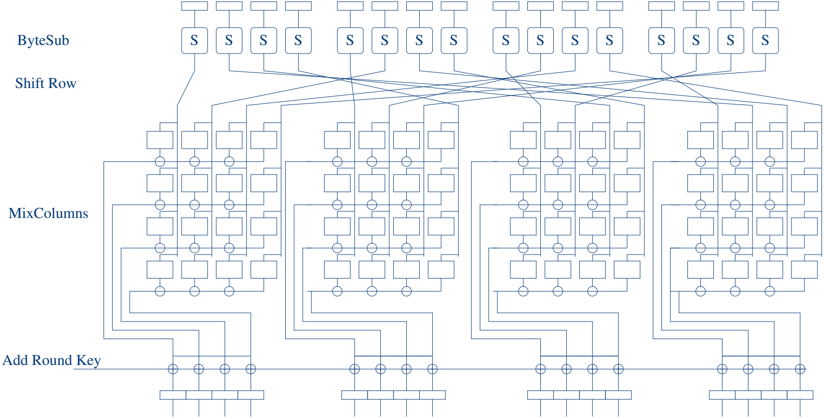

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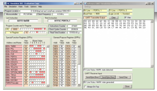

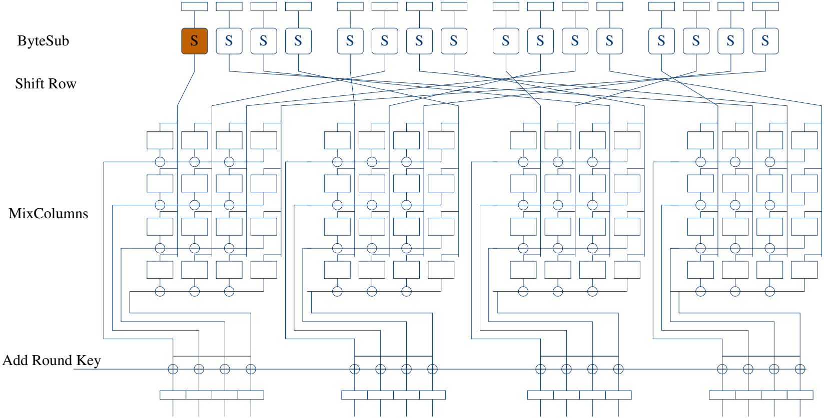

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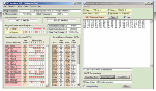

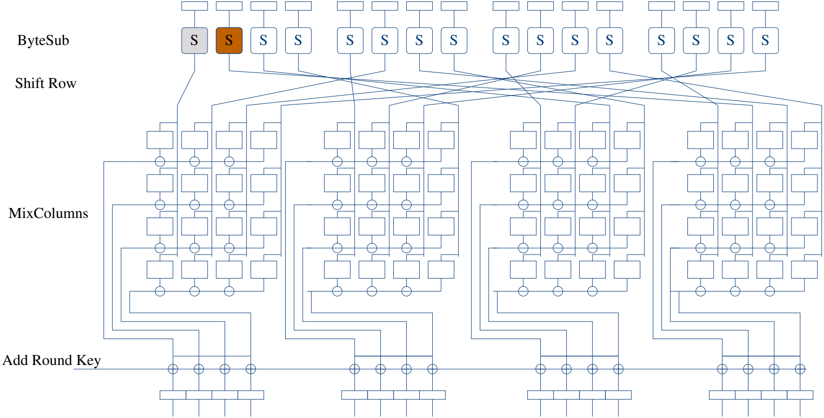

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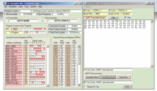

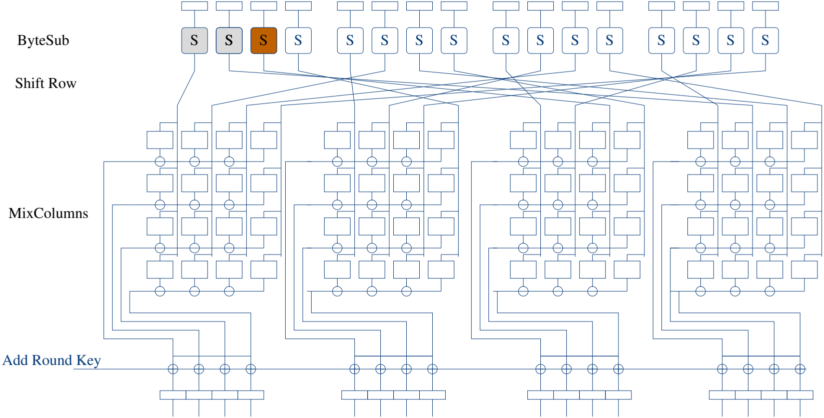

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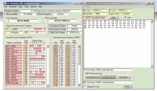

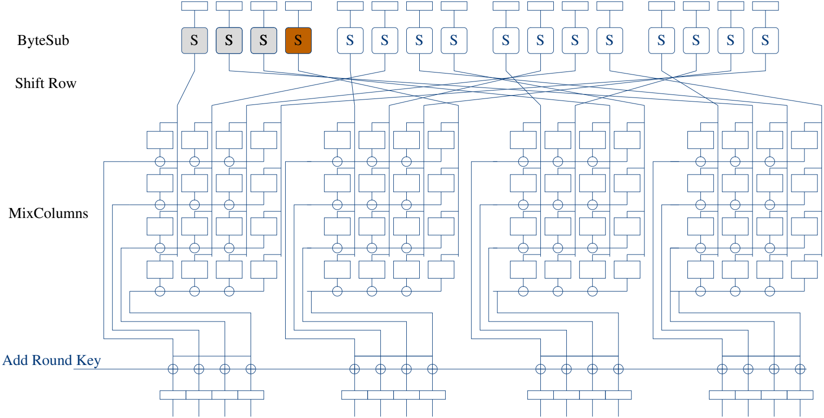

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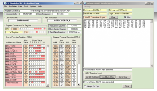

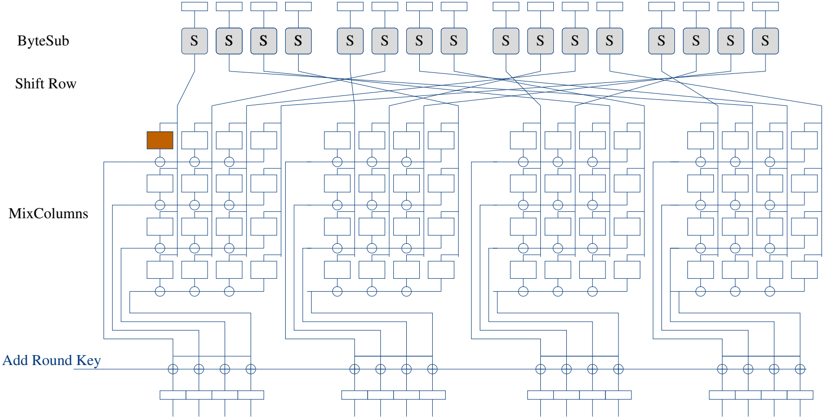

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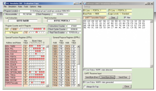

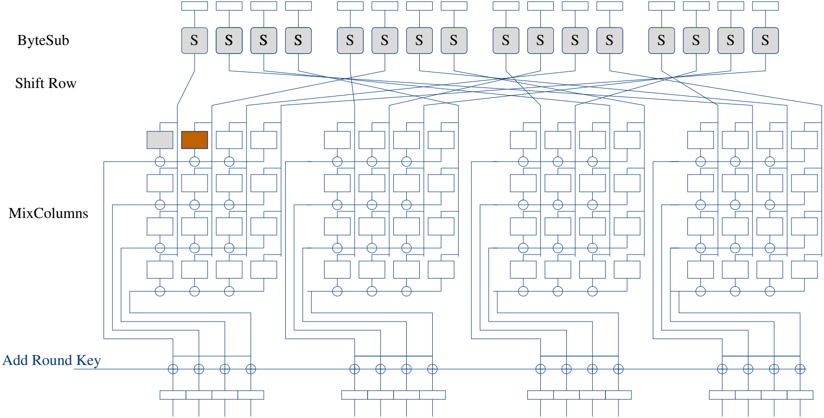

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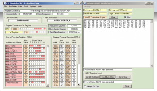

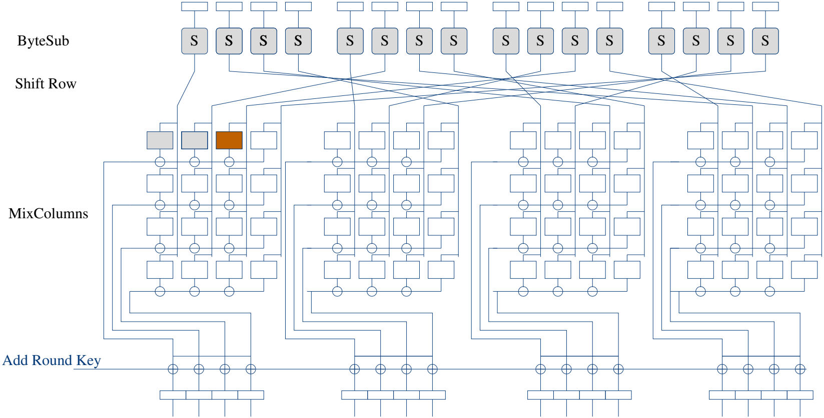

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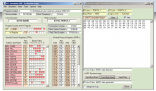
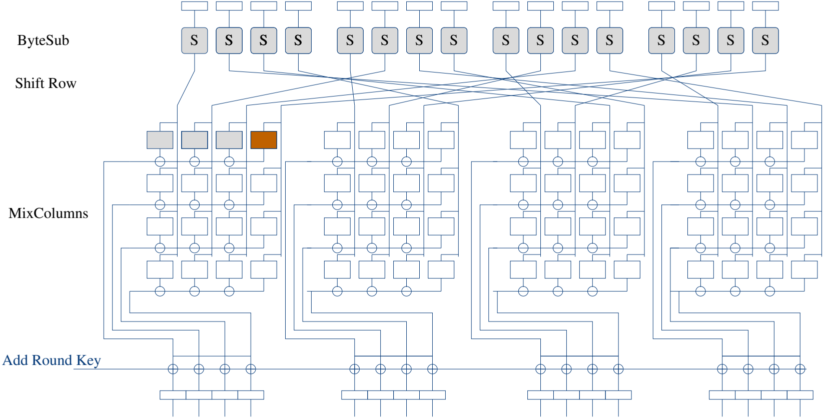
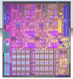
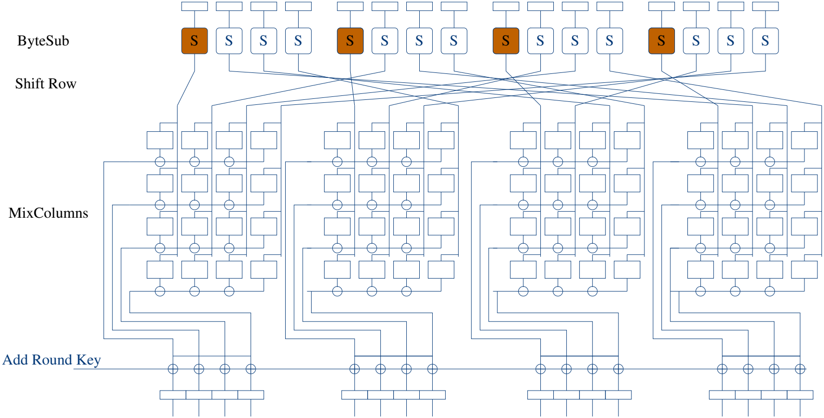




Advanced Side Channel Attacks (DPA like attacks)

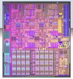
AES Round - Parallel Hardware Implementation

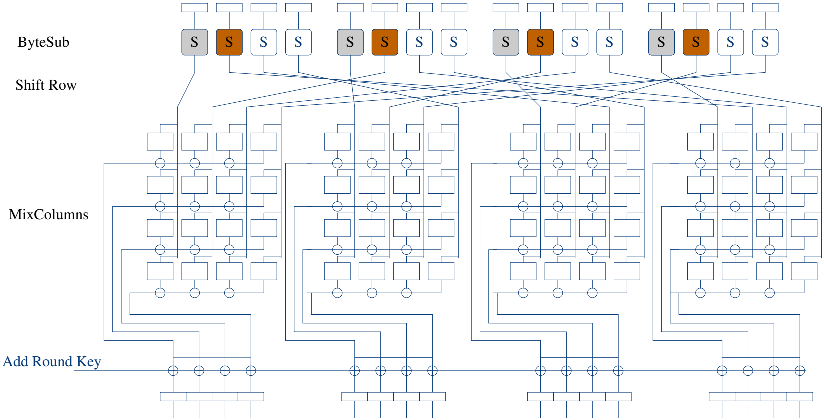




Advanced Side Channel Attacks (DPA like attacks)

AES Round - Parallel Hardware Implementation





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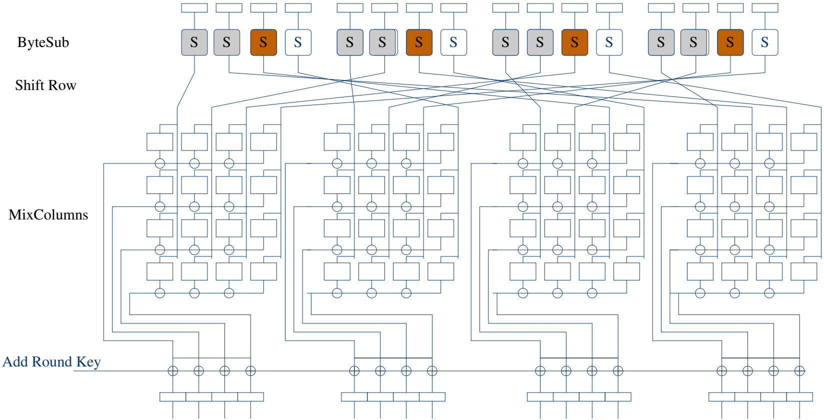

Advanced Side Channel Attacks (DPA like attacks)

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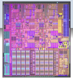
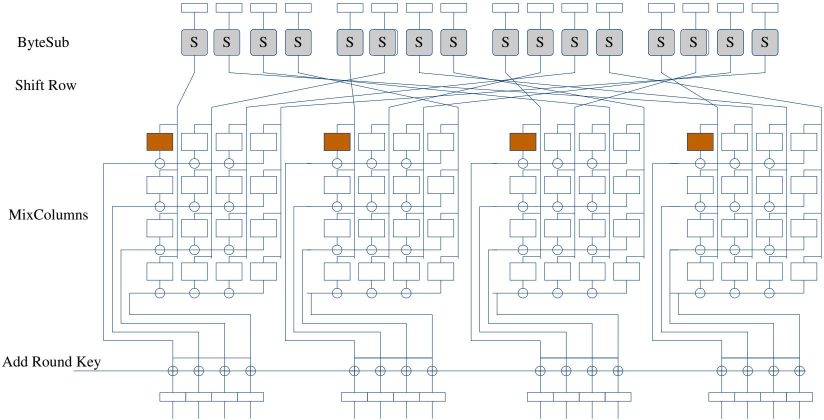
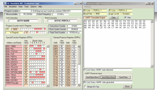
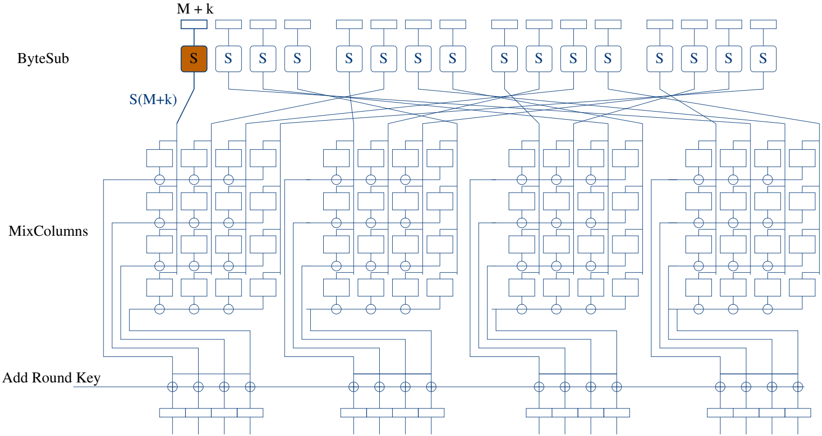




Advanced Side Channel Attacks (DPA like attacks)

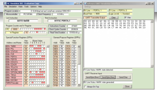
AES Round - Software Implementation – SCA attack

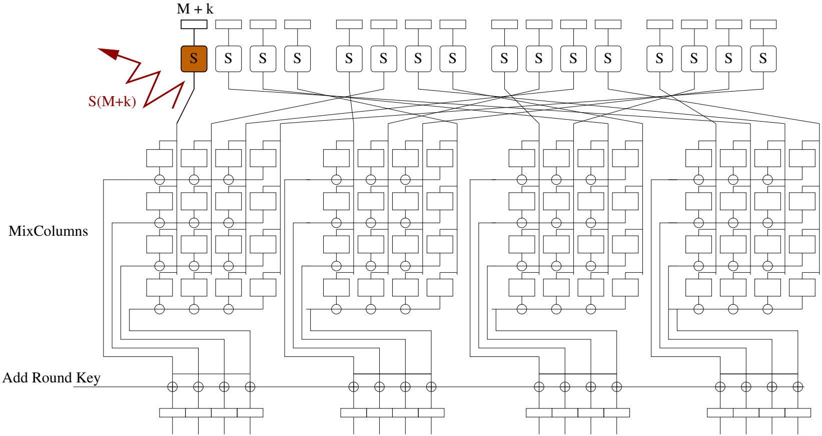




Advanced Side Channel Attacks (DPA like attacks)

AES Round - Software Implementation – SCA attack





Advanced Side Channel Attacks (DPA like attacks)

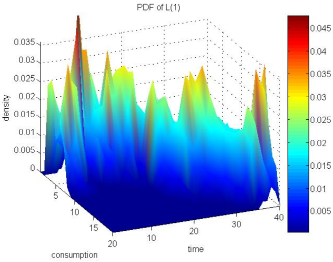
Main Observation

Example: pdf of the leakage for a device processing...

... $\texttt{AES-Sbox}(X + \mathbf{K})$ with $\mathbf{K} = 1$ and $X = \text{cst}$.

For each time (abs.) and each value ℓ in a finite interval (ord.) we plotted in z-axis:

$$Pr[\text{leakage} = \ell] \sim pdf_{\text{leakage}}(\ell)$$



Advanced Side Channel Attacks (DPA like attacks)

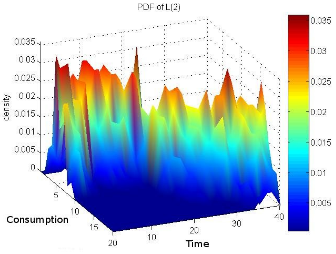
Main Observation

Example: pdf of the leakage for a device processing...

... $\texttt{AES-Sbox}(X + \mathbf{K})$ with $\mathbf{K} = 2$ and $X = \text{cst}$.

For each time (abs.) and each value ℓ in a finite interval (ord.) we plotted in z-axis:

$$Pr[\text{leakage} = \ell] \sim pdf_{\text{leakage}}(\ell)$$



Advanced Side Channel Attacks (DPA like attacks)

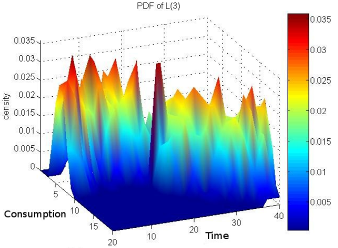
Main Observation

Example: pdf of the leakage for a device processing...

... $\texttt{AES-Sbox}(X + \mathbf{K})$ with $\mathbf{K} = 3$ and $X = \text{cst}$.

For each time (abs.) and each value ℓ in a finite interval (ord.) we plotted in z-axis:

$$Pr[\text{leakage} = \ell] \sim pdf_{\text{leakage}}(\ell)$$



Advanced Side Channel Attacks (DPA like attacks)

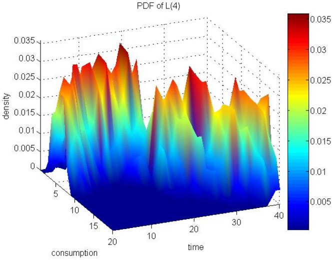
Main Observation

Example: pdf of the leakage for a device processing...

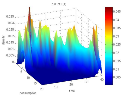
... $\texttt{AES-Sbox}(X + \mathbf{K})$ with $\mathbf{K} = 4$ and $X = \text{cst}$.

For each time (abs.) and each value ℓ in a finite interval (ord.) we plotted in z-axis:

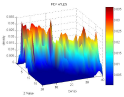
$$Pr[\text{leakage} = \ell] \sim pdf_{\text{leakage}}(\ell)$$



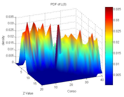
- [Pre-computation] For every possible key $k^\star$ pre-compute the pdf of the leakage $\mathbf{L}$.



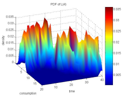
$k^\star = 1$



$k^\star = 2$



$k^\star = 3$



$k^\star = 4$

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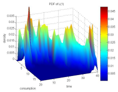
  - [Necessary Condition] Have an open access to a copy of the target device and be able to choose the key value.

- [Measurement] Measure the consumption for the target device and estimate the pdf of $\mathbf{L}$ for this target.



$k^\star = ?$

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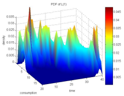

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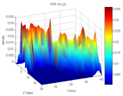

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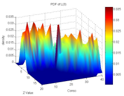

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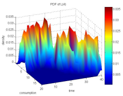

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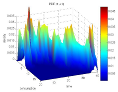

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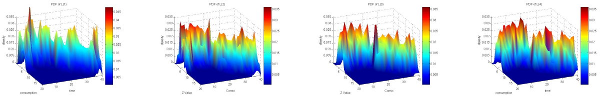

$k^\star = 1$ $k^\star = 2$ $k^\star = 3$ $k^\star = 4$ ....

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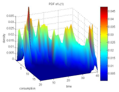

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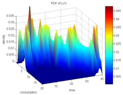

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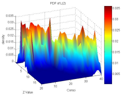

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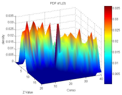

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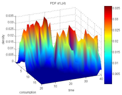

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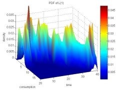

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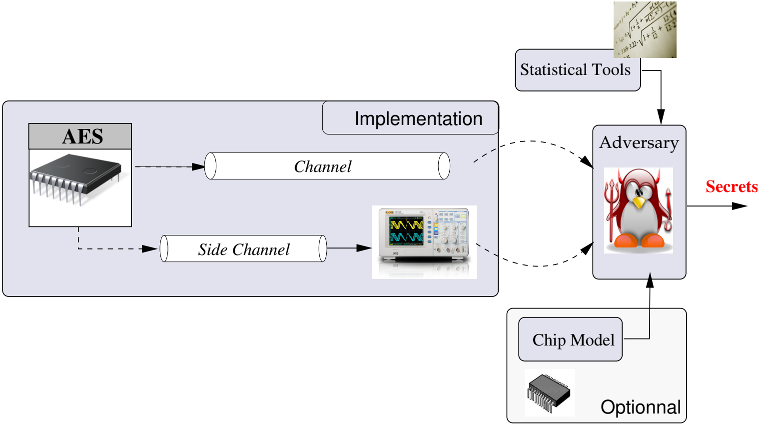
Advanced Side Channel Attacks (DPA like attacks)

Side Channel Analysis: General Framework.



Advanced Side Channel Attacks (DPA like attacks)

Side Channel Analysis: General Framework (Theoretical)

Context: attack during the manipulation of $S(X + k)$.

1. Measurement :
   - get a leakages sample $(\ell_{k,i})_i$ related to a sample $(x_i)_i$ of plaintexts.
2. Model Selection :
   - Design/Select a function $\mathbf{m}(\cdot)$.
3. Prediction :
   - For every $\hat{k}$, compute $m_{\hat{k},i} = \mathbf{m}(S(x_i + \hat{k}))$.
4. Distinguisher Selection :
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Advanced Side Channel Attacks (DPA like attacks)

Side Channel Analysis: attack Description Sheet/Form

Attack Description Sheet/Form

Type of Leakage: *e.g. power consumption or electromagnetic emanation*
Model Function: *e.g. one bit of Z or its Hamming weight*
Statistical Distinguisher: *e.g. difference of means, correlation or entropy*
Key Candidate Selection: *e.g. the candidate the maximizes the scores*

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Advanced Side Channel Attacks (DPA like attacks)

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Side Channel Analysis: define a model for the consumption.

Goal: define the kind of dependency between the manipulated data and the device behaviour.

- First solution (template/profiled attacks principle):
  - use an exact copy of the attacked device and estimate the pdf of L for every possible pair (X, k).
  - see [Chari et al at CHES 2002].
- Second solution (unprofiled attacks principle):
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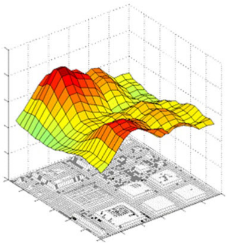

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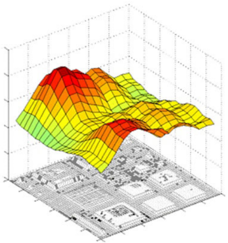

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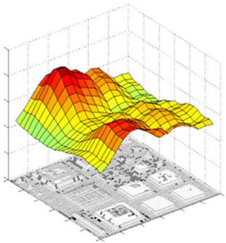

Advanced Side Channel Attacks (DPA like attacks)

Side Channel Analysis: modelling for unprofiled attacks.

Independent Noise Assumption (INA)

The random variable L related to the manipulation of Z equals $Y + B$, where Y is a function of Z and B is independent of Z.

- B is usually called the noise and is viewed as a continuous random variable.
- We usually assume $B \sim \mathcal{N}(0, \sigma^2)$. (Gaussian Noise Assumption).
- Usually, we have $Z = S(X + K)$ where
  - X is known,
  - k is the secret to recover
  - $S(\cdot)$ is a known cryptographic primitive (*e.g. an s-box*).

New problem statement

Modelling = recover the function φ s.t. $Y = \varphi(Z)$.

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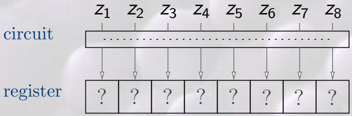

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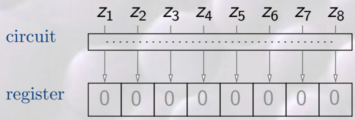

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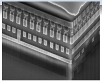

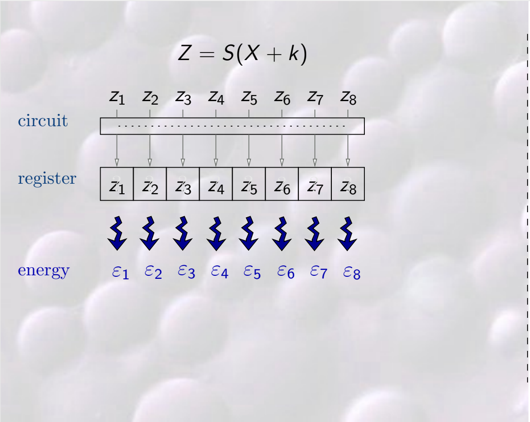

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the ε_i's are indep.

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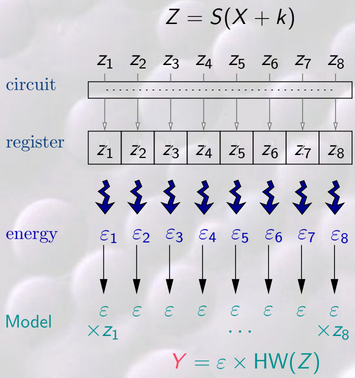

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Hamming Weight Model

Advanced Side Channel Attacks (DPA like attacks)

Side Channel Analysis: modelling for unprofiled attacks.

$$L \leftarrow Y + B = \varphi(Z) + B$$

To sum up

- The deterministic part Y in a leakage L may be viewed as a multivariate polynomial in the bit-coordinate z_i of Z with coefficients in $\mathbb{R}$.
  - $\varphi(Z)$ is a polynomial in $\mathbb{R}[z_1, \cdots, z_n]$ and this polynomial is *a priori* unknown to the adversary.
- The modelling problem hence reduces to a problem of polynomial interpolation in noisy context:
  - from noisy observations of $\varphi(Y)$, we want to recover the coefficients ε_0, ε_1, ... such that:

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- Usually, we assume the polynomial $\varphi(Z)$ is of degree 1.
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- The latter assumption (called Hamming Weight) is today pertinent for almost all smart card technologies.
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Advanced Side Channel Attacks (DPA like attacks)

Side Channel Analysis: the statistical distinguisher

Under INA assumption, the pdf f_L of L is a Gaussian Mixture:

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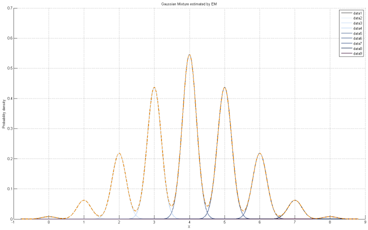

Figure : No noise ($\sigma = 0.2$)

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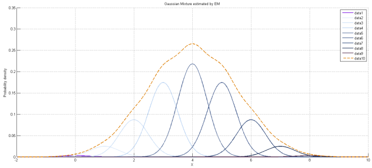

Figure : Small noise ($\sigma = 0.5$)

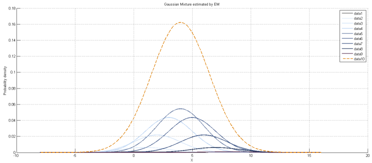

Figure : Medium noise ($\sigma = 2$)

Advanced Side Channel Attacks (DPA like attacks)

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Question: which property of this mixture depends on the secret k?

Note: difficult question since the adversary does not know φ but a model $\mathbf{m}$ for it!

Many proposals have been done in the literature:

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DPA attack Kocher et al at CRYPTO 96.

Attack Description Sheet/Form: DPA

Type of Leakage: no restriction.
Model Function: the function $\mathbf{m} : Z \mapsto z_i$ for some index i.
Statistical Distinguisher: difference of means Test.
Key Candidate Selection: the candidate the maximizes the scores.

Score value $\Delta_{\hat{k}}$: a statistical estimator of

$$\Delta_{\hat{k}} = \mathbf{E}(L \mid M_{\hat{k}} = 1) - \mathbf{E}(L \mid M_{\hat{k}} = 0)$$

with $M_{\hat{k}}$ equal to the ith bit of $Z = S(X + \hat{k})$.

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Side Channel Analysis: the statistical distinguisher

DPA attack Kocher et al at CRYPTO 96. Why does it work?

$$\begin{aligned}\Delta_{\hat{k}} &= \mathbf{E}(L \mid M_{\hat{k}} = 1) - \mathbf{E}(L \mid M_{\hat{k}} = 0) \\ &= \mathbf{E}(\varphi(Z) + B \mid M_{\hat{k}} = 1) - \mathbf{E}(\varphi(Z) + B \mid M_{\hat{k}} = 0)\end{aligned}$$

Since the noise B is independent of Z,

$$\begin{aligned}\Delta_{\hat{k}} &= \mathbf{E}(\varphi(Z) \mid M_{\hat{k}} = 1) - \mathbf{E}(\varphi(Z) \mid M_{\hat{k}} = 0) \\ &= \mathbf{E}(\varepsilon_i z_i + (\varphi(Z) - \varepsilon_i z_i) \mid M_{\hat{k}} = 1) - \mathbf{E}(\varepsilon_i z_i + (\varphi(Z) - \varepsilon_i z_i) \mid M_{\hat{k}} = 0)\end{aligned}$$

Let us assume that $(\varphi(Z) - \varepsilon_i z_i)$ is independent of z_i and $M_{\hat{k}}$ (true in practice).

$$\boxed{\Delta_{\hat{k}} = \varepsilon_i \left(\mathbf{E}(z_i \mid M_{\hat{k}} = 1) - \mathbf{E}(z_i \mid M_{\hat{k}} = 0)\right)}$$

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where

- z_i is the ith bit of $S(M + k)$
- $M_{\hat{k}}$ is the ith bit of $S(M + \hat{k})$

If $k = \hat{k}$, then $z_i = M_{\hat{k}}$ and :

$$\Delta_{\hat{k}} = \varepsilon_i \,(1 - 0) = \varepsilon_i$$

If $k = \hat{k}$, then z_i and $M_{\hat{k}}$ are independent (due to properties of S) and

$$\Delta_{\hat{k}} = \varepsilon_i \,(\mathbf{E}(z_i) - \mathbf{E}(z_i)) = 0$$

Advanced Side Channel Attacks (DPA like attacks)

Side Channel Analysis: the statistical distinguisher

DPA attack Kocher et al at CRYPTO 96.

- Pros: no need for assumption on the device properties, quite efficient in practice.
- Cons: does not use all the information in the trace and attack each bit of the target separately.

Advanced Side Channel Attacks (DPA like attacks)

Side Channel Analysis: the statistical distinguisher

Multi-bit DPA attack Messerges in his PhD Thesis.

Attack Description Sheet/Form: Multi-bit DPA

Type of Leakage: no restriction.
Model Function **m**: the Hamming weight function.
Statistical Distinguisher: difference of means for a parameter τ.
Key Candidate Selection: the candidate the maximizes the scores.

Distinguishing value $\Delta_{\hat{k}}$: a statistical estimator of

$$\Delta_{\hat{k}} = \mathbf{E}(L \mid M_{\hat{k}} \leq \tau) - \mathbf{E}(L \mid M_{\hat{k}} > \tau)$$

with $M_{\hat{k}}$ equal to the $\mathrm{HW}[S(X + \hat{k})]$.

- Pros: exploit more information than the DPA.
- Cons: need assumption (Hamming weight) on the device behaviour.

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Side Channel Analysis: the statistical distinguisher

CPA attack Brier et al at CHES 2004.

Attack Description Sheet/Form: CPA

Type of Leakage: no restriction.
Model Function **m**: possibly any function (in practice HW).
Statistical Distinguisher: linear correlation coefficient.
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$$\Delta_{\hat{k}} = \rho(L, M_{\hat{k}})$$

- Pros: exploit more information than the previous ones and is more powerful
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Advanced Side Channel Attacks (DPA like attacks)

Side Channel Analysis: the statistical distinguisher

MIA attack Gierlichs et al at CHES 2008.

Attack Description Sheet/Form: MIA

Type of Leakage: no restriction.
Model Function **m**: any non-injective function (in practice HW).
Statistical Distinguisher: mutual information (MI).
Key Candidate Selection: the candidate the maximizes the scores.

Distinguishing value $\Delta_{\hat{k}}$: a statistical estimator of

$$\Delta_{\hat{k}} = MI(L; M_{\hat{k}}) = \text{entropy}(L) - \text{entropy}(L \mid M_{\hat{k}})$$

- Pros: theoretically able to detect any kind of dependency whatever the quality of the model if the function $x \mapsto \mathbf{m} \circ S(x+k)$ is non-injective!
- Cons: need for efficient estimators of the entropy (currently less efficient than the CPA) Batina *et al, Journal of Cryptology 2011*.

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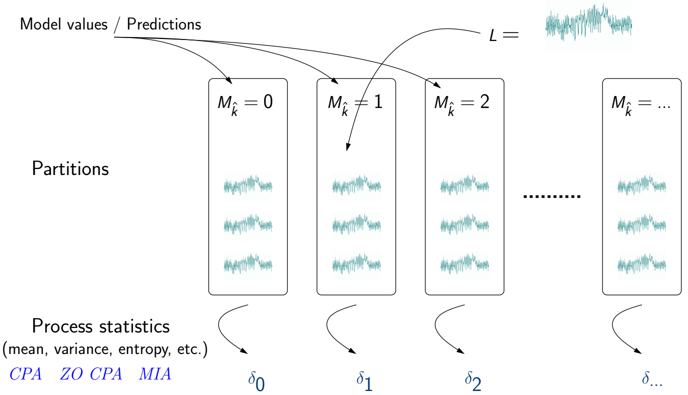
Advanced Side Channel Attacks (DPA like attacks)

Other attacks

- Stochastic attacks: See Schindler et al. at CHES 2005 or Doget et al at JCEN 2011.
  - Good alternative when classical (*e.g. HW) models fail.*
  - *Amounts to process an Euclidean distance between the leakage values and the estimations in the regressed model.*
- Kolmogorov-Smirnov Based attacks: Whitnall et al. at CARDIS 2011.
  - Good alternative to the MIA.
- PPA, EPA, VPA, etc: other attacks exist **but are often very ad hoc ones with no clear advantage to the "classical" ones**.
- Works comparing the attacks:
  - "How to Compare Profiled Side-Channel Attacks?" Standaert *et al, ACNS 2009*.
  - "A fair evaluation framework for comparing side-channel distinguishers" by Withnall *et al, JCEN 2011*.
  - "Univariate Side Channel Attacks and Leakage Modeling" by Doget *et al, JCEN 2011*.

Distinguishers Processing

... a partitioning description.



Combine the statistics

$$\Delta_{\hat{k}} = \sum_i \delta_i \times \mathrm{P}[M_{\hat{k}} = i]$$

Advanced Side Channel Attacks (DPA like attacks)

Attack Efficiency Consideration

Attack Efficiency

The efficiency of an SCA given a success rate β is the smallest value N such that:

$$\Pr(\text{Attack succeeds in recovering } k \text{ with } N \text{ measurements}) \geq \beta \ .$$

Particular case: the attack involves correlation coefficient ($i.e. \Delta = \rho$):

$$\Pr\left(\hat{\rho}_k(N) > \max_{\hat{k} \neq k} \hat{\rho}_{\hat{k}}(N)\right) \geq \beta \ .$$

where $\hat{\rho}_k(N)$ denotes the estimation of ρ_k based on N.

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Advanced Side Channel Attacks (DPA like attacks)

Attack Efficiency Consideration

- Fisher: when $\hat{\rho}_{\hat{k}}(N)$ is computed between samples that have a joint normal distribution, $Z_{N,\hat{k}} = \frac{1}{2}\ln\left(\frac{1+\hat{\rho}_k(N)}{1-\hat{\rho}_{\hat{k}}(N)}\right)$ has a normal distribution with parameters

$$\mathbf{E}(Z_{N,\hat{k}}) = \frac{1}{2}\ln\left(\frac{1+\rho_k}{1-\rho_{\hat{k}}}\right) \text{ and } \mathrm{Var}(Z_{N,\hat{k}}) = (N-3)^{-2}.$$

- [Mangard at CT-RSA 2004] So, $\Pr(\hat{\rho}_k(N) > \hat{\rho}_{\hat{k}}(N)) = \beta$ implies:

$$N = 3 + 8\left(\frac{\Phi^{-1}(\beta)}{\ln\left(\frac{1+\rho_k}{1-\rho_{\hat{k}}}\right)}\right)^2 ,$$

where Φ denotes the pdf of $\mathcal{N}(0,1)$.

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- [Mangard at CT-RSA 2004] Assuming $\rho_{\hat{k}}(N) = 0$ we get:

$$\boxed{N \approx 8 \times \Phi^{-1}(\beta)^2 \times \rho_k^{-2}} \ ,$$

since $\ln(1+x) \approx x$ if $|x| < 1$.

Advanced Side Channel Attacks (DPA like attacks)

Link between Efficiency and Signal to Noise Ratio (SNR)

- Let us define the SNR by:

$$\mathrm{SNR} = \frac{\mathbf{Var}[\mathbf{L}] - \mathbf{E}[\mathbf{Var}[\mathbf{L} \mid Z]]}{\mathbf{E}[\mathbf{Var}[\mathbf{L} \mid Z]]} = \frac{\mathbf{Var}[\varphi(Z)]}{\mathbf{E}[\mathbf{Var}[\mathbf{L} \mid Z]]}$$

  Note: can be computed without knowing φ!

- [Mangard at CT-RSA 2004] If SNR $\ll 1$, we have

$$\rho_{\hat{k}}(N) = \mathrm{SNR} \times \rho^0_{\hat{k}}(N)$$

  where $\rho^0_{\hat{k}}(N)$ denotes the correl. when there is no stoch. noise.

- Consequently,

$$N \sim \frac{1}{SNR}$$

$SNR = 0.01$	$\rightarrow$	around 100 traces	$\rightarrow$	*few seconds*
$SNR = 0.001$	$\rightarrow$	around 1000 traces	$\rightarrow$	*less than $^1/_4$ hour*
$SNR = 0.0001$	$\rightarrow$	around $1O^5$ traces	$\rightarrow$	*several hours*
$SNR = 10^{-6}$	$\rightarrow$	several millions of traces	$\rightarrow$	*several days*

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Advanced Side Channel Attacks (DPA like attacks)

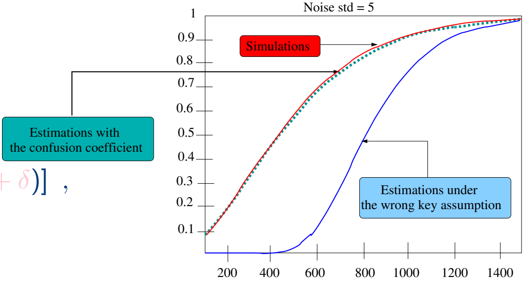
More accurate efficiency evaluations

- Core Idea: relax the assumption $\rho_{\hat{k}}(N) = 0$ for any $\hat{k} \neq 0$.
- Note: this assumption contradicts the ghost Peaks phenomenon ... which is however observed in practice!
- Recent works on this subject: Rivain, SAC 2008, Fei, Luo, Ding, CHES 2012, Thillard, Prouff, Roche, CHES 2013.

Use the notion of Confusion Coefficient, defined for every δ by:

$$\kappa_\delta = \mathbf{E}[\mathrm{m}\circ S(X+k)\times \mathrm{m}\circ S(X+k+\delta)] \ ,$$

where m is the model used in the attack.



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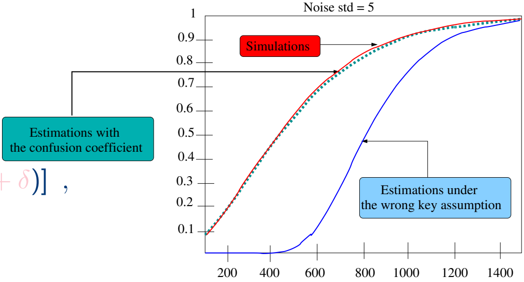

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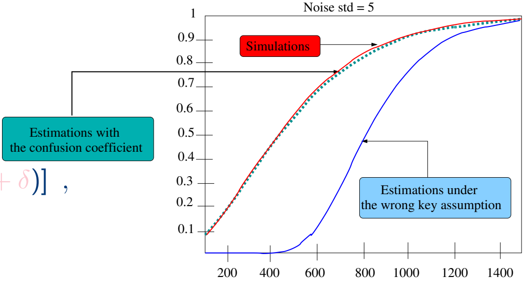

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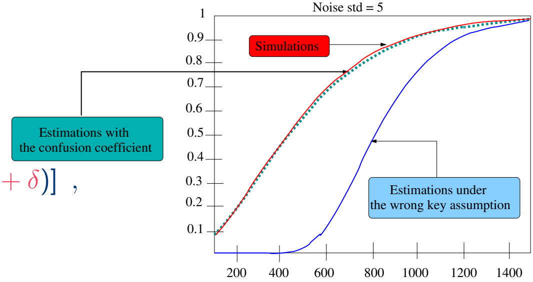

Advanced Side Channel Attacks (DPA like attacks)

Efficiency of Other Attacks (MIA, Templates, etc.)

- When provided with the same *a priori* information about the leakage, CPA, MIA, DPA and Gaussian template attacks are asymptotically equivalent Mangard *et al, IET Information Security 2011*.
  - $\Longrightarrow$ Efficiency formula $N \approx 8 \times \Phi^{-1}(\beta)^2 \times \Delta_k^{-2}$ stays true for the corresponding distinguishers.
- Note: for Template attacks, the cost of the on-line phase may be constant but the cost of the off-line templates building will be linear in SNR^{-1}.
- In conclusion, adding security consists in finding efficient way(s) to decrease Δ_k as much as possible.
  - *i.e.*specify the algorithm implementation such that for any instantaneous leakage L, for any key part k and for any function g:

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where X is some plaintext part and ε is a security parameter.

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Part III

Software Countermeasures for AES and HOSCA

Outline

Plan

SCA Countermeasures

- Masking [IBM Team at CRYPTO 1999].
  - Efficient against SCA in practice.
  - Difficult to implement for non-linear transformations.
- Shuffling [Researchers from Graz University at ACNS 2006].
  - Less efficient against SCA in practice.
  - Easy to implement for every transformation.
- Whitening [Kocher Jaffe June, CRYPTO 1999].
  - Less efficient than masking when used alone and costly in Hardware.
  - Easy to implement for every transformation.

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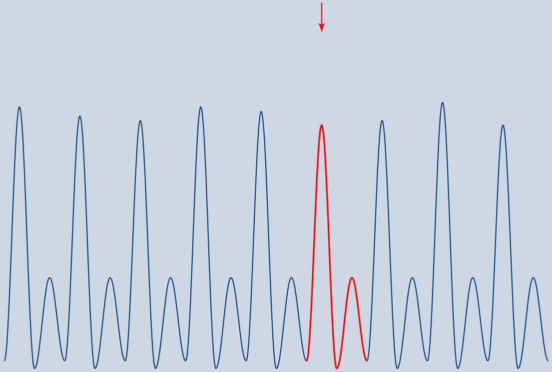
SCA Countermeasures

Core Ideas



SCA Countermeasures

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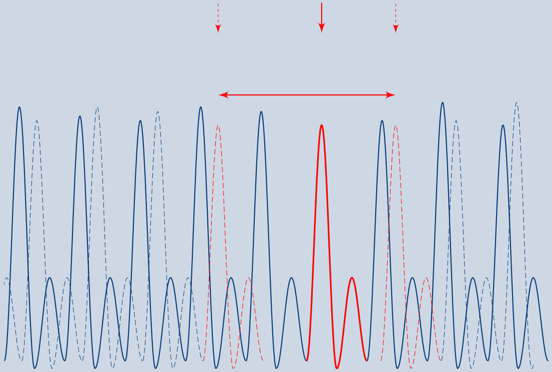
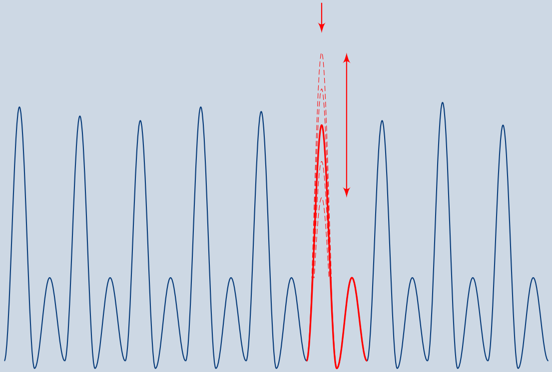


Desynchronisation = different points, same amplitutde

SCA Countermeasures

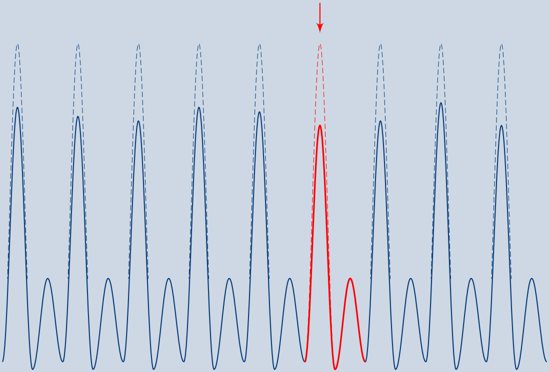
Core Ideas



Masking = same point, random amplitude

SCA Countermeasures

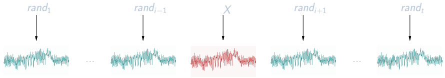
Core Ideas



Balanced Logic = same point, constant amplitude

SCA Countermeasures
Shuffling Method

- **Core Idea:** spread the sensitive signal related to Z over t different signals $S_1, \ldots, S_t$ leaking at different times.
- Select an index at random:



- Impact: decreases the attack efficiency by a factor of t (*i.e.* $\Delta_k^2 \longrightarrow \Delta_k^2/t$)
- Asset: can be used to protect any operation Op on Z.
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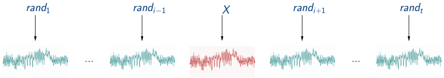

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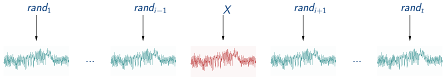

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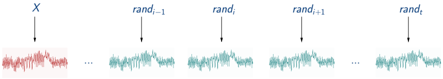

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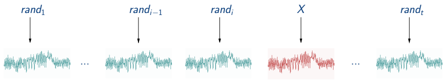

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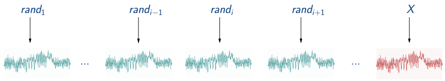

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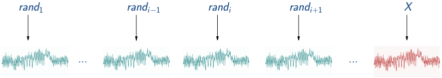

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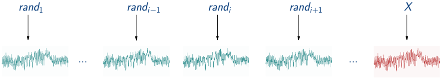

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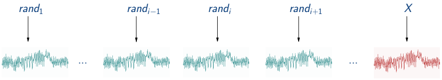

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Masking Method

- Core idea: randomly split Z into $d+1$ shares $M_0, \ldots, M_d$ s.t

$$M_0 \oplus \cdots \oplus M_d = Z \ .$$

- Impact: attack efficiency decreases exponentially w.r.t. d, the base of the exponential being the noise std [CJRR99].
- Impact: for $d = 2$ the distinguisher value Δ_k of first-order SCA is reduced to 0!
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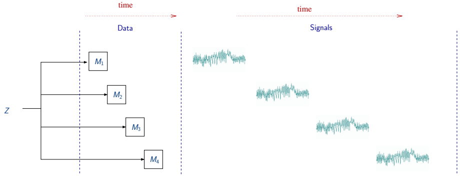

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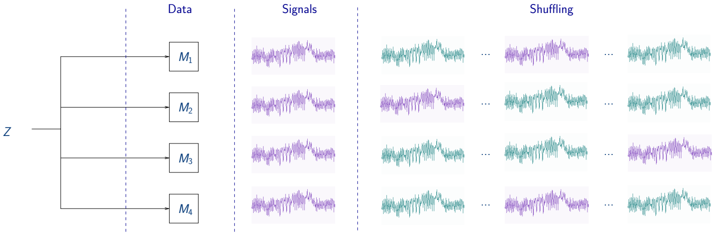
SCA Countermeasures
Masking-and-Shuffling Method



$Z = M_1 \oplus M_2 \oplus M_3 \oplus M_4$

Plan

SCA Countermeasures
Masking Scheme for Block Ciphers

- SPN networks (*e.g. DES, AES*)
- *The different transformations must satisfy:*

Completeness

The masked variable M_0 and the masks M_i must verify:

$$M_0 \oplus \cdots \oplus M_d = Z \ .$$

Security

All the shares M_i must be manipulated at different times.

SCA Countermeasures

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SCA Countermeasures

Masking Scheme for Block Ciphers

- Propagation through **linear transformation**

$$
\begin{tikzcd}
M_0 \arrow[d] & M_1 \arrow[d] & \cdots & M_d \arrow[d] \\
\boxed{\mathrm{L}} \arrow[d] & \boxed{\mathrm{L}} \arrow[d] & & \boxed{\mathrm{L}} \arrow[d] \\
L(M_0) & L(M_1) & \cdots & L(M_d)
\end{tikzcd}
$$

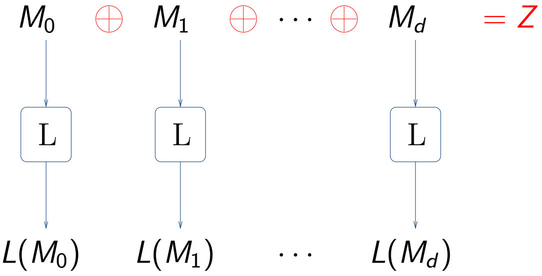
Issue

How to compute $(M_0', \ldots, M_d')$ from $(M_0, \ldots, M_d)$ in a secure way?

SCA Countermeasures

Masking Scheme for Block Ciphers

- Propagation through **linear transformation**



Issue

How to compute $(M'_0, \ldots, M'_d)$ from $(M_0, \ldots, M_d)$ in a secure way?

SCA Countermeasures
Masking Scheme for Block Ciphers

- Propagation through **linear transformation**

$$M_0 \oplus M_1 \oplus \cdots \oplus M_d = Z$$

$$L(M_0) \oplus L(M_1) \oplus \cdots \oplus L(M_d) = L(Z)$$

Issue

How to compute $(M'_0, \ldots, M'_d)$ from $(M_0, \ldots, M_d)$ in a secure way?

SCA Countermeasures

Masking Scheme for Block Ciphers

- Propagation through **s-box**

$$M_0 \oplus M_1 \oplus \cdots \oplus M_d = Z$$

$$S(M_0) \oplus S(M_1) \oplus \cdots \oplus S(M_d) \neq S(Z)$$

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Masking Scheme for Block Ciphers

- Propagation through **s-box**

$$
\begin{array}{ccccccccc}
M_0 & \oplus & M_1 & \oplus & \cdots & \oplus & M_d & & = Z \\
\downarrow & & \downarrow & & & & \downarrow & & \\
?? & & ?? & & & & ?? & & \\
\downarrow & & \downarrow & & & & \downarrow & & \\
M'_0 & \oplus & M'_1 & \oplus & \cdots & \oplus & M'_d & & = S(Z)
\end{array}
$$

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How to compute $(M'_0, \ldots, M'_d)$ from $(M_0, \ldots, M_d)$ in a secure way?

SCA Countermeasures
Masking Scheme for first order

- Method by table recomputation for $d = 1$

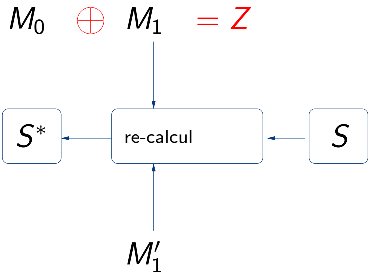

Table Recomputation

For every x: $S^*(x) \leftarrow S(x \oplus M_1) \oplus M'_1$

- $M'_0 \leftarrow S^*(M_0)$

SCA Countermeasures

Masking Scheme for first order

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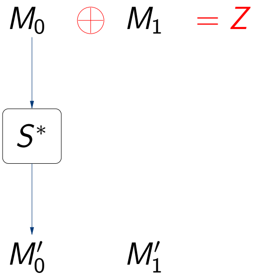

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\begin{array}{ccccc}
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\downarrow & & & & \\
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$$

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SCA Countermeasures

Masking Scheme for first order

Algo Firs-Order Masking Pre-processing

INPUT(S) : A table representation of the function S, an input mask M_1 and an output mask M_1'

OUTPUT(S) : The table representation of the function $X \mapsto S(X \oplus M_1) \oplus M_1'$

1: **for** $x = 0$ to $2^n - 1$ **do**

2: $\quad T[x \oplus M_1] \leftarrow T[x] \oplus M_1'$

3: **return** T

Algo Firs-Order Masking of an s-box processing

INPUT(S) : A masked input $Z + M_1$ (*e.g.* $Z = S(M \oplus k)$)

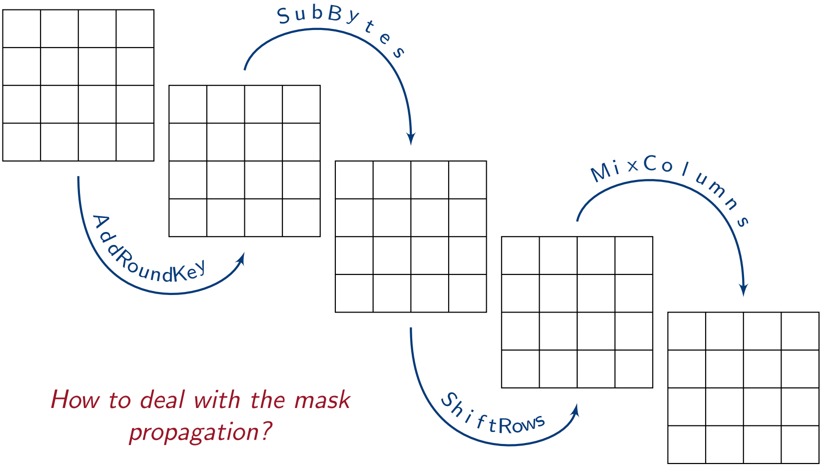
OUTPUT(S) : *The value* $Y = S(Z) \oplus N_1$ *where* M_1' *is a known random value*

1: $Y \leftarrow T^\star(Z)$

2: **return** Y

SCA Countermeasures

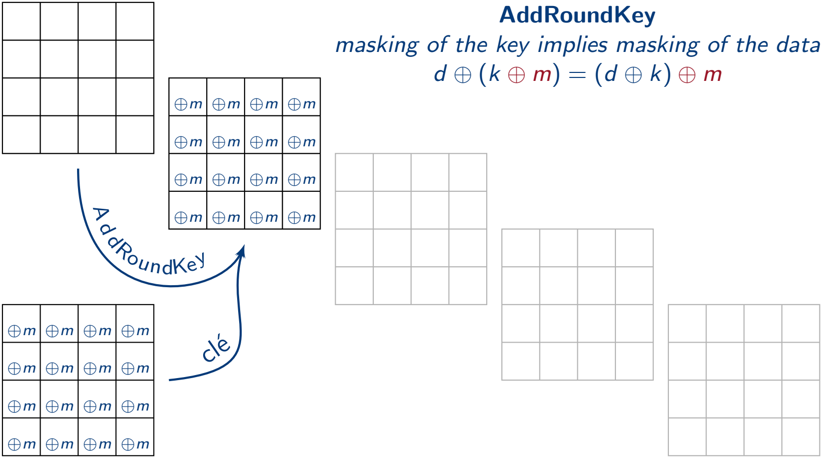
Illustration with a software AES Herbst et al., ACNS 2006



How to deal with the mask propagation?

SCA Countermeasures

Illustration with a software AES Herbst et al., ACNS 2006



AddRoundKey

masking of the key implies masking of the data

$$d \oplus (k \oplus m) = (d \oplus k) \oplus m$$

SCA Countermeasures

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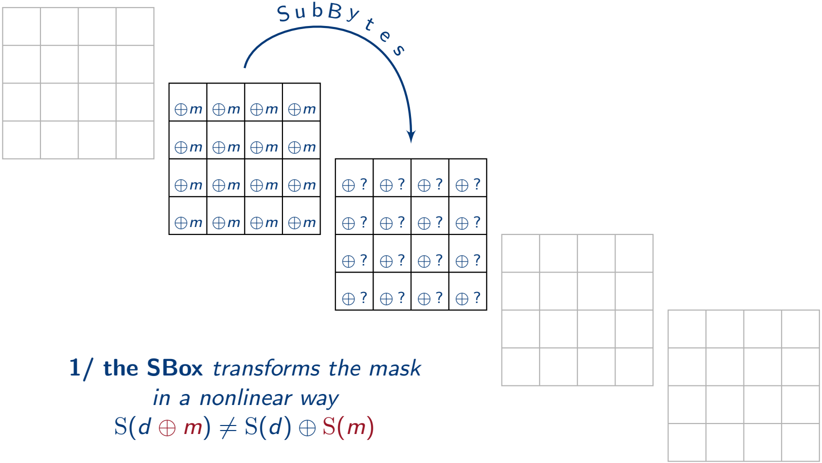


1/ the SBox *transforms the mask in a nonlinear way*

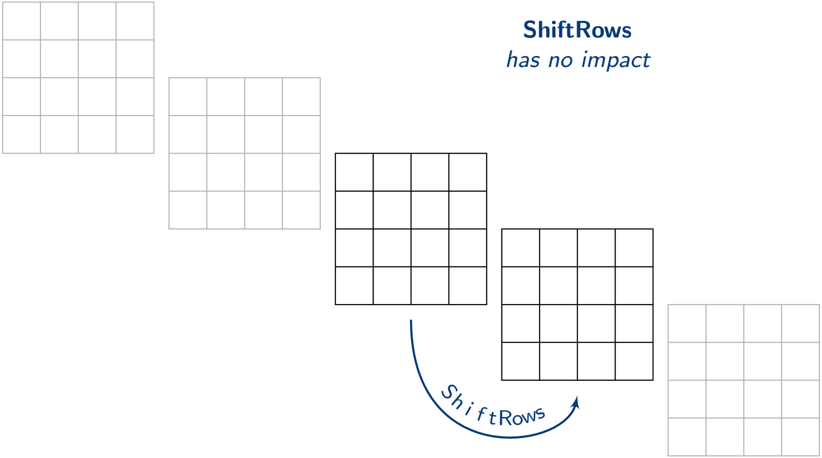
$$\mathrm{S}(d \oplus m) \neq \mathrm{S}(d) \oplus \mathrm{S}(m)$$

SCA Countermeasures

Illustration with a software AES Herbst et al., ACNS 2006

ShiftRows

has no impact



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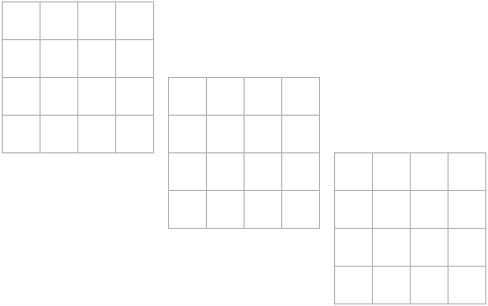
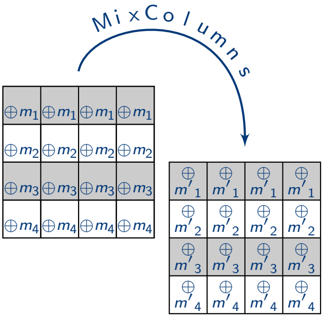




MixColumns

performs a linear combination of the col. elts

$\rightarrow$ *requires at least 2 masks*

$\rightarrow$ *better (perfs) to use 4 masks, one per line*

SCA Countermeasures

Illustration with a software AES Herbst et al., ACNS 2006

To sum-up

1. Generate 6 masks
   - m et m' will be resp. the input and output mask of the **sbox**
   - m_1, m_2, m_3 et m_4 will be used for **MixColumns**
2. Pre-compte a new **sbox** $\mathrm{S}_{m,m'}$ s.t.

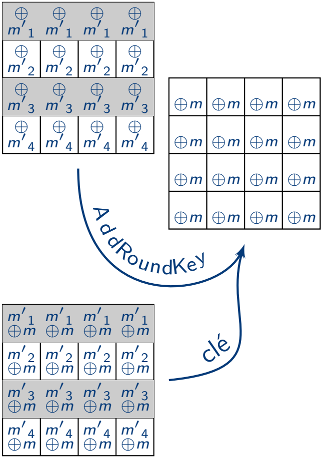
$$\mathrm{S}_{m,m'}(d \oplus m) = \mathrm{S}(d) \oplus m' \quad \textbf{(costly in RAM \ldots)}$$

3. At each MixColumns processing, apply MixColumns to the masked data and to (m_1, m_2, m_3, m_4) s.t.

$$(m'_1, m'_2, m'_3, m'_4) = \mathrm{MixColumns}(m_1, m_2, m_3, m_4)$$

SCA Countermeasures

Illustration with a software AES Herbst et al., ACNS 2006

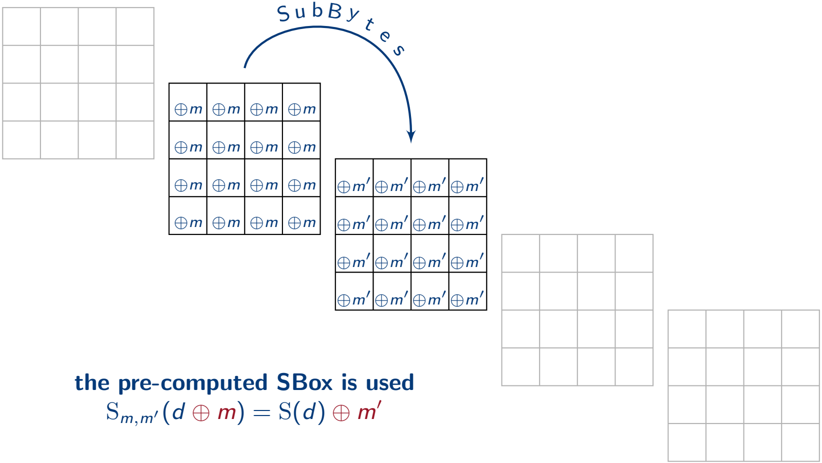


AddRoundKey

- each line of the sate is masked with (m'_1, m'_2, m'_3, m'_4)
- each line of the key state is masked with $(m'_1 \oplus m, m'_2 \oplus m, m'_3 \oplus m, m'_4 \oplus m)$

SCA Countermeasures

Illustration with a software AES Herbst et al., ACNS 2006



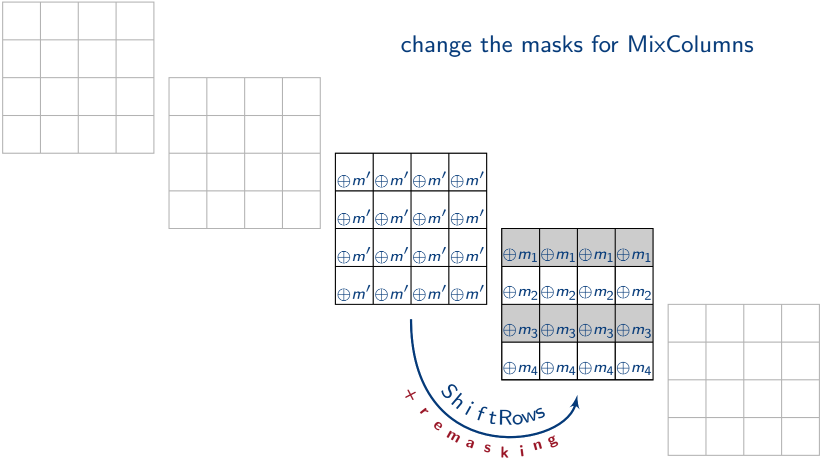
the pre-computed SBox is used

$$\mathrm{S}_{m,m'}(d \oplus m) = \mathrm{S}(d) \oplus m'$$

SCA Countermeasures

Illustration with a software AES Herbst et al., ACNS 2006

change the masks for MixColumns



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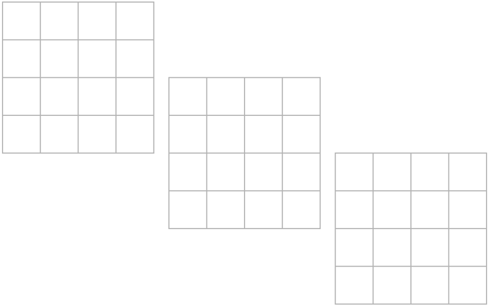
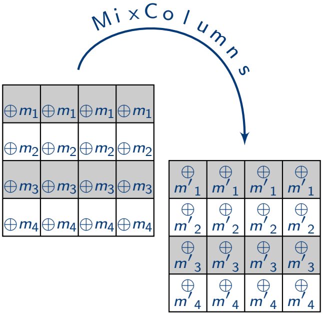


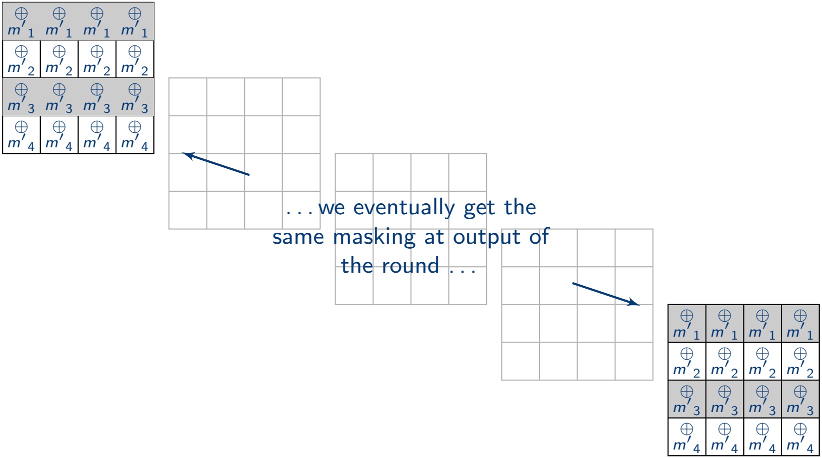


Such that

$$\begin{pmatrix} m'_1 \\ m'_2 \\ m'_3 \\ m'_4 \end{pmatrix} = \textbf{MixColumns} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{pmatrix}$$

SCA Countermeasures

Illustration with a software AES Herbst et al., ACNS 2006



. . . we eventually get the same masking at output of the round . . .

SCA Countermeasures

Masking Schemes for first order: other proposals...

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Note: all those masking does not lead to perfect security against first-order SCA (*i.e.* $\Delta_k \neq 0$).
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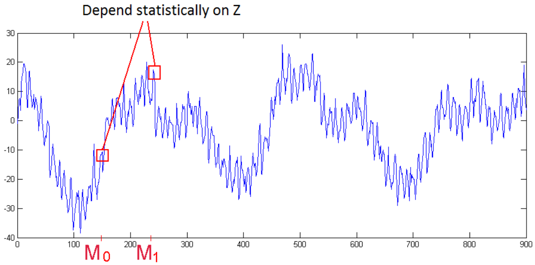
Plan

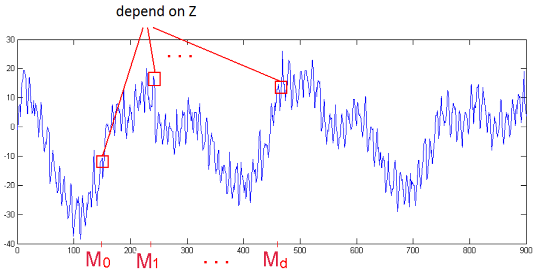
Higher Order Side Channel Attacks
Core Principle

- First Order Masking: $M_0 = Z \oplus M_1$
- $\Longrightarrow$ Second Order SCA:



Higher Order Side Channel Attacks
Core Principle

- Masking of order d: $M_0 = Z \oplus M_1 \oplus \cdots \oplus M_d$
- Attack of order $d+1$:



Higher Order Side Channel Attacks
Introduction

Advanced SCA have been defined to target each CM

- d^{th}-order Masking: HO-SCA
  - [Messerges in his PhD Thesis]
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Higher Order Side Channel Attacks

Introduction - General Principle

All the previous SCA follow the same outlines.

1. Input: set of observations for the signals $(L_i)_i$ related to a sensitive datum Z
2. Choose a statistical distintguisher Δ and a pre-processing function f
3. From the observations, estimate $f(L_i)$
4. For every hypothesis $\mathrm{HW}[S(M+\hat{k})]$ on Z, estimate

$$\Delta_{\hat{k}} = |\Delta(\mathrm{HW}[S(M+\hat{k})], f((L_i)_i))| \ .$$

5. Select the hypothesis that maximizes the estimation of $\Delta_{\hat{k}}$.

Note: if the mutual information is used instead of the correlation coefficient, there is not need for a pre-processing function f.
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Example: if $Z = S(M+k)$ and $M_{\hat{k}} = \mathrm{HW}[S(M+\hat{k})]$, we have ...

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HO-SCA against Higher Order Masking

Illustration with Δ being Pearson' Correlation Coefficient

Context: sensitive variable Z split into $d+1$ shares M_0, ..., M_d
Notation: L_i is the signal related to M_i.

Function f is a normalized product:

$$f(L_0, \cdots, L_d) = \prod_{i=0}^{d} (L_i - \mathbf{E}(L_i)) \ .$$

In the Hamming Weight Model, the efficiency satisfies:

$$\rho_k = \frac{cst_1}{\left(\sqrt{1 + cst_2 \cdot \sigma^2}\right)^{d+1}} \ .$$

It is denoted by $\rho(d, \sigma)$.

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Integrated SCA Against Shuffling

Illustration with Δ being Pearson' Correlation Coefficient

Context: the signal S containing information about Z is randomly spread over t different signals L_1, ..., L_t.

Function f is an Integrated signal:

$$f(L_1, \cdots, L_t) = L_1 + L_2 + \ldots + L_t$$

Note: the sum always contains the term S.
In the Hamming Weight Model, the efficiency satisfies:

$$\boxed{\rho_k = \frac{1}{\sqrt{t}\sqrt{1 + cst_2 \cdot \sigma^2}} .}$$

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Part IV

Deafeating HOSCA and Proven Security

Outline

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Need for security proofs?

A brief history of countermeasures and attacks

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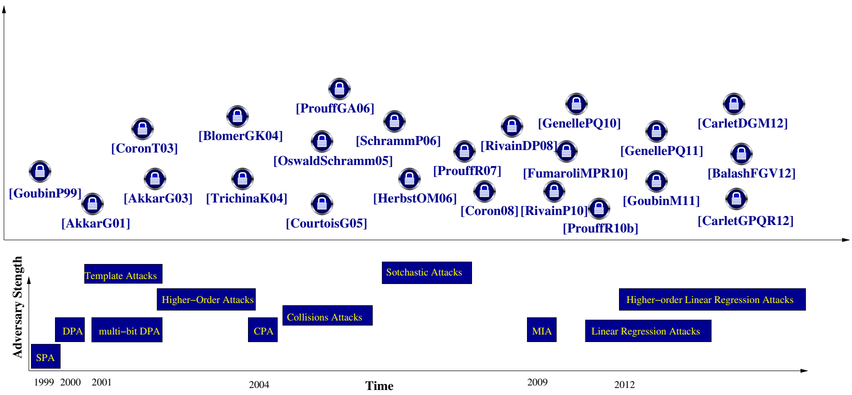

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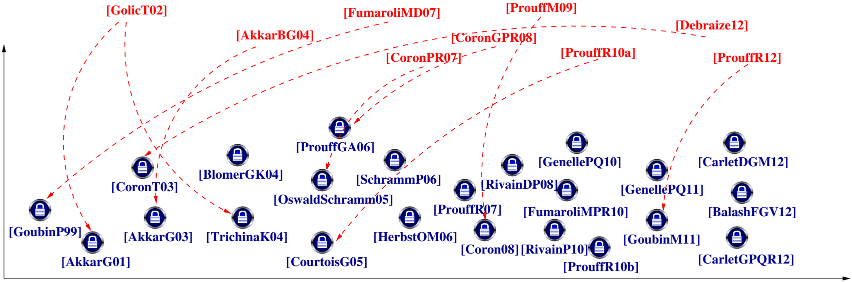
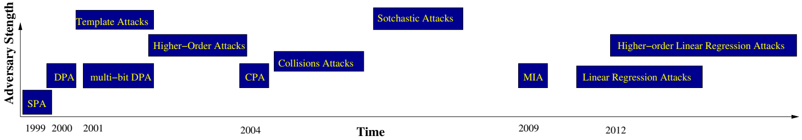
A brief history of countermeasures and attacks





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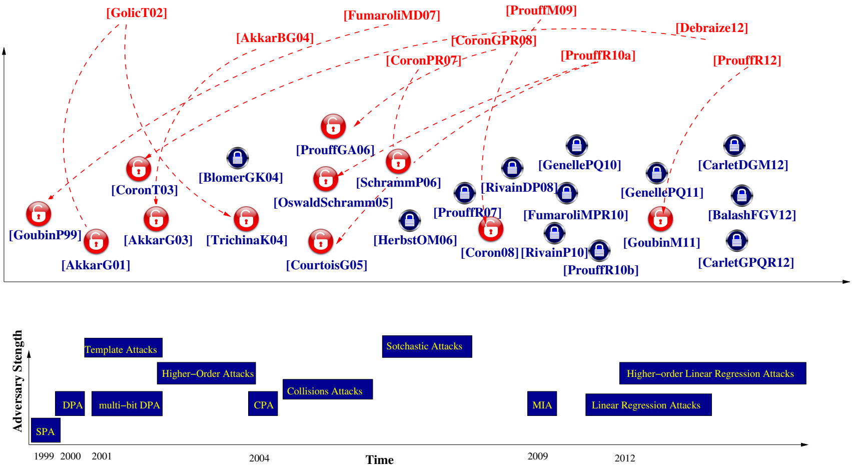

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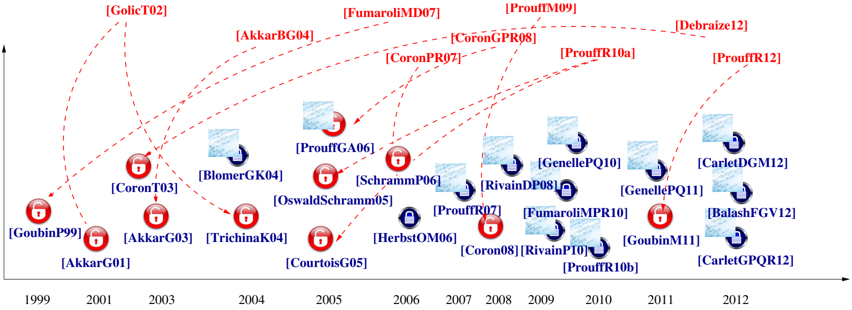

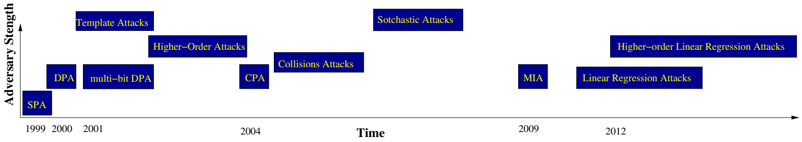

Which security guaranty?

Provable security for embedded systems. Two main approaches...

First approach consists in designing cryptosystems that can be proved secure for some leakage models.

- Recent interest from the crypto theory community (start with DziembowskiPietrzak2007).
- Proofs are given for some leakage models:
    - Bounded Retrieval Model (BRM): the overall sensitive leakage is bounded.
    - (coutinuous) Leakage-resilient cryptography (LRC): the leakage is limited for each invocation only.
- BRM primitives are insecure against DPA and its practical relevence is still under discussion.
- LRC primitives aims at DPA-security
    - Based on re-keying techniques
    - The kind of adversary catched by those models is too strong, which strongly impacts the efficiency.

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Which security guaranty?

Provable security for embedded systems (second approach)

Second approach consists in securing the implementation using secret sharing techniques.

- First Ideas in GoubinPatarin99 and ChariJutlaRaoRohatgi99.
- Soundness based on the following remark:

- Bit x masked $\mapsto x_0, x_1, \ldots, x_d$
- Leakage : $L_i \sim x_i + \mathcal{N}(\mu, \sigma^2)$
- Number q of leakage samples to test $((L_i)_i | x = 0) \stackrel{?}{=} ((L_i)_i | x = 1)$:

$$q \geq O(1)\sigma^d$$

- Until now, two options exist to prove the security:
  - the probing Adversary model
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Probing Adversary Model

IshaiSahaiWagner, CRYPTO 2003

- A d^{th}-order probing adversary is allowed to observe **at most** d intermediate results during the overall algorithm processing.
  - Hardware interpretation: d is the maximum of wires observed in the circuit.
  - Software interpretation: d is the maximum of different timings during the processing.
- d^{th}-order probing adversary = d^{th}-order SCA as introduced in Messerges99.
- Countermeasures proved to be secure against a d^{th}-order probing adv.:
  - $d = 1$: KocherJaffeJune99, BlömerGuajardoKrummel04, ProuffRivain07.
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Probing Adversary Model
Proofs Outlines

To prove the security of an implementation...

- for $d = 1, 2$: list all the intermediate variables and check that none of them is sensitive.
- for $d \geq 3$: the method above starts is too costly!
- Issue: how to prove that a scheme can be made d^{th}-order secure for any given d?
- Ishai-Sahai-Wagner's approach:
  - Two players: the **Adversary** who can observe any d-tuple of intermediate results and an **Oracle** with no access to the implementation
  - The game: for any d-tuple, prove that the oracle can simulate the adversary's view of the implementation execution.
- Method works well for simple schemes (*e.g. the implementation of a multiplication), it is difficult to apply otherwise!*

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Information Bounded Model

Chari *et al, CRYPTO 1999* and Prouff and Rivain, Eurocrypt 2013

- Implementation Model. Micali-Reyzin, TCC 2004

  Implementation = seq. of elem. computations producing a list of interm. results $(Z_i)_i$.

- Leakage Model. The leakage on each Z_i is modelled by a probabilistic function f_i s.t.

  $$\mathrm{MI}(Z_i; f_i(Z_i) \leq O(1/\psi)\ ,$$

  where ψ is a security parameter which only depends on the stochastic noise.
- Security Proof goal: find a deterministic function P s.t. the following bound is as tight as possible:

  $$\mathrm{MI}((X,k); (f_i(Z_i)))_i \leq P(1/\psi)$$

  where X is the plaintext and k is the key.
- Example of scheme benefiting from such a security bound:
  - $\longrightarrow$ Ishai-Sahai-Wagner d^{th}-order Boolean Masking of AES and its extensions Ishai,Sahai and Wagner, CRYPTO 2004 and Rivain-Prouff, CHES 2010.

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Plan

Higher-Order Masking Schemes

Achieving security in the probing adversary model

Definition

A *dth-order masking scheme* for an encryption algorithm $c \leftarrow \mathcal{E}(m, k)$ is an algorithm

$$(c_0, c_1, \ldots, c_d) \leftarrow \mathcal{E}'((m_0, m_1, \ldots, m_d), (k_0, k_1, \ldots, k_d))$$

- Completeness: there exists R s.t.:

$$R(c_0, \cdots, c_d) = \mathcal{E}(m, k)$$

- Security: $\forall\{iv_1, iv_2, \ldots, iv_d\} \subseteq \{\text{intermediate var. of } \mathcal{E}'\}$:

$$\Pr(k \mid iv_1, iv_2, \ldots, iv_d) = \Pr(k)$$

For SPN (*eg.* DES, AES) the main issue is masking the S-box.

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Masking a S-box

Original work of Ishai, Sahai and Wagner

Main idea: split the S-box computation into elementary fields operations and protect each of them individually.

- Original idea limited to GF(2), then extended to any field in RivainProuff2010 and FaustRabinReyzinTromerVaikuntanathan2011.
- Data are split by bitwise addition: $x \longrightarrow x_0, \cdots, x_d$ s.t. $x_i \leftarrow \$$, $i > 0$, and $x_0 = \bigoplus_i x_i$.
- `NOT` gate masking (*a.k.a. encoding*):

$$\begin{array}{cccc} x_0 & x_1 & \cdots & x_d \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \mathit{NOT}\,(x_0) & \mathit{NOT}\,(x_1) & \cdots & \mathit{NOT}\,(x_d) \end{array}$$

- `AND` gate masking: issue since the operations cannot be done on each shares separately.

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Ishai-Sahai-Wagner (ISW) Scheme

Masking an AND gate

- AND gates encoding:
  - Input: $(a_i)_i$, $(b_i)_i$ s.t. $\bigoplus_i a_i = a$, $\bigoplus_i b_i = b$
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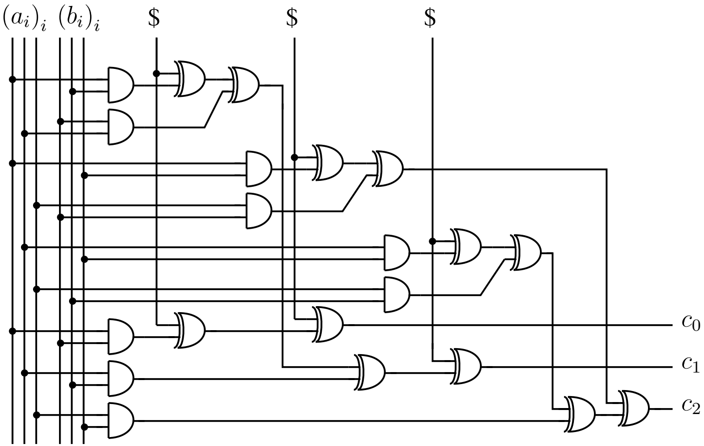
Example: AND gate for $d = 2$



Ishai-Sahai-Wagner (ISW) Scheme

Practical Issues

- Important area overhead for the masked circuit
  - A wire is encoded by $d+1$ wires
  - One AND gate encoded by
    - $(d+1)^2$ ANDs $+$ $2d(d+1)$ XORs $+$ $d(d+1)/2$ \$
  - Example: AES S-box circuit

	ISW		
No masking	$d=1$	$d=2$	$d=3$
200 gates	500 gates	1.1 Kgates	2 Kgates

- Not suitable for software implementations
- Idea: apply ISW in larger fields

ProuffRivain10,FaustRabinReyzinTromerVaikuntanathan2011.

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Application to AES in Software

RivainProuff10

- AES S-box: $\mathrm{S} = \mathrm{Af} \circ \mathrm{Exp}$:
  - Af: affine transformation over $\mathrm{GF}(2)^8$
  - $\mathrm{Exp} : x \mapsto x^{254}$ over $\mathrm{GF}(2^8)$

- Masking Af is efficient:

$$\begin{array}{cccc} x_0 & x_1 & \cdots & x_d \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \mathrm{Af}(x_0) & \mathrm{Af}(x_1) & \cdots & \mathrm{Af}(x_d) \end{array}$$

- Masking Exp:
  - masked square: easy since $x^2 = x_0^2 \oplus x_1^2 \oplus \cdots \oplus x_d^2$
  - masked multiplications : apply ISW on $\mathrm{GF}(2^8)$
  - To minimize the number of multiplications: find a small **addition chain** for 254
    - done in RivainProuff10: only 4 multiplications (and 7 squares).

Application to AES in Software

RivainProuff10

- AES S-box: $\mathrm{S} = \mathrm{Af} \circ \mathrm{Exp}$:
  - Af: affine transformation over $\mathrm{GF}(2)^8$
  - $\mathrm{Exp} : x \mapsto x^{254}$ over $\mathrm{GF}(2^8)$
- Masking Af is efficient:

$$\begin{array}{cccc} x_0 & x_1 & \cdots & x_d \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \mathrm{Af}(x_0) & \mathrm{Af}(x_1) & \cdots & \mathrm{Af}(x_d) \end{array}$$

- Masking Exp:
  - masked square: easy since $x^2 = x_0^2 \oplus x_1^2 \oplus \cdots \oplus x_d^2$
  - masked multiplications : apply ISW on $\mathrm{GF}(2^8)$
  - To minimize the number of multiplications: find a small **addition chain** for 254
    - done in RivainProuff10: only 4 multiplications (and 7 squares).

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Extension to Any S-Box

CarletGoubinProuffQuisquaterRivain12

- Write the s-box $\mathrm{S} : \{0,1\}^n \to \{0,1\}^m$ as a polynomial function over $\mathrm{GF}(2^n)$:

$$\mathrm{S}(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{2^n-1} x^{2^n-1}$$

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General Method

- Four kinds of operations over $\mathrm{GF}(2^n)$:
  1. additions
  2. scalar multiplications (*i.e.* by constants)
  3. squares
  4. regular multiplications $\Rightarrow$ *nonlinear multiplications*
- Masking is efficient for the 3 first kinds
  - $(x + y) = (x_0 + y_0) + (x_1 + y_1) + \cdots + (x_d + y_d)$
  - $x^2 = x_0^2 + x_1^2 + \cdots + x_d^2$
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Masking Complexity

- Masking an operation $\in$ {addition, square, scalar mult.}
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  $\Rightarrow$ $(d+1)^2$ mult. $+$ $2d(d+1)$ add. $+$ $nd(d+1)/2$ random bits

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The *masking complexity* of a (n,m) s-box is the minimal number of nonlinear multiplications required to evaluate its polynomial representation over $\mathrm{GF}(2^n)$.

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Straightforward schemes

- Goal: evaluate $\mathrm{S}(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{2^n-1} x^{2^n-1}$
- first solution :
  - compute $\mathrm{S}(x) = a_0 + x(a_1 + x(a_2 + x(\cdots)))$
  - $\Rightarrow 2^n - 2$ nonlinear multiplications
- second solution :
  - first compute x^2, x^3, x^4, ... then evaluate $\mathrm{S}(x)$
  - $x^j \leftarrow (x^{j/2})^2$ when j even, $x^j \leftarrow x \cdot x^{j-1}$ when j odd
  - $\Rightarrow 2^{n-1} - 1$ nonlinear multiplications
- But we can do much better Carlet, Goubin, Prouff, Quisquater, Prouff, FSE 2013 and Roy and Vivek, CHES 2013!
  - Optimal methods for power functions.
  - Efficient heuristic for the general case: around $2^{n-r-1} + 2^r - 2$ multiplications where 2^r is the degree of the polynomial.

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Optimal Masking of Power Functions

Problem

For a given $\alpha \in [1; 2^n - 1]$ compute x^α with the least number of nonlinear multiplications.

$$\Leftrightarrow$$

Problem

Find the shortest 2-addition chain for α (modulo $2^n - 1$).

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- If $\beta \in C_\alpha$ ($\Leftrightarrow C_\beta = C_\alpha$)
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  - $x \to x^2, x^4, x^8, \ldots$ (0 nonlinear multiplications)
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    - $x^3 = x \cdot x^2 \to x^6, x^{12}, x^{24}, \ldots$
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  - x^α and x^β have the same masking complexity
- Exhaustive search for best 2-addition chains
  - $x \rightarrow x^2, x^4, x^8, \ldots$ (0 nonlinear multiplications)
  - with 1 nonlinear multiplication
    - $x^3 = x \cdot x^2 \rightarrow x^6, x^{12}, x^{24}, \ldots$
    - $x^5 = x \cdot x^4 \rightarrow x^{10}, x^{20}, x^{40}, \ldots$
    - *etc.*
  - with 2 nonlinear multiplications
    - $x^7 = x^3 \cdot x^4 \rightarrow x^{14}, x^{28}, \ldots$
    - $x^{11} = x^3 \cdot x^8 \rightarrow x^{22}, x^{44}, \ldots$
    - *etc.*

Optimal Masking of Power Functions

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    - *etc.*

k	Cyclotomic classes in $\mathcal{M}_k^n$
	$n = 4$
0	$C_0 = \{0\},\ C_1 = \{1, 2, 4, 8\}$
1	$C_3 = \{3, 6, 12, 9\},\ C_5 = \{5, 10\}$
2	$C_7 = \{7, 14, 13, 11\}$
	$n = 6$
0	$C_0 = \{0\},\ C_1 = \{1, 2, 4, 8, 16, 32\}$
1	$C_3 = \{3, 6, 12, 24, 48, 33\},\ C_5 = \{5, 10, 20, 40, 17, 34\},\ C_9 = \{9, 18, 36\}$
2	$C_7 = \{7, 14, 28, 56, 49, 35\},\ C_{11} = \{11, 22, 44, 25, 50, 37\},\ C_{13} = \{13, 26, 52, 41, 19, 38\},$ $C_{15} = \{15, 30, 29, 27, 23\},\ C_{21} = \{21, 42\},\ C_{27} = \{27, 54, 45\}$
3	$C_{23} = \{23, 46, 29, 58, 53, 43\},\ C_{31} = \{31, 62, 61, 59, 55, 47\}$
	$n = 8$
0	$C_0 = \{0\},\ C_1 = \{1, 2, 4, 8, 16, 32, 64, 128\}$
1	$C_3 = \{3, 6, 12, 24, 48, 96, 192, 129\},\ C_5 = \{5, 10, 20, 40, 80, 160, 65, 130\},$ $C_9 = \{9, 18, 36, 72, 144, 33, 66, 132\},\ C_{17} = \{17, 34, 68, 136\}$
2	$C_7 = \{7, 14, 28, 56, 112, 224, 193, 131\},\ C_{11} = \{11, 22, 44, 88, 176, 97, 194, 133\},$ $C_{13} = \{13, 26, 52, 104, 208, 161, 67, 134\},\ C_{15} = \{15, 30, 60, 120, 240, 225, 195, 135\},$ $C_{19} = \{19, 38, 76, 152, 49, 98, 196, 137\},\ C_{21} = \{21, 42, 84, 168, 81, 162, 69, 138\},$ $C_{25} = \{25, 50, 100, 200, 145, 35, 70, 140\},\ C_{27} = \{27, 54, 108, 216, 177, 99, 198, 141\},$ $C_{37} = \{37, 74, 148, 41, 82, 164, 73, 146\},\ C_{45} = \{45, 90, 180, 105, 210, 165, 75, 150\},$ $C_{51} = \{51, 102, 204, 153\},\ C_{85} = \{85, 170\}$
3	$C_{23} = \{23, 46, 92, 184, 113, 226, 197, 139\},\ C_{29} = \{29, 58, 116, 232, 209, 163, 71, 142\},$ $C_{31} = \{31, 62, 124, 248, 241, 227, 199, 143\},\ C_{39} = \{39, 78, 156, 57, 114, 228, 201, 147\},$ $C_{43} = \{43, 86, 172, 89, 178, 101, 202, 149\},\ C_{47} = \{47, 94, 188, 121, 242, 229, 203, 151\},$ $C_{53} = \{53, 106, 212, 169, 83, 166, 77, 154\},\ C_{55} = \{55, 110, 220, 185, 115, 230, 205, 155\},$ $C_{59} = \{59, 118, 236, 217, 179, 103, 206, 157\},\ C_{61} = \{61, 122, 244, 233, 211, 167, 79, 158\},$ $C_{63} = \{63, 126, 252, 249, 243, 231, 207, 159\},\ C_{87} = \{87, 174, 93, 186, 117, 234, 213, 171\},$ $C_{91} = \{91, 182, 109, 218, 181, 107, 214, 173\},\ C_{95} = \{95, 190, 125, 250, 245, 235, 215, 175\},$ $C_{111} = \{111, 222, 189, 123, 246, 237, 219, 183\},\ C_{119} = \{119, 238, 221, 187\}$
4	$C_{127} = \{127, 254, 253, 251, 247, 239, 223, 191\}$

Cyclotomic Method

$$
\begin{aligned}
\mathrm{S}(x) &= a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 \\
&\quad + a_8x^8 + a_9x^9 + a_{10}x^{10} + a_{11}x^{11} + a_{12}x^{12} + \ldots \\
&= a_0 + a_1x + a_2x^2 + a_4x^4 + a_8x^8 + \ldots \\
&\quad + a_3x^3 + a_6x^6 + a_{12}x^{12} + a_{24}x^{24} + \ldots \\
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&\quad + \ldots \\
&= a_0 + L_1(x) + L_3(x^3) + L_5(x^5) + \ldots
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where

- $L_1(X) = a_1X + a_2X^2 + a_4X^4 + a_8X^8 + \ldots$
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Cyclotomic Method

1. Compute one power per cyclotomic class x, x^3, x^5, x^7, ...
2. Evaluate the corresponding linearized polynomials $L_1(x)$, $L_3(x^3)$, $L_5(x^5)$, $L_7(x^7)$, ...
3. Compute the sum $\mathrm{S}(x) = a_0 + L_1(x) + L_3(x^3) + L_5(x^5) + L_7(x^7) + \ldots$

Number of nonlinear multiplication

$$=$$

$$\#\{\textbf{cyclotomic classes}\}\backslash(C_0 \cup C_1)$$

n	3	4	5	6	7	8	9	10
# nlm	1	3	5	11	17	33	53	105

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n	3	4	5	6	7	8	9	10
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Parity-Split Method

$$\mathrm{S}(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7 + a_8 x^8 + a_9 x^9 + a_{10} x^{10} + a_{11} x^{11} + a_{12} x^{12} + \ldots$$

where $X = x^2$

- Nonlinear mult. : 1
- and the evaluation of 2^{r+1} polynomials in $X = x^{2^r}$
  - we derive X^j for $j < 2^{n-r}$
  - $2^{n-r-1} - 1$ nonlinear mult.

$$\Rightarrow 2^{n-r-1} + 2^r - 2 \text{ nonlinear mult.}$$

Parity-Split Method

$$\mathrm{S}(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7 + a_8 x^8 + a_9 x^9 + a_{10} x^{10} + a_{11} x^{11} + a_{12} x^{12} + \ldots$$

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Parity-Split Method

$$
\begin{aligned}
S(x) \quad = \quad & a_0 + a_2x^2 + a_4x^4 + a_6x^6 + a_8x^8 + \ldots \\
& a_1x + a_3x^3 + a_5x^5 + a_7x^7 + a_9x^9 + \ldots
\end{aligned}
$$

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$$\Rightarrow 2^{n-r-1} + 2^r - 2 \quad \text{nonlinear mult.}$$

Parity-Split Method

$$
\begin{aligned}
\mathrm{S}(x) &= a_0 + a_2x^2 + a_4x^4 + a_6x^6 + a_8x^8 + \ldots \\
&\quad (a_1 + a_3x^2 + a_5x^4 + a_7x^6 + a_9x^8 + \ldots) \cdot x
\end{aligned}
$$

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Parity-Split Method

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Parity-Split Method

$$\mathrm{S}(x) = a_0 + a_4X^2 + a_8X^4 + \ldots + a_2X + a_6X^3 + \ldots$$
$$(a_1 + a_5X^2 + a_9X^4 + \ldots + a_3x^2 + a_7X^3 + \ldots) \cdot x$$

where $X = x^2$

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- and the evaluation of 2^{r+1} polynomials in $X = x^{2^r}$
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  - $2^{n-r-1} - 1$ nonlinear mult.

$$\Rightarrow 2^{n-r-1} + 2^r - 2 \text{ nonlinear mult.}$$

Parity-Split Method

$$\begin{aligned}\mathrm{S}(x) \;=\;& a_0 + a_4X^2 + a_8X^4 + \ldots + (a_2 + a_6X^2 + \ldots)\cdot X + \\ & (a_1 + a_5X^2 + a_9X^4 + \ldots + (a_3 + a_7X^2 + \ldots)\cdot X)\cdot x\end{aligned}$$

where $X = x^2$

- Nonlinear mult. : 1+2
- and the evaluation of 2^{r+1} polynomials in $X = x^{2^r}$
  - we derive X^j for $j < 2^{n-r}$
  - $2^{n-r-1} - 1$ nonlinear mult.

$$\Rightarrow 2^{n-r-1} + 2^r - 2 \text{ nonlinear mult.}$$

Parity-Split Method

$$\begin{aligned} \mathrm{S}(x) \quad = \quad & a_0 + a_4 x^4 + a_8 x^8 + \ldots + (a_2 + a_6 x^4 + \ldots) \cdot x^2 + \\ & (a_1 + a_5 x^4 + a_9 x^8 + \ldots + (a_3 + a_7 x^4 + \ldots) \cdot x^2) \cdot x \end{aligned}$$

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Comparison

Number of nonlinear multiplications w.r.t. the evaluation method

Method \ n	3	4	5	6	7	8	9	10
Cyclotomic	1	3	5	11	17	33	53	105
Parity-Split	2	4	6	10	14	22	30	46

- For PRESENT ($n = 4$), we shall prefer the cyclotomic method
- For DES ($n = 6$), we shall prefer the parity-split method

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... for algorithms combining only affine and power functions

- Idea: Mix additive with multiplicative masking defined on the same field.
- Recall (Additive masking):
  $x \in \mathrm{GF}(2^n) \mapsto (x_0, \cdots x_d) \in \mathrm{GF}(2^n)^{d+1}$ s.t.

$$\sum_i x_i = x \ .$$

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- Issue 1: convert additive masking into multiplicative masking without leaking information in the d^{th}-order probing model?
  - Solution: conversions algorithms proposed in GenelleProuffQuisquater11 (complexity: d^2 additions and $d(3+d)/2$ multiplications).
- Issue 2: multiplicative is only sound in the multiplicative group! How to deal with the 0 value problem?
  - Solution: map the sharing of 0 into the sharing of 1 and keep trace of this modification for further correction.
  - Amounts to secure the processing of the function

$$x \mapsto x \oplus \delta_0(x) \text{ with } \delta_0(x) = x_0 \texttt{ AND } x_1 \texttt{ AND } \dots \texttt{ AND } x_n \ .$$

  - Soundness: for any power e, we have

$$(x \oplus \delta_0(x))^e = x^e \oplus \delta_0(x)$$

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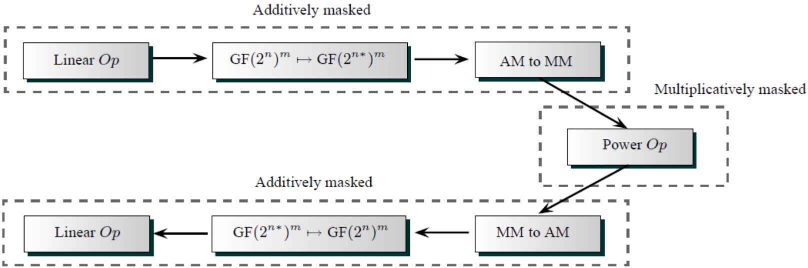

Performances

Table : Comparison of secure AES implementations

	Method	cycles (10^3)	RAM (bytes)
	Unprotected Implementation		
1.	No Masking	2	32
	First Order Masking		
2.	Re-computation	10	256
3.	Tower Field in $\mathrm{GF}(2^2)$	77	42
4.	Multiplicative Masking	22	256
5.	Secure exponentiation for $d = 1$	73	24
6.	Additive and Multiplicative Masking for $d = 1$	25	50
	Second Order Masking		
7.	Double Re-computations	594	$512 + 28$
8.	Single Re-computation	672	$256 + 22$
9.	Secure exponentiation for $d = 2$	189	48
10.	Additive and Multiplicative Masking f for $d = 2$	69	86
	Third Order Masking		
11.	Secure exponentiation for $d = 3$	326	72
12.	Additive and Multiplicative Masking f for $d = 3$	180	128

Masking Schemes for Block Ciphers

Still other alternatives...

- Apply Tower Field Approach: $\mathrm{GF}(2^8) \sim \mathrm{GF}(2^4)[X]/p(X) \sim \mathrm{GF}(2^2)[X]/p'(X) \sim \mathrm{GF}(2)[X]/p''(X)$.
  - see *e.g.* *OswaldMangardPramstallerRijmen05*.
  - *sometimes lead to efficiency improvement as some operations can be tabulated.*
- Split exponentiation in more complex sequences than simply squarings and multiplications.
  - *e.g. also consider bilinear operations as* $x \mapsto x \times L(x)$ *where* L *is linear* *ProuffRivainRoche13*.
- Develop masking schemes for security in presence of glitches.
  - current propositions based on MPC techniques NikovaRijmenSchläffer11,ProuffRoche11.
- Find alternatives to reduce the consumption of random values.

Conclusion

- Security of current implementations is usually only evaluated w.r.t. first-order (a.k.a. univariate) SCA.
  - Against those attacks efficient and effective solutions exist: shuffling + first-order masking + noise addition.
- Higher-order SCA start to be also considered by security evaluators.
  - Effective countermeasures exist but their efficiency is low (see the masking schemes presented here).
  - Best alternative for software AES: shuffling + additive/multiplicative masking + noise.
  - For other block ciphers: only ISW extensions may be applied but they are costly. **This topic needs more studies.**
- Security of today implementations must be formally proved
  - Give upper bound on the information leakage, obtained with sound models. **This topic needs more studies.**
  - Prove the security against some classes of adversaries.
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Thank you for your attention!

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