Destroying Fault Invariant with Randomization -A Countermeasure for AES against Differential Fault Attacks

Harshal Tupsamudre, Shikha Bisht, Debdeep Mukhopadhyay (IIT KHARAGPUR)

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Preliminaries

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Image: A matrix

AES128



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 $\begin{pmatrix} l_0 & l_4 & l_8 & l_{12} \\ l_1 & l_5 & l_9 & l_{13} \\ l_2 & l_6 & l_{10} & l_{14} \\ l_3 & l_7 & l_{11} & l_{15} \end{pmatrix}$

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$$\begin{pmatrix} l_{0} & l_{4} & l_{8} & l_{12} \\ l_{1} & l_{5} & l_{9} & l_{13} \\ l_{2} & l_{6} & l_{10} & l_{14} \\ l_{3} & l_{7} & l_{11} & l_{15} \end{pmatrix} - S - \begin{pmatrix} S[l_{0}] & S[l_{4}] & S[l_{8}] & S[l_{12}] \\ S[l_{1}] & S[l_{6}] & S[l_{6}] & S[l_{6}] \\ S[l_{2}] & S[l_{6}] & S[l_{6}] & S[l_{6}] \\ S[l_{2}] & S[l_{6}] & S[l_{6}] & S[l_{6}] \\ S[l_{3}] & S[l_{7}] & S[l_{7}] & S[l_{7}] \\ S[l_{3}] & S[l_{7}] & S[l_{7}] & S[l_{7}] \\ \end{bmatrix} - SR - \begin{pmatrix} S[l_{0}] & S[l_{4}] & S[l_{6}] & S[l_{12}] \\ S[l_{5}] & S[l_{6}] & S[l_{6}] \\ S[l_{10}] & S[l_{6}] & S[l_{6}] \\ S[l_{10}] & S[l_{6}] & S[l_{7}] \\ S[l_{6}] & S[l_{6}] \\ S[l$$

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$$\begin{pmatrix} l_{0} & l_{4} & l_{8} & l_{12} \\ l_{1} & l_{5} & l_{0} & l_{13} \\ l_{2} & l_{6} & l_{10} & l_{14} \\ l_{3} & l_{7} & l_{11} & l_{15} \end{pmatrix} \longrightarrow \\ -MC - \begin{pmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{pmatrix} \longrightarrow \\ \begin{pmatrix} S[l_{0}] & S[l_{4}] & S[l_{3}] & S[l_{1}] \\ S[l_{3}] & S[l_{7}] & S[l_{11}] & S[l_{13}] \\ S[l_{7}] & S[l_{7}] & S[l_{7}] & S[l_{7}] \\ S[l_{7}] & S[l_{7}]$$

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Fault Attack



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Fault Attack



Only one fault sufficient to retrieve the entire secret key of AES.

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Fault Attack

Fault models to model the strength of adversary

- Bit flip Fault Model : Affects a bit of the intermediate result
- Onstant Byte Fault Model : Requires control over fault value and position
- **③** Random Byte Fault Model : No control over fault value and position
- Attacks that require both the correct and faulty ciphertext are known as differential fault attacks

Countermeasures Against Fault Attacks

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Detection Countermeasure



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Infection Countermeasure



Redundant Round

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LatinCrypt 2012 Infection Countermeasure

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LatinCrypt 2012 Infection Countermeasure SNLF operates on a byte and SNLF(0) = 0



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LatinCrypt 2012 Infection Countermeasure

Dummy rounds occur randomly



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LatinCrypt 2012 Infection Countermeasure RoundFunction(β , k^0) = β



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• Fault f in l_1^{10} , *i.e.*, first byte of the second row in the input of 10^{th} cipher round of AES128

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- Fault *f* in I_1^{10} , *i.e.*, first byte of the second row in the input of 10^{th} cipher round of AES128
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 - After the execution of 10th cipher round

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- Countermeasure infects the faulty computation twice
 - After the execution of 10th cipher round
 - After the execution of compulsory dummy round



Image: A math a math



Image: A math a math





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FDTC 2013 Attack: Infection Caused by the 10th Cipher Round

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After Infection Step, the difference is:

$$R_0 \oplus R_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon \oplus SNLF[\varepsilon] \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

where $\boldsymbol{\varepsilon} = S[I_1^{10} \oplus \boldsymbol{f}] \oplus S[I_1^{10}]$

③ The differential of R_2 and β is:

$$R_2 \oplus \beta = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & SNLF[\varepsilon] \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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6 \therefore RoundFunction $(R_2, k^0) \oplus \beta =$

$$\begin{pmatrix}
0 & 0 & \Delta_1 & 0 \\
0 & 0 & \Delta_2 & 0 \\
0 & 0 & \Delta_3 & 0 \\
0 & 0 & \Delta_4 & 0
\end{pmatrix}$$

(2) Infection caused by compulsory dummy round does not affect ε .

$$C \oplus C^* = \begin{pmatrix} 0 & 0 & \Delta_1 & 0 \\ 0 & 0 & \Delta_2 & \varepsilon \oplus SNLF[\varepsilon] \\ 0 & 0 & \Delta_3 & 0 \\ 0 & 0 & \Delta_4 & 0 \end{pmatrix}$$

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- **(3)** Infection SNLF[ε] caused by 10th cipher round is ineffective.
- Attacker uses the value of $\varepsilon = S[I_1^{10} \oplus f] \oplus S[I^{10}]$ to make hypotheses on I_1^{10} and key byte k_{13}^{11} .
- Repeat this process with two more pairs of faulty and correct ciphertexts, using constant byte fault model.

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- Repeat this process with two more pairs of faulty and correct ciphertexts, using constant byte fault model.
- **1** The attack targets **last three rows** of the 10th round input.
- Recover remaining 4 bytes of top row using brute force search.

Flaws Exploited by FDTC 2013 attack

• The last cipher round is always the penultimate round: The attacker can verify target round using side channel.

Flaws Exploited by FDTC 2013 attack

- The last cipher round is always the penultimate round: The attacker can verify target round using side channel.
- A fault in last three rows of 10^{th} round \implies Infection caused by compulsory dummy round does not affect the erroneous byte.

Remark

What happens if the infection caused by compulsory dummy round affects the erroneous byte of 10^{th} round??

Further Loop Holes in LatinCrypt 2012 Countermeasure

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 - After the execution of compulsory dummy round



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where $\varepsilon = S[I_0^{10} \oplus f] \oplus S[I_0^{10}]$

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• RoundFunction(R_2, k^0) $\oplus \beta =$

$$\begin{pmatrix} \alpha_1 & 0 & 0 & 0 \\ \alpha_2 & 0 & 0 & 0 \\ \alpha_3 & 0 & 0 & 0 \\ \alpha_4 & 0 & 0 & 0 \end{pmatrix}$$

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$$C \oplus C^* = \begin{pmatrix} \alpha_1 \oplus \varepsilon \oplus SNLF[\varepsilon] & 0 & 0 & 0 \\ \alpha_2 & 0 & 0 & 0 \\ \alpha_3 & 0 & 0 & 0 \\ \alpha_4 & 0 & 0 & 0 \end{pmatrix}$$

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We show that α_i are interrelated and infection caused by compulsory dummy round is ineffective.

A Major Flaw in the Infection Scheme

Since *RoundFunction*(β , k^0) = β we can write:

RoundFunction(R_2, k^0) $\oplus \beta = RoundFunction(R_2, k^0) \oplus RoundFunction(\beta, k^0)$

A Major Flaw in the Infection Scheme

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 $\begin{aligned} \text{RoundFunction}(R_2, k^0) \oplus \beta &= \text{RoundFunction}(R_2, k^0) \oplus \text{RoundFunction}(\beta, k^0) \\ &= MC(SR(S(R_2))) \oplus k^0 \oplus MC(SR(S(\beta))) \oplus k^0 \end{aligned}$
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When R₂ = β, RoundFunction(R₂, k⁰) ⊕ β = 0
 When R₂ ≠ β, RoundFunction(R₂, k⁰) ⊕ β ≠ 0

③ The differential of R_2 and β is:

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Therefore we can write the difference between correct and faulty computation as:

$$C \oplus C^* = \begin{pmatrix} 2y \oplus \varepsilon \oplus SNLF[\varepsilon] & 0 & 0 & 0 \\ 1y & 0 & 0 & 0 \\ 1y & 0 & 0 & 0 \\ 3y & 0 & 0 & 0 \end{pmatrix}$$

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- **2** y can be unmasked.

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- 2y can be unmasked.
- And the attack of FDTC 2013 can be mounted.
- Now, this attack can target any 12 bytes of 10th round input.

FDTC 2013 Attack Extended to the Top Row



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Piret and Quisquater's Attack

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- The attack can retrieve only last 3 rows of k¹¹ using 12*3 = 36 faults.
- Solution The top row of k^{11} has to be recoverd using brute force search.

 The attack targets the penultimate round of AES, e.g, in case of AES128, input of 9th round is the target.

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 - After the execution of 9th cipher round

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 - After the execution of 9th cipher round
 - After the execution of 10th cipher round
 - After the execution of compulsory dummy round

Differential after 9th round

Without Countermeasure

$$R_0 \oplus R_1 = \begin{pmatrix} 2f' & 0 & 0 & 0 \\ f' & 0 & 0 & 0 \\ f' & 0 & 0 & 0 \\ 3f' & 0 & 0 & 0 \end{pmatrix}$$

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Differential after 9th round

Without Countermeasure

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With Countermeasure

$$R_0 \oplus R_1 = \begin{pmatrix} 2f' \oplus SNLF[2f'] & 0 & 0 & 0 \\ f' \oplus SNLF[f'] & 0 & 0 & 0 \\ f' \oplus SNLF[f'] & 0 & 0 & 0 \\ 3f' \oplus SNLF[3f'] & 0 & 0 & 0 \end{pmatrix}$$

Differential after 10th round

Without Countermeasure

$$R_0 \oplus R_1 = \begin{pmatrix} S[l_0^{10}] \oplus S[l_0^{10} \oplus P_0] & 0 & 0 & 0\\ 0 & 0 & 0 & S[l_1^{10}] \oplus S[l_1^{10} \oplus P_1] \\ 0 & 0 & S[l_2^{10}] \oplus S[l_2^{10} \oplus P_2] & 0 \\ 0 & S[l_3^{10}] \oplus S[l_3^{10} \oplus P_3] & 0 & 0 \end{pmatrix}$$

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Differential after 10th round

Without Countermeasure

$$R_0 \oplus R_1 = \begin{pmatrix} S[l_0^{10}] \oplus S[l_0^{10} \oplus P_0] & 0 & 0 & 0 \\ 0 & 0 & 0 & S[l_1^{10}] \oplus S[l_1^{10} \oplus P_1] \\ 0 & 0 & S[l_2^{10}] \oplus S[l_2^{10} \oplus P_2] & 0 \\ 0 & S[l_3^{10}] \oplus S[l_3^{10} \oplus P_3] & 0 & 0 \end{pmatrix}$$

With Countermeasure

$$R_0 \oplus R_1 = \begin{pmatrix} z_0 \oplus SNLF[z_0] & 0 & 0 & 0\\ 0 & 0 & 0 & z_1 \oplus SNLF[z_1] \\ 0 & 0 & z_2 \oplus SNLF[z_2] & 0\\ 0 & z_3 \oplus SNLF[z_3] & 0 & 0 \end{pmatrix}$$

where $z_i = S[I_i^{10}] \oplus S[I_i^{10} \oplus P_i \oplus SNLF[P_i]], i \in \{0, \ldots, 3\}.$

Equations for the keys

Without Countermeasure

$$\begin{aligned} 2 \cdot f' &= S^{-1}[T_0 \oplus k_0^{11}] \oplus S^{-1}[T_0^* \oplus k_0^{11}] \\ 1 \cdot f' &= S^{-1}[T_{13} \oplus k_{13}^{11}] \oplus S^{-1}[T_{13}^* \oplus k_{13}^{11}] \\ 1 \cdot f' &= S^{-1}[T_{10} \oplus k_{10}^{11}] \oplus S^{-1}[T_{10}^* \oplus k_{10}^{11}] \\ 3 \cdot f' &= S^{-1}[T_7 \oplus k_7^{11}] \oplus S^{-1}[T_7^* \oplus k_7^{11}] \end{aligned}$$

where T and T^* is correct and faulty ciphertext resp.

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Equations for the keys

Without Countermeasure

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With Countermeasure

 $2 \cdot f' \oplus SNLF[2 \cdot f'] = S^{-1}[T_0 \oplus k_0^{11}] \oplus S^{-1}[T_0^* \oplus k_0^{11}]$ $1 \cdot f' \oplus SNLF[1 \cdot f'] = S^{-1}[T_{13} \oplus k_{13}^{11}] \oplus S^{-1}[T_{13}^* \oplus k_{13}^{11}]$ $1 \cdot f' \oplus SNLF[1 \cdot f'] = S^{-1}[T_{10} \oplus k_{10}^{11}] \oplus S^{-1}[T_{10}^* \oplus k_{10}^{11}]$ $3 \cdot f' \oplus SNLF[3 \cdot f'] = S^{-1}[T_7 \oplus k_7^{11}] \oplus S^{-1}[T_7^* \oplus k_7^{11}]$

where T and T^* is correct and faulty ciphertext resp.

Infection of Compulsory dummy round

Oue to the presence of compulsory dummy round, the difference between the final faulty and correct ciphertext:

$$T \oplus T^* = \begin{pmatrix} m_0 \oplus cdr_0 & cdr_4 & cdr_8 & cdr_{12} \\ cdr_1 & cdr_5 & cdr_9 & m_1 \oplus cdr_{13} \\ cdr_2 & cdr_6 & m_2 \oplus cdr_{10} & cdr_{14} \\ cdr_3 & m_3 \oplus cdr_7 & cdr_{11} & cdr_{15} \end{pmatrix}$$
$$m_j = z_j \oplus SNLF[z_j], j \in \{0, \dots, 3\}.$$

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$$m_j = z_j \oplus SNLF[z_j], j \in \{0,\ldots,3\}.$$

2 Using the relation: $RoundFunction(R_2, k^0) \oplus \beta = MC(SR(S(R_2) \oplus S(\beta)))$ we have:

 $T \oplus T^* = \begin{pmatrix} m_0 \oplus g_1(F_1, F_2) & 1F_3 & h_1(F_4, F_5, F_6) & 3F_7 \\ g_2(F_1, F_2) & 1F_3 & h_2(F_4, F_5, F_6) & m_1 \oplus 2F_7 \\ g_3(F_1, F_2) & 3F_3 & m_2 \oplus h_3(F_4, F_5, F_6) & 1F_7 \\ g_4(F_1, F_2) & m_3 \oplus 2F_3 & h_4(F_4, F_5, F_6) & 1F_7 \end{pmatrix}$

 $F_i, i \in \{1, ..., 7\}$ is infection caused by compulsory dummy round and g_j and $h_j, j \in \{1, ..., 4\}$ are linear functions.

P&Q's Attack on LatinCrypt 2012 Countermeasure: Infection Removal

After removing infection caused by compulsory dummy round we obtain:

$$T \oplus T^* = \begin{pmatrix} m_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_1 \\ 0 & 0 & m_2 & 0 \\ 0 & m_3 & 0 & 0 \end{pmatrix}$$

where $m_j = z_j \oplus SNLF[z_j], j \in \{0, \ldots, 3\}.$

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• We can deduce z_j (two possibilities) from m_j which gives 2^4 possibilities for T^* .

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where $m_j = z_j \oplus SNLF[z_j], j \in \{0, \ldots, 3\}.$

We can deduce z_j(two possibilities) from m_j which gives 2⁴ possibilities for T^{*}.

3 Now, we can make hypotheses on 4 bytes of last round key k^{11} .

$$2 \cdot f' \oplus SNLF[2 \cdot f'] = S^{-1}[T_0 \oplus k_0^{11}] \oplus S^{-1}[T_0^* \oplus k_0^{11}]$$

$$1 \cdot f' \oplus SNLF[1 \cdot f'] = S^{-1}[T_{13} \oplus k_{13}^{11}] \oplus S^{-1}[T_{13}^* \oplus k_{13}^{11}]$$

$$1 \cdot f' \oplus SNLF[1 \cdot f'] = S^{-1}[T_{10} \oplus k_{10}^{11}] \oplus S^{-1}[T_{10}^* \oplus k_{10}^{11}]$$

$$3 \cdot f' \oplus SNLF[3 \cdot f'] = S^{-1}[T_7 \oplus k_7^{11}] \oplus S^{-1}[T_7^* \oplus k_7^{11}]$$
Complexity Analysis



2⁴ values of T^* gives $2^4 * 1036$ candidate values for 4 bytes of k^{11} .

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- **Q** 2^4 values of T^* gives $2^4 * 1036$ candidate values for 4 bytes of k^{11} .
- Repeating the attack with another pair of faulty and correct ciphertext gives atmost 2 candidate values.
- Solution Total 8 faulty ciphertexts required to retrieve all 16 bytes of k^{11} .



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Number of random dummy rounds : d

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- **3** Target round of fault injection : $(22 + d 2)^{th}$ RoundFunction.
- $(22+d)^{th}$ RoundFunction: 10^{th} cipher round.
- S ∴ The probability of $(22 + d 2)^{th}$ RoundFunction being a 9th cipher round: $\frac{(19+d)!/((19)!\cdot(d)!)}{(21+d)!/((21)!\cdot(d)!)}$

- Number of random dummy rounds : d
- 2 Total number of rounds : 22 + d + 1
- **③** Target round of fault injection : $(22 + d 2)^{th}$ RoundFunction.
- $(22+d)^{th}$ RoundFunction: 10^{th} cipher round.
- Solution The probability of (22 + d − 2)th RoundFunction being a 9th cipher round: (19+d)!/((19)! · (d)!)/((21+d)!/((21)! · (d)!))
- If d = 20 then the probability that 40^{th} RoundFunction is a 9^{th} cipher round is nearly 0.26.

Simulation Results



• The last cipher round is always the penultimate round: The attacker can verify target round using side channel.

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- A fault in last three rows of 10^{th} round \implies Infection caused by compulsory dummy round does not affect the erroneous byte.

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- A fault in last three rows of 10^{th} round \implies Infection caused by compulsory dummy round does not affect the erroneous byte.
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- The effect of infection varies for different rounds.

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- Fault injection in any of the cipher, redundant or dummy round ⇒ Every byte in the resulting ciphertext is infected with a different value.
- ② The resulting infected faulty ciphertext is completely random.
- Image of the second second second and the second second
- The improved countermeasure protects both SPN ciphers and Feistel ciphers.

Summary & Conclusion

The infection mechanism of LatinCrypt 2012 countermeasure is shown to be ineffective.

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Summary & Conclusion

- The infection mechanism of LatinCrypt 2012 countermeasure is shown to be ineffective.
- An improved countermeasure is developed, which outputs a completely random value in case of fault injection so that fault attack is impossible.

Thank You !

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