Very-Efficient Simulatable Flipping of Many Coins into-a-Well

(and a New Universally-Composable Commitment Scheme)

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PORTUGAL

Roadmap

1. **Simulatable coin-flipping and commitments**

2. **Protocol #1: coin-flipping (simulatable with rewinding)**

3. **Protocol #2: UC Commitment Scheme**

4. **Open questions / research directions**

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An ideal coin-flipping

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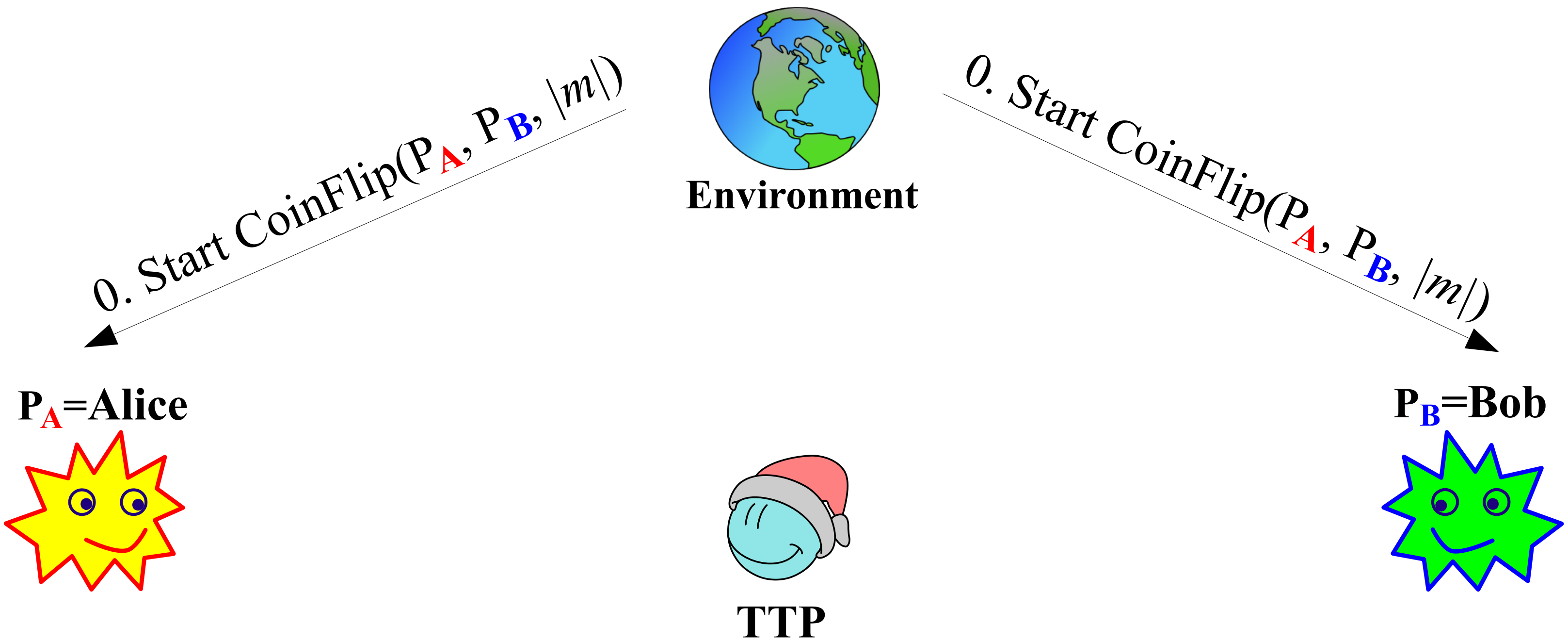
TTP = trusted third party



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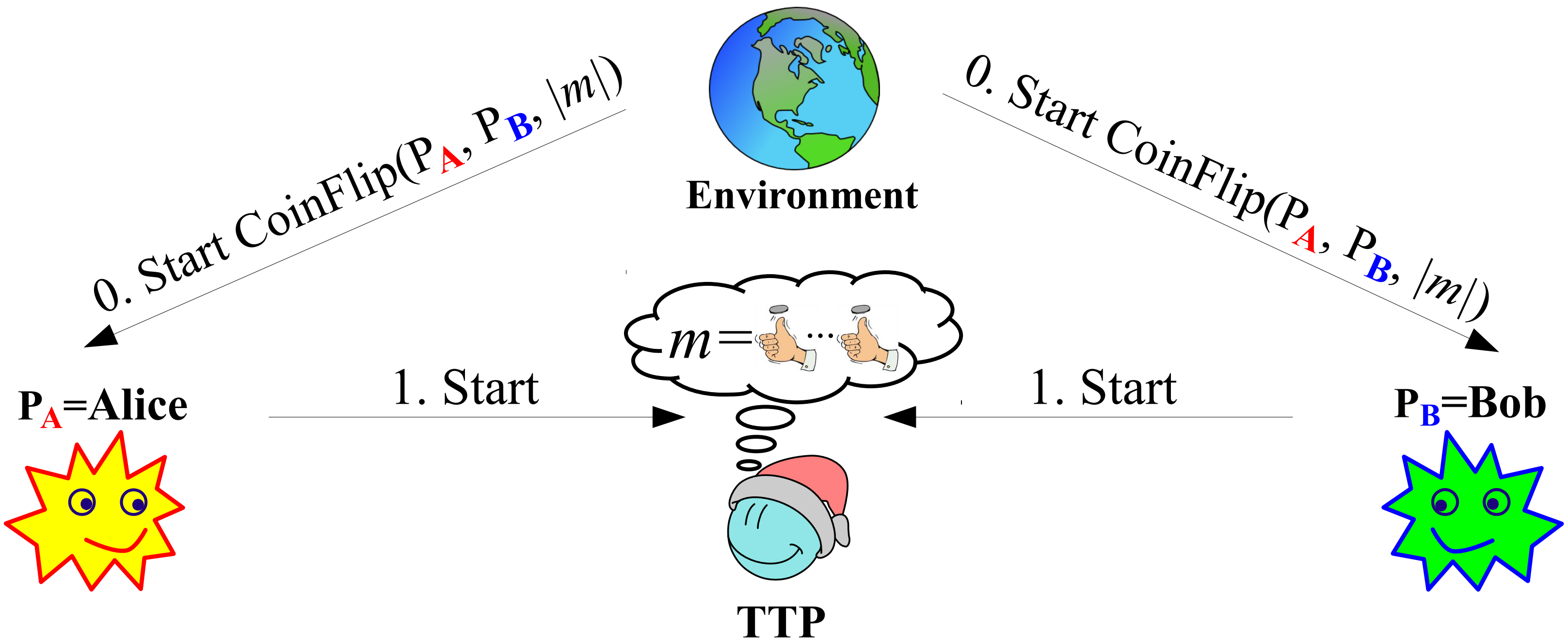

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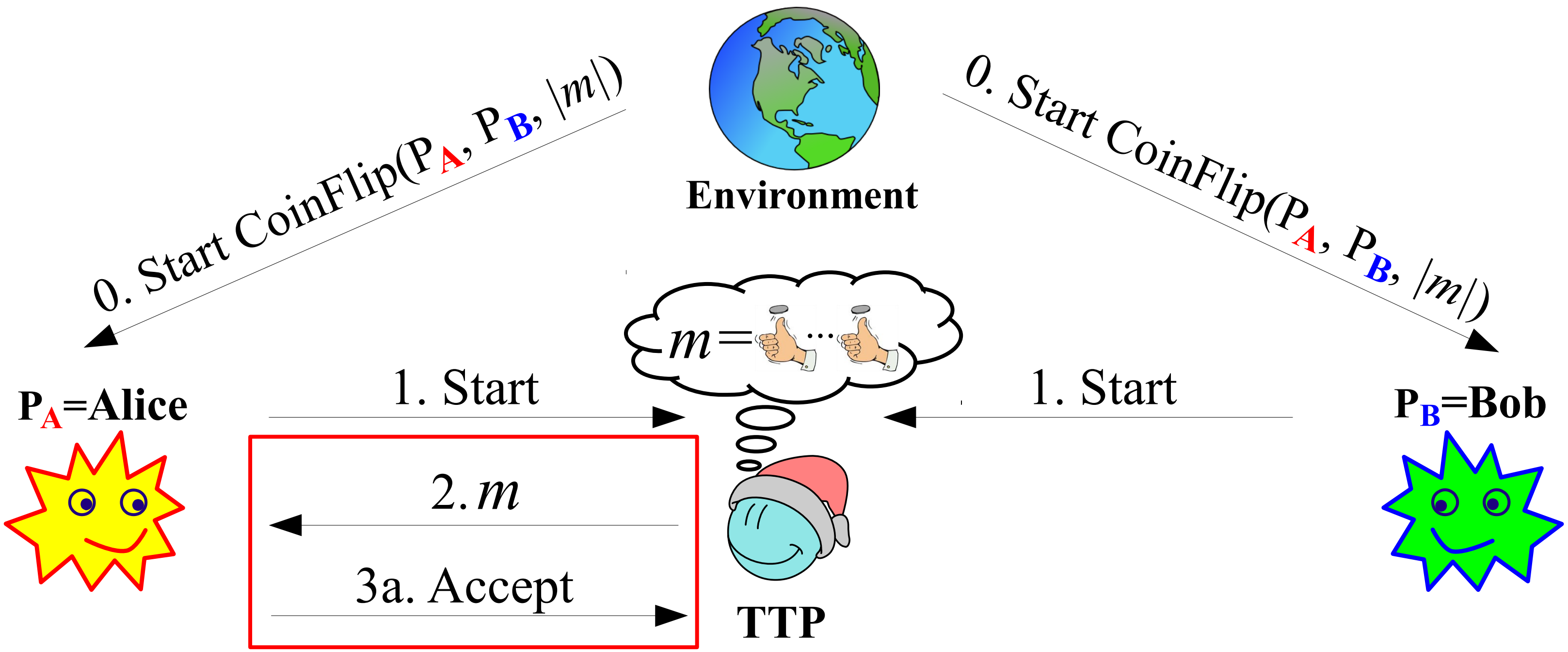

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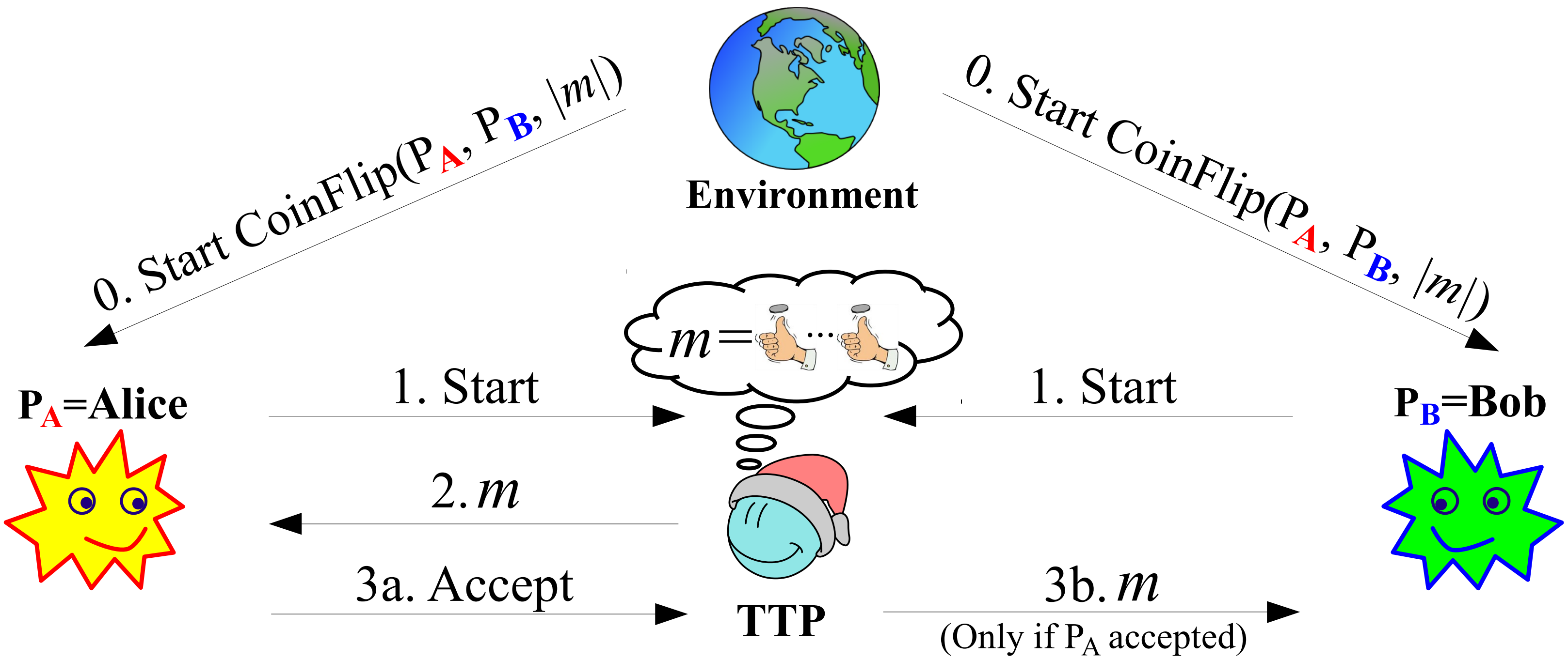

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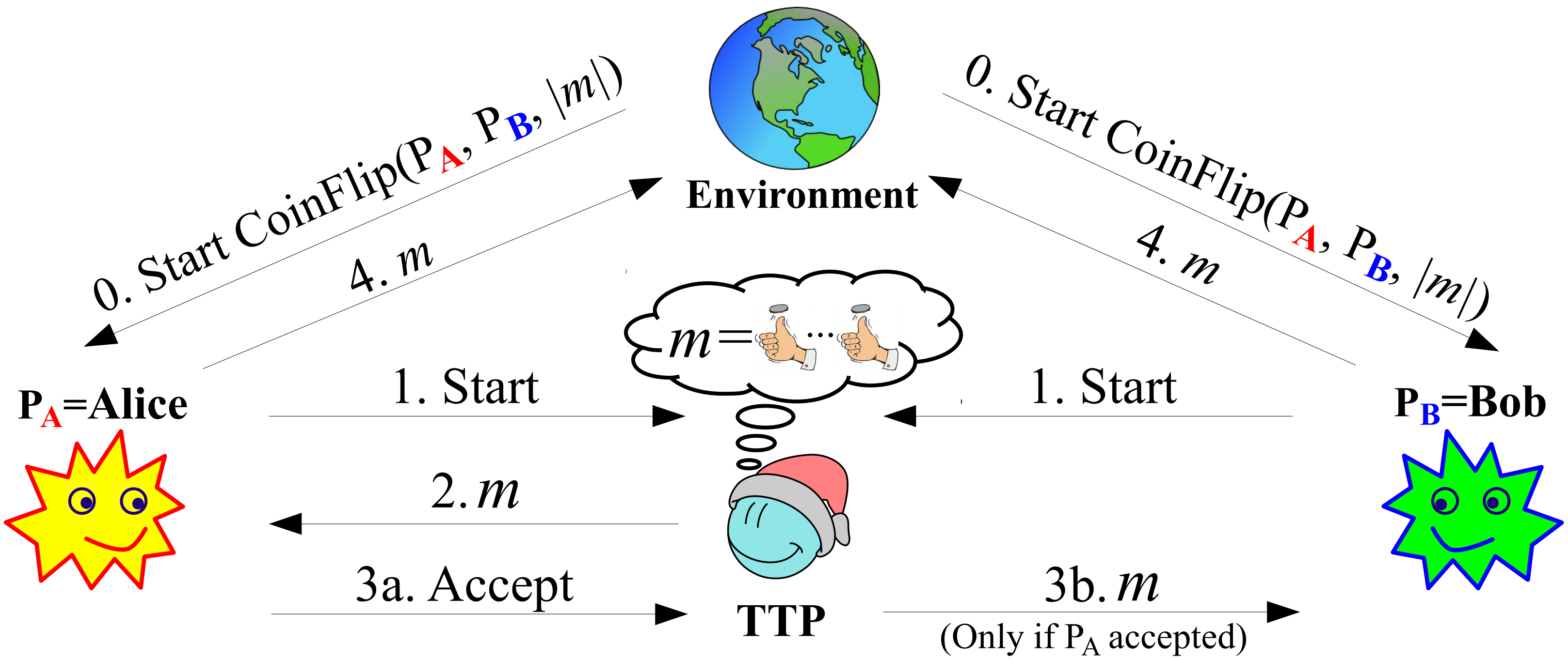

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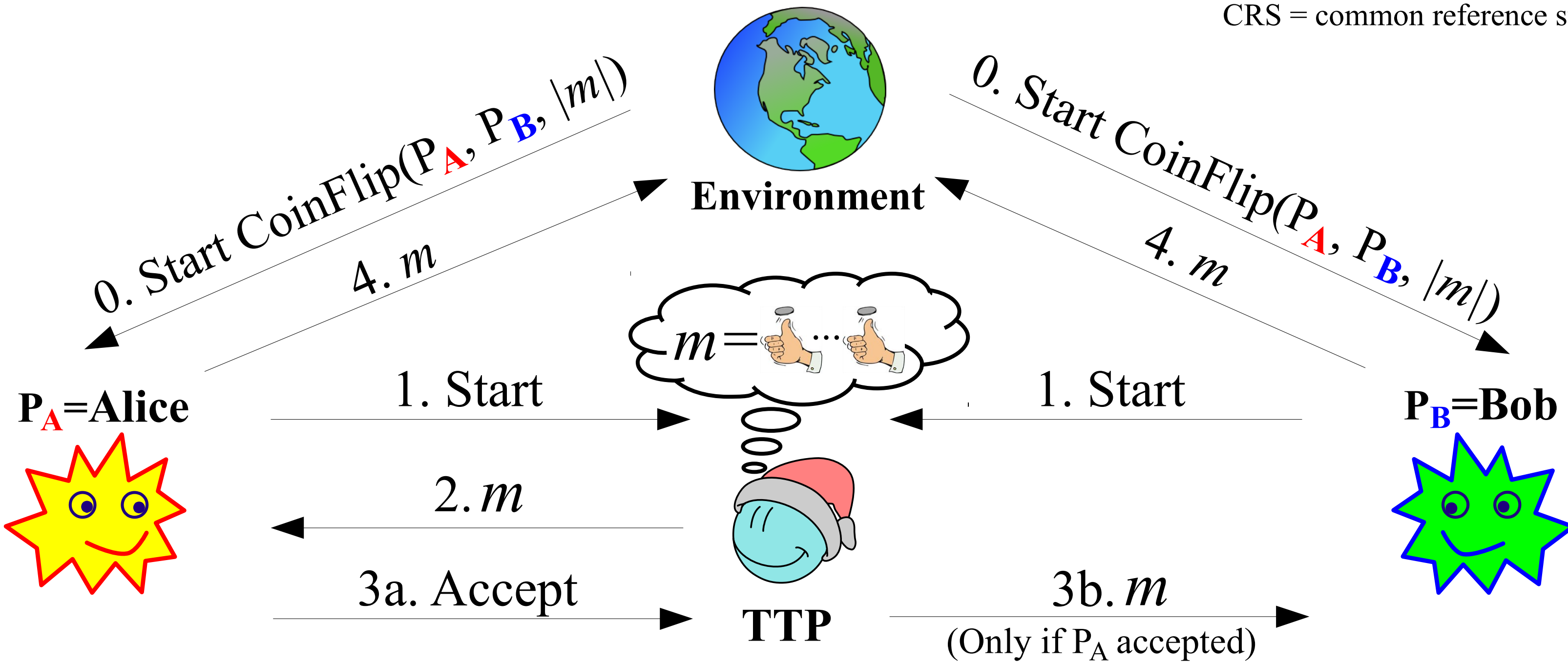

Example motivations

- Real world decisions (e.g., who gets the car? [Blum83])
- Enable probabilistic output of external two-party protocol
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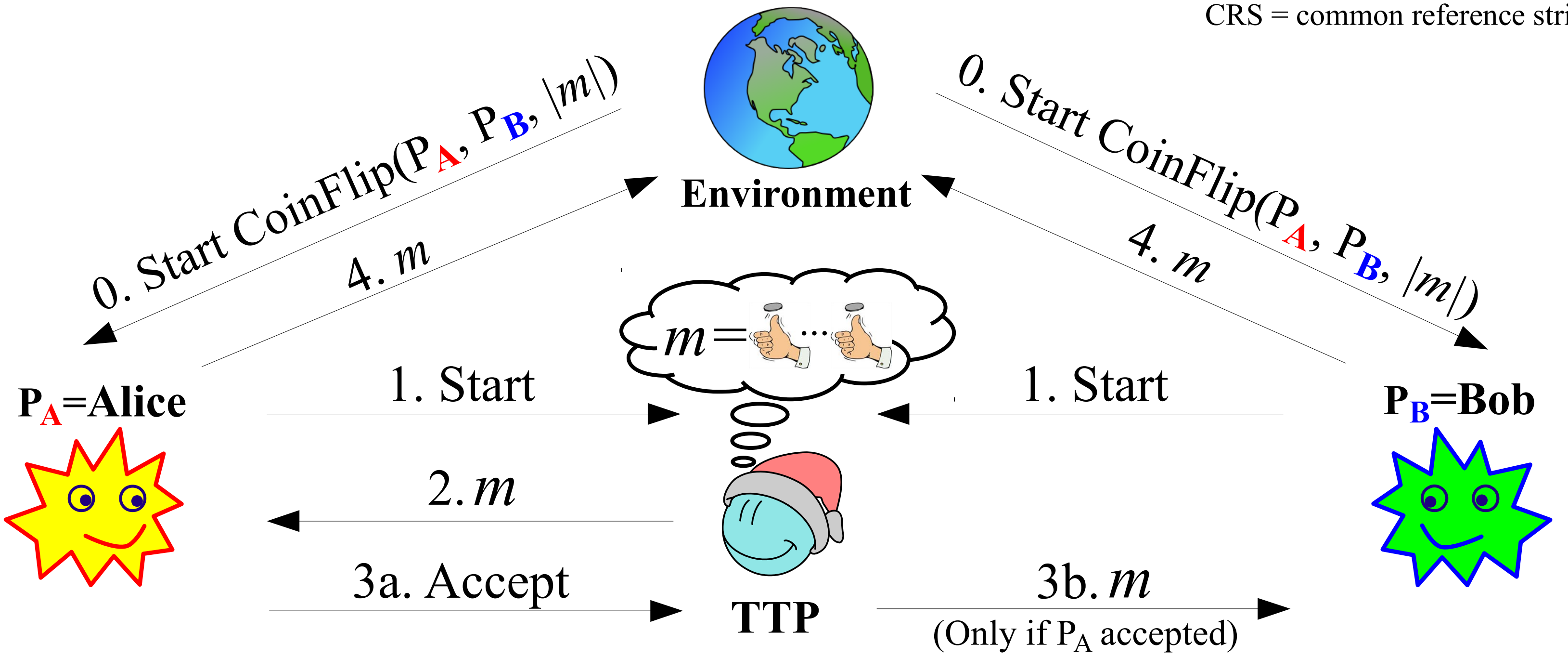

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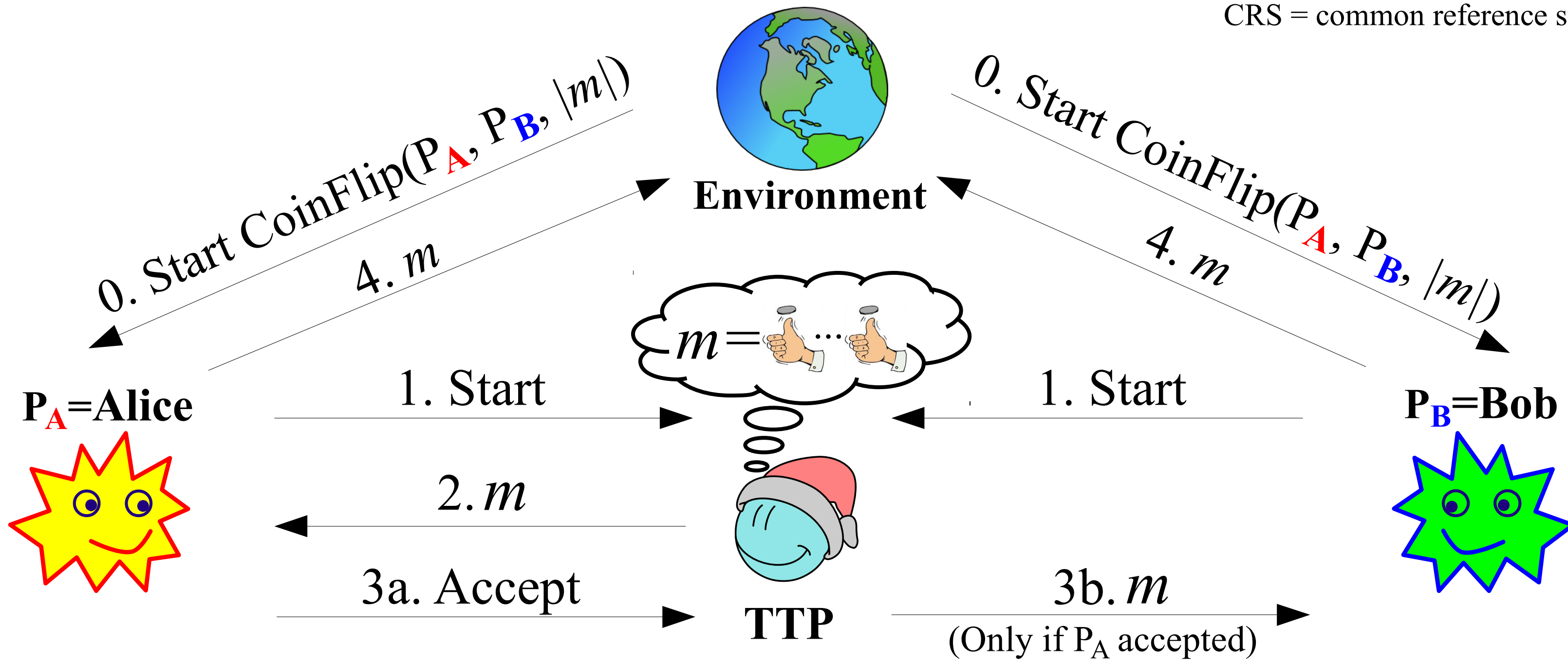

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(Adversarial model: static, malicious, computational)

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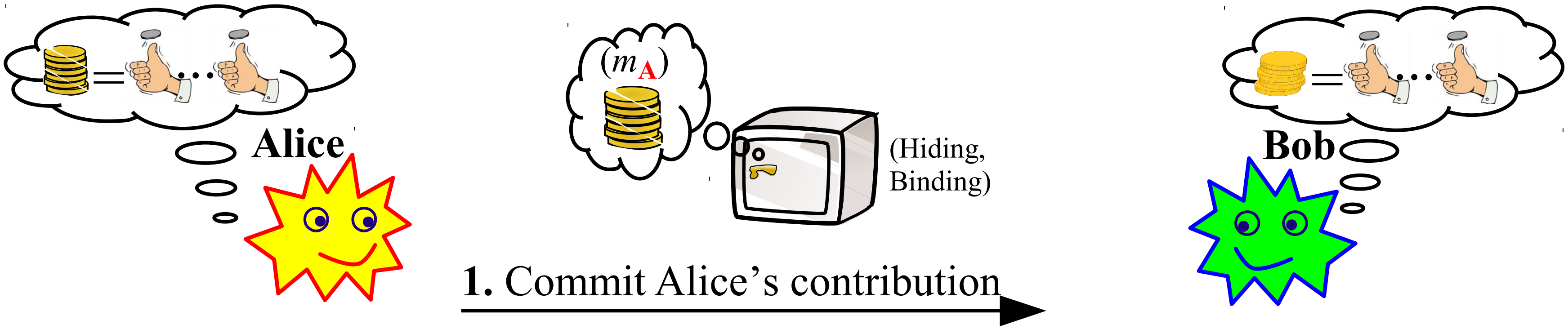

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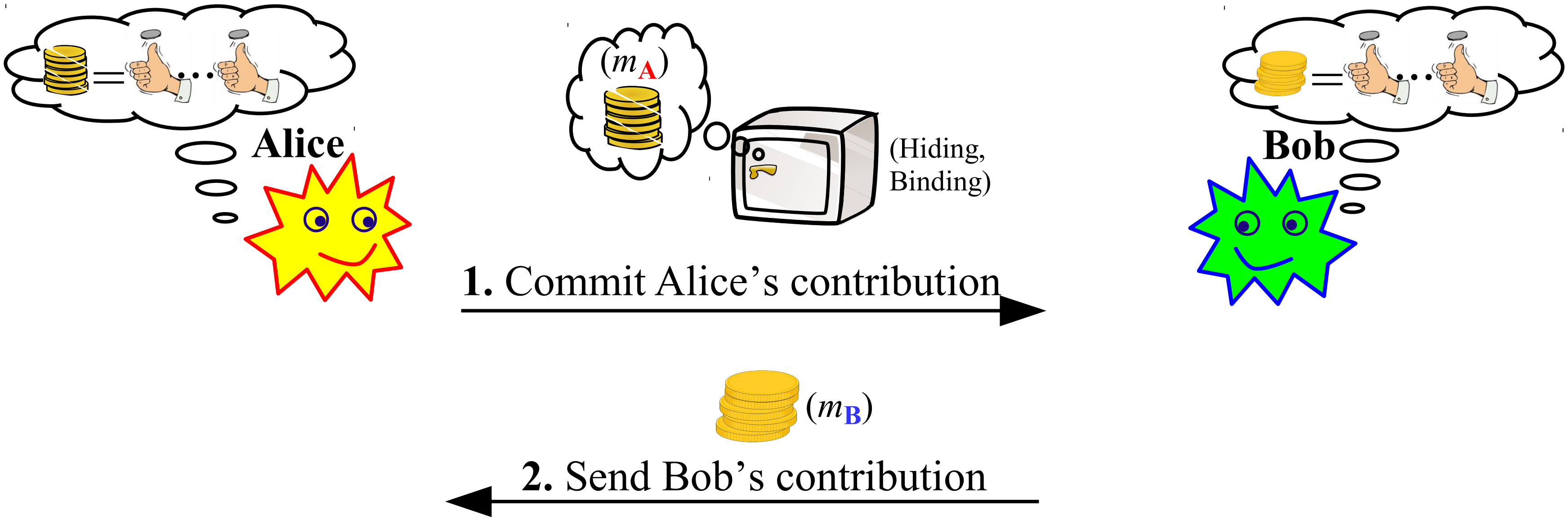

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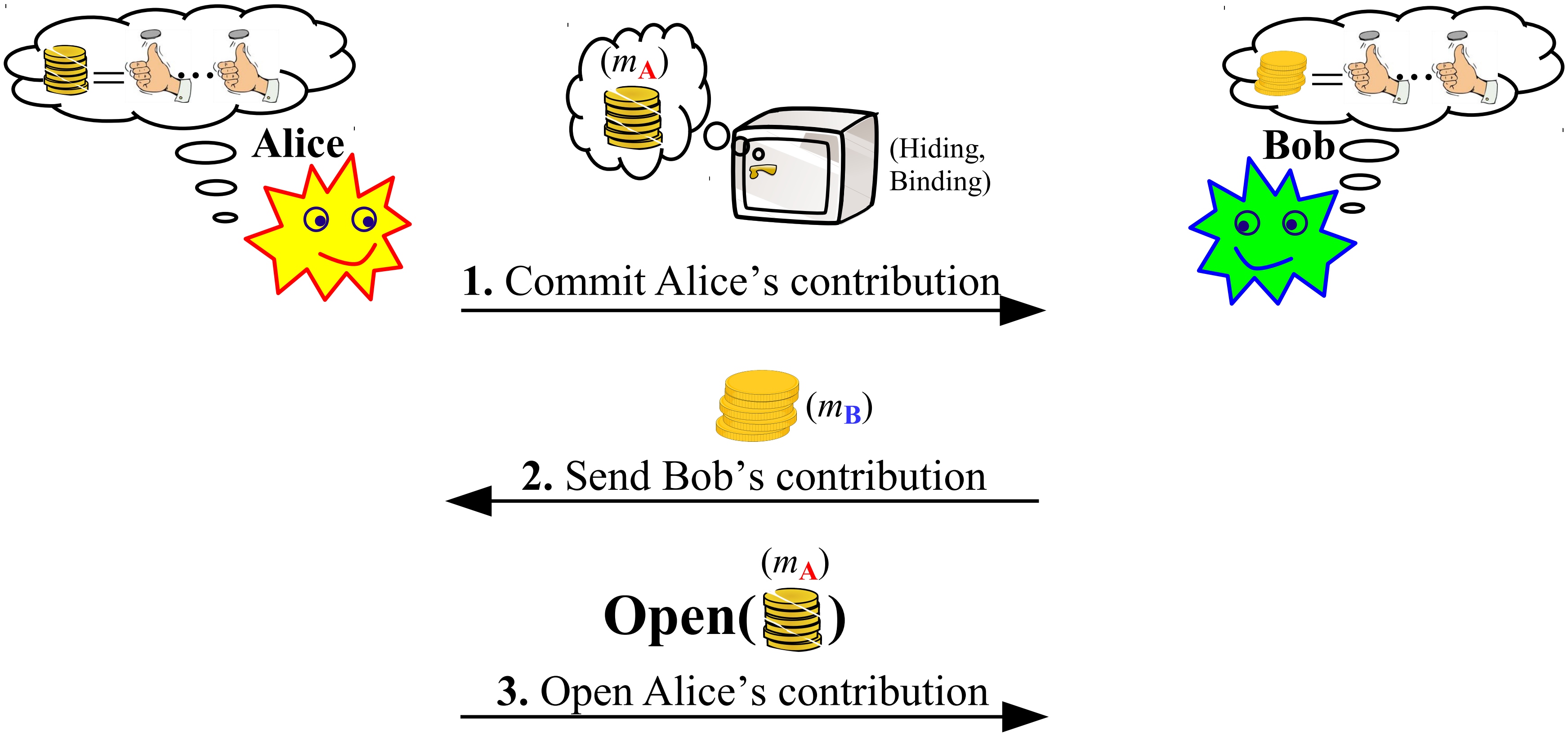

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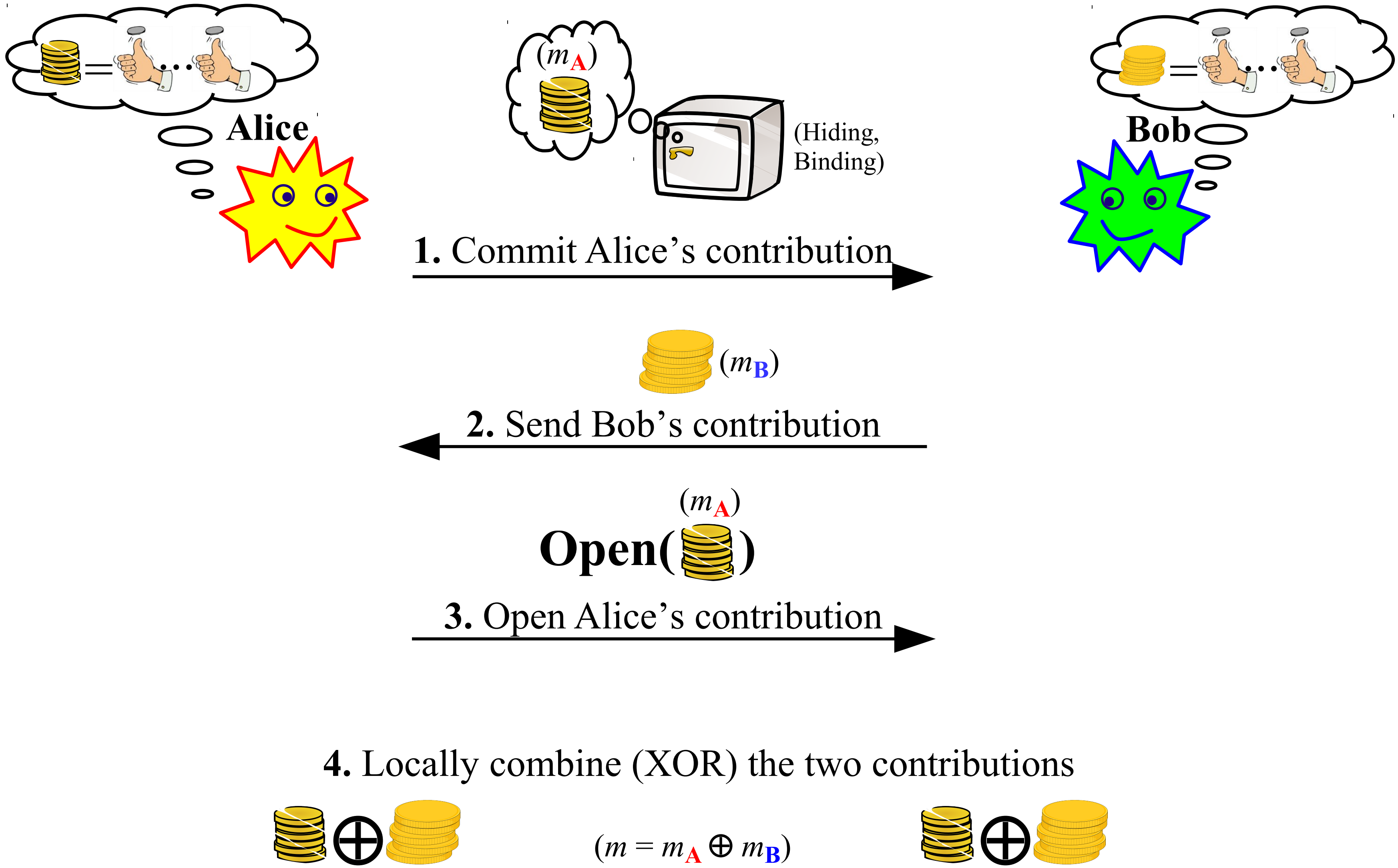

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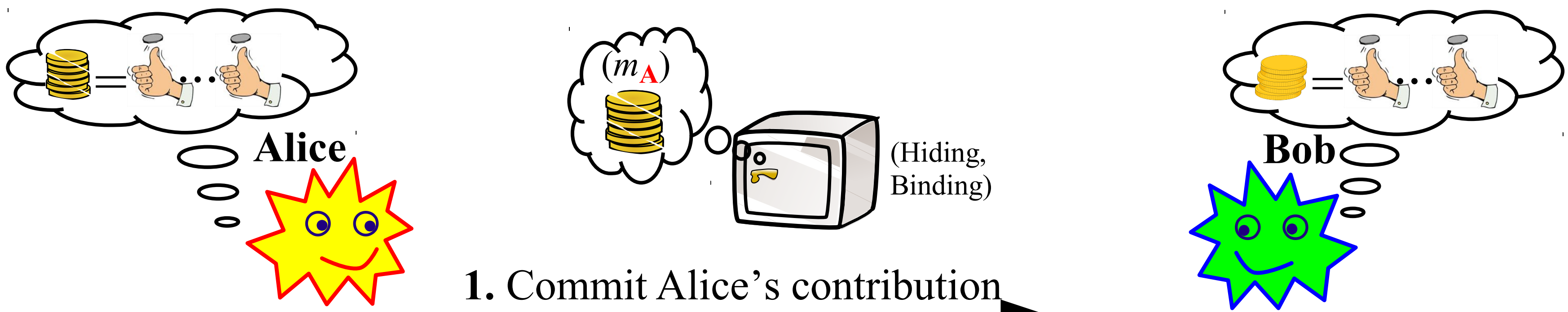


1. Commit Alice's contribution →

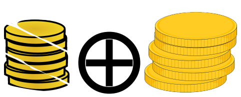
← **2.** Send Bob's contribution

(m_A)

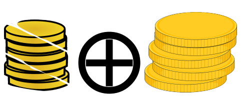
Open(**)**

3. Open Alice's contribution →

4. Locally combine (XOR) the two contributions



$(m = m_A \oplus m_B)$



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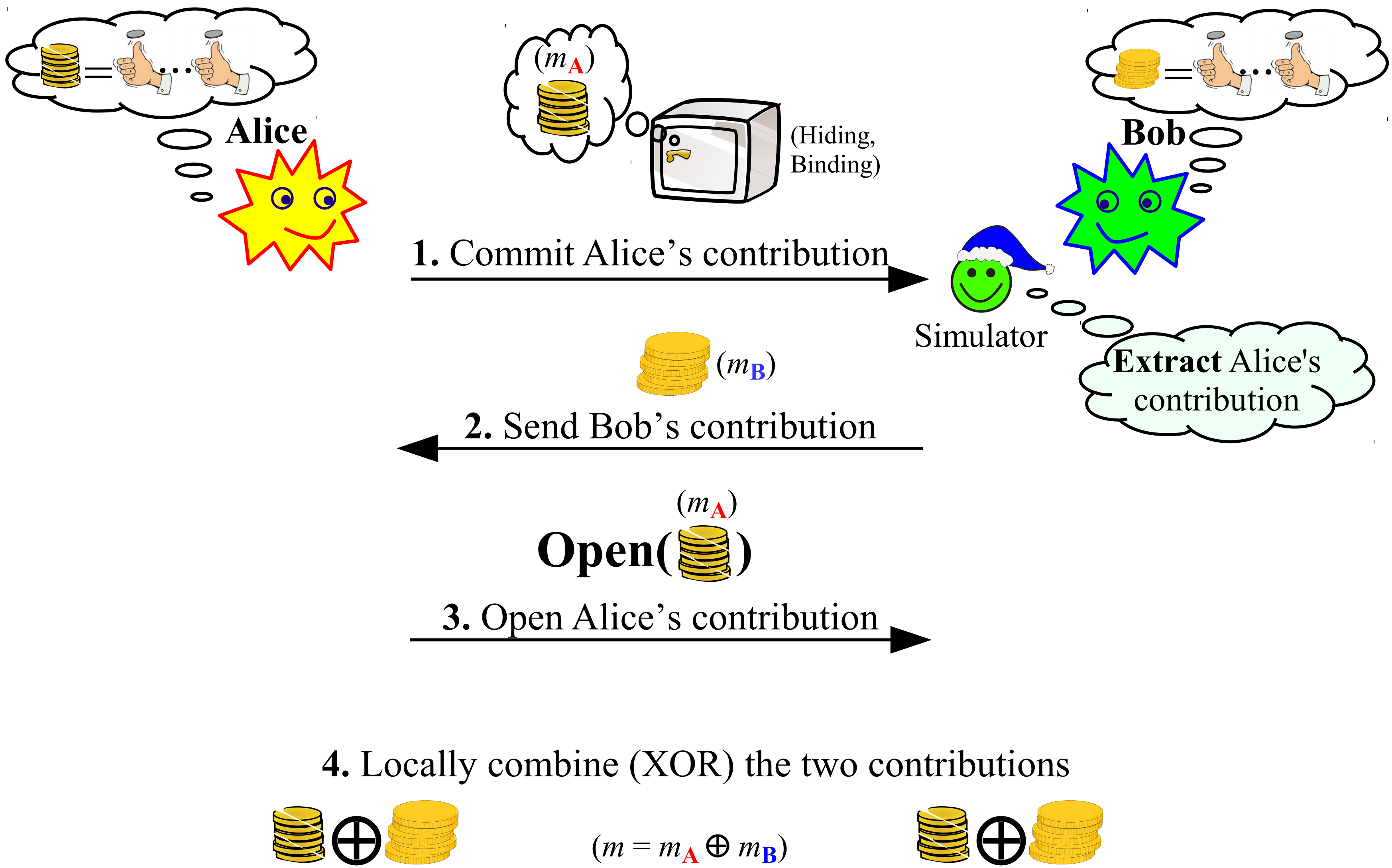

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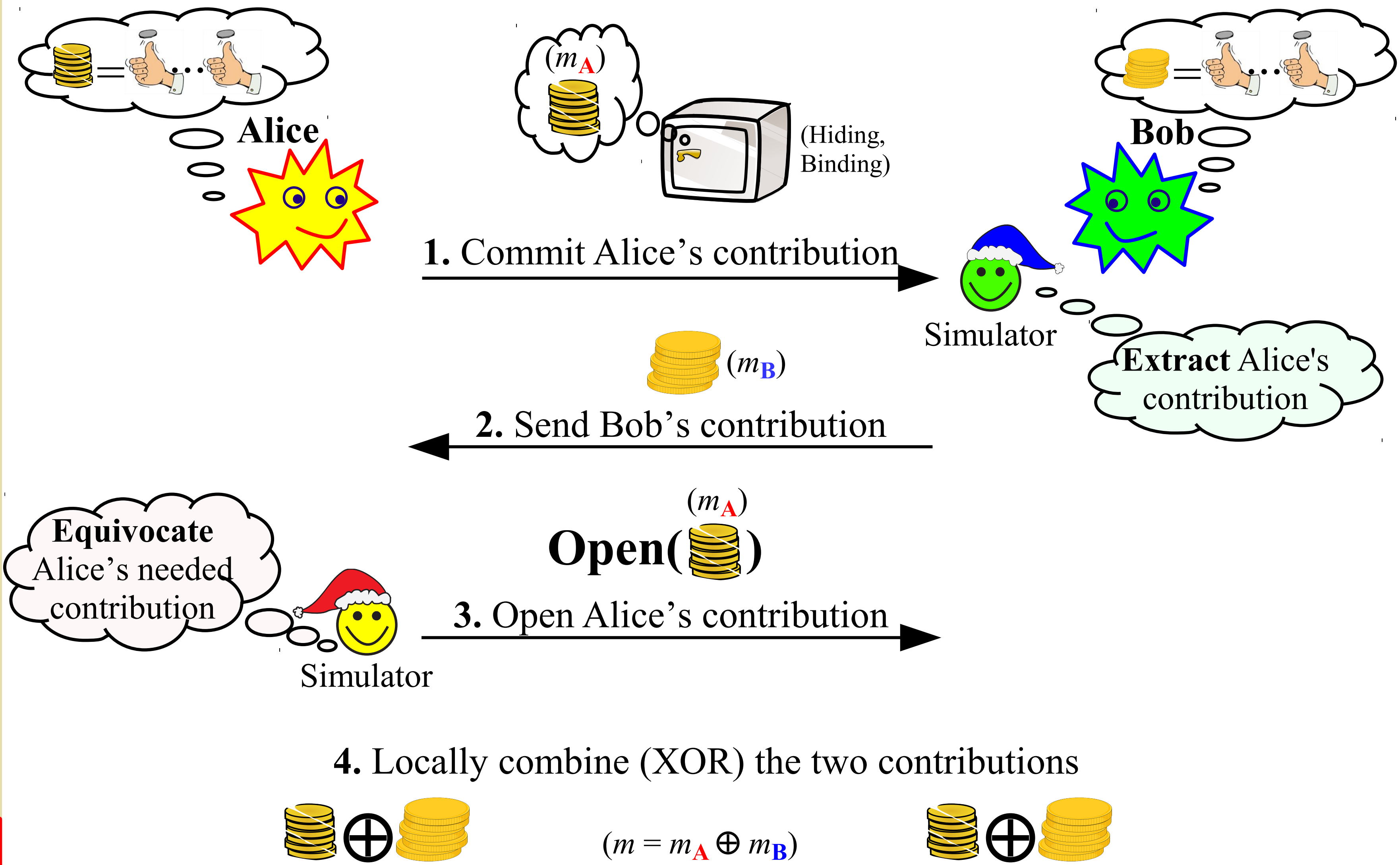

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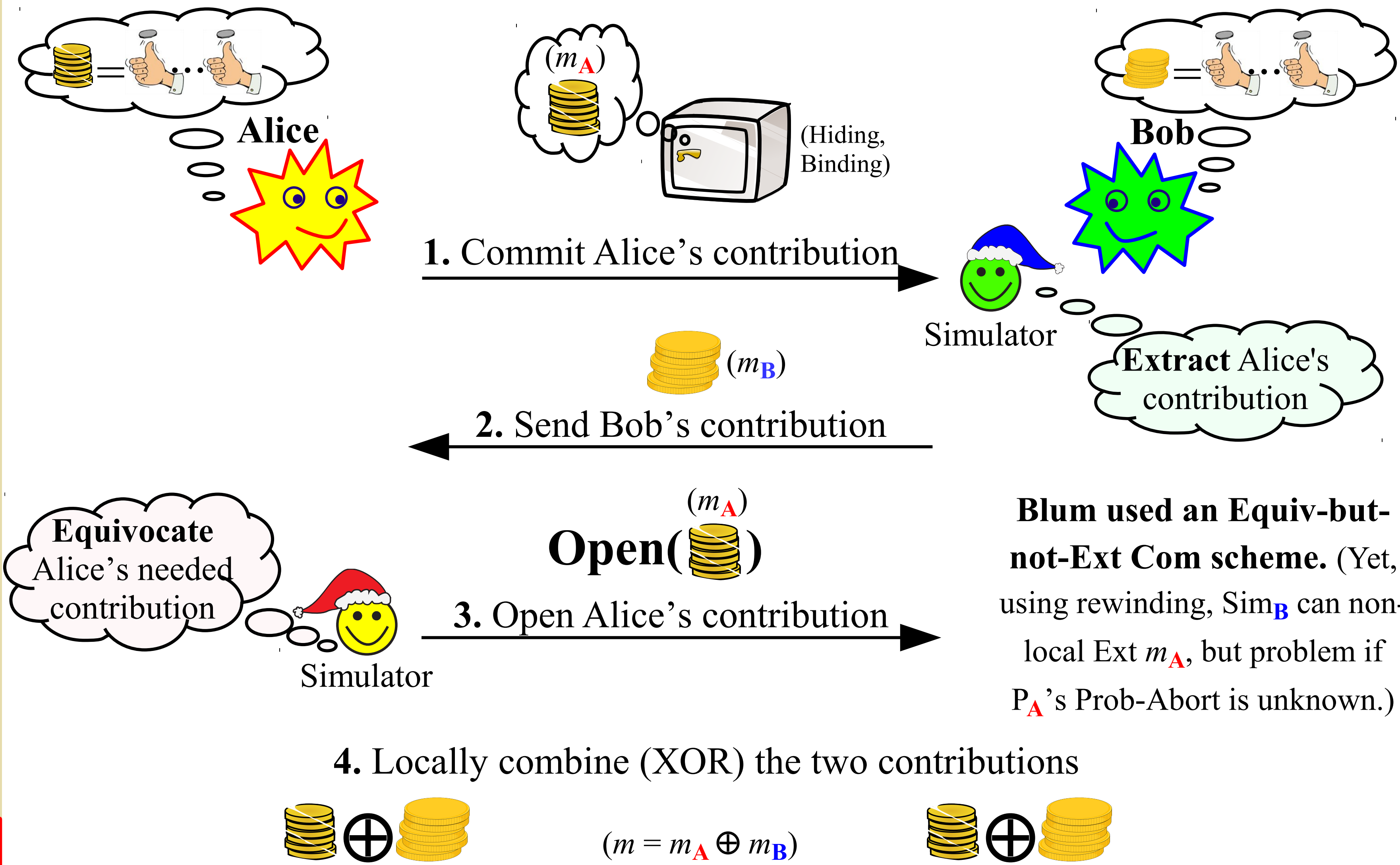

Blum used an Equiv-but-not-Ext Com scheme. (Yet, using rewinding, Sim_B can non-local Ext m_A, but problem if P_A's Prob-Abort is unknown.)

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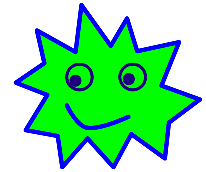
Receiver



Commit phase:

Open phase:

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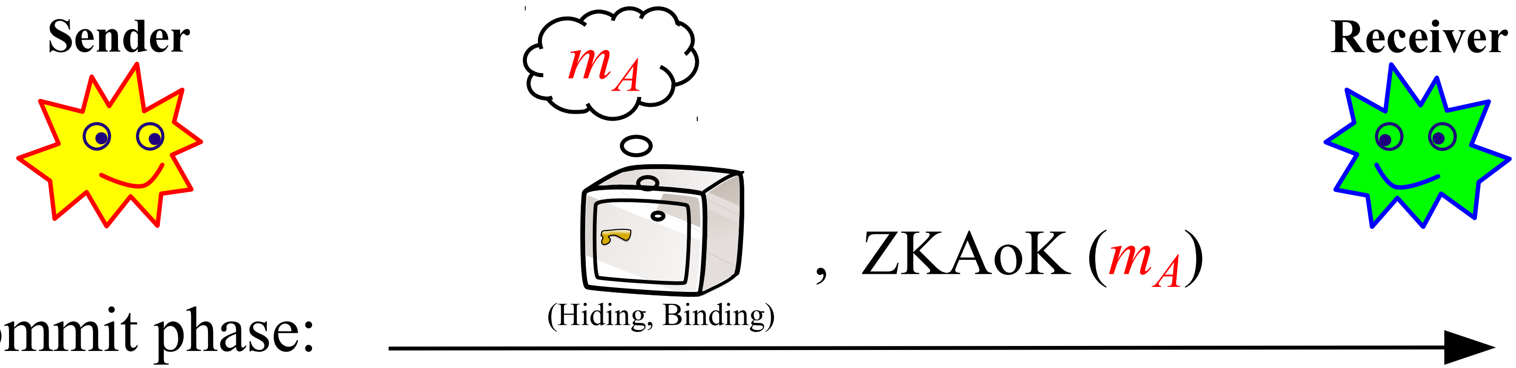

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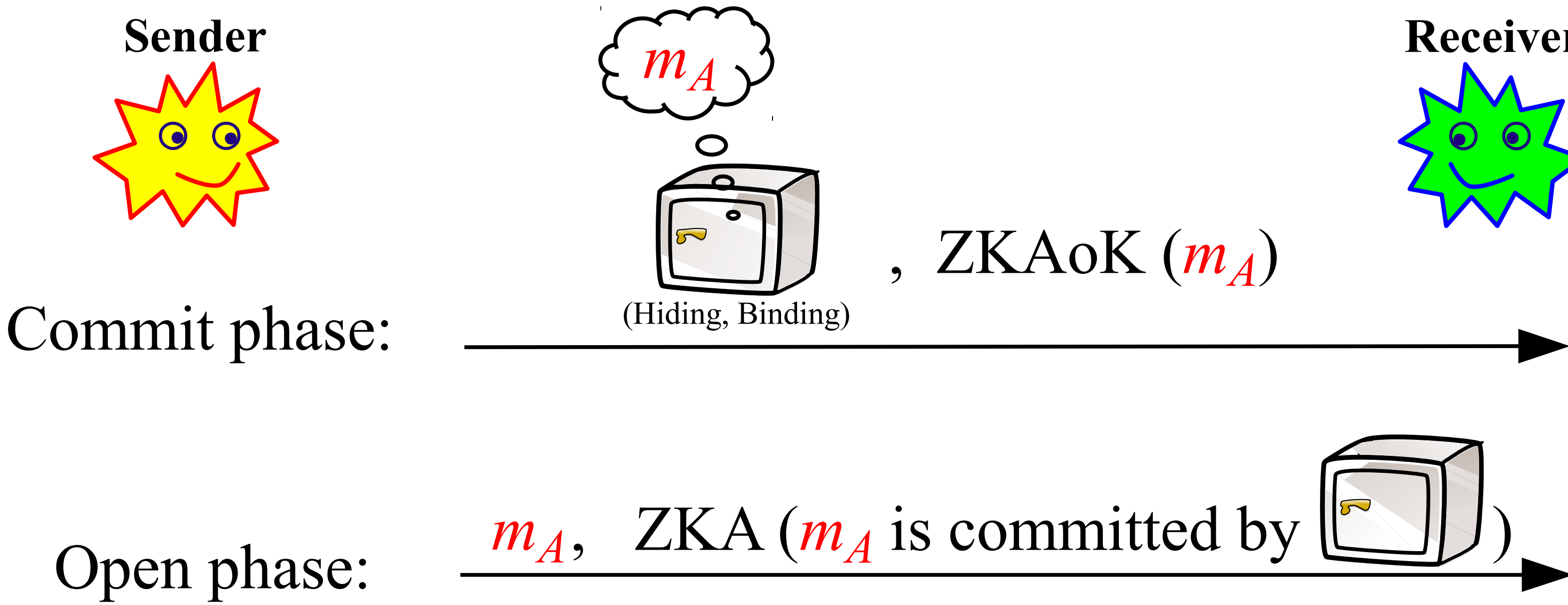

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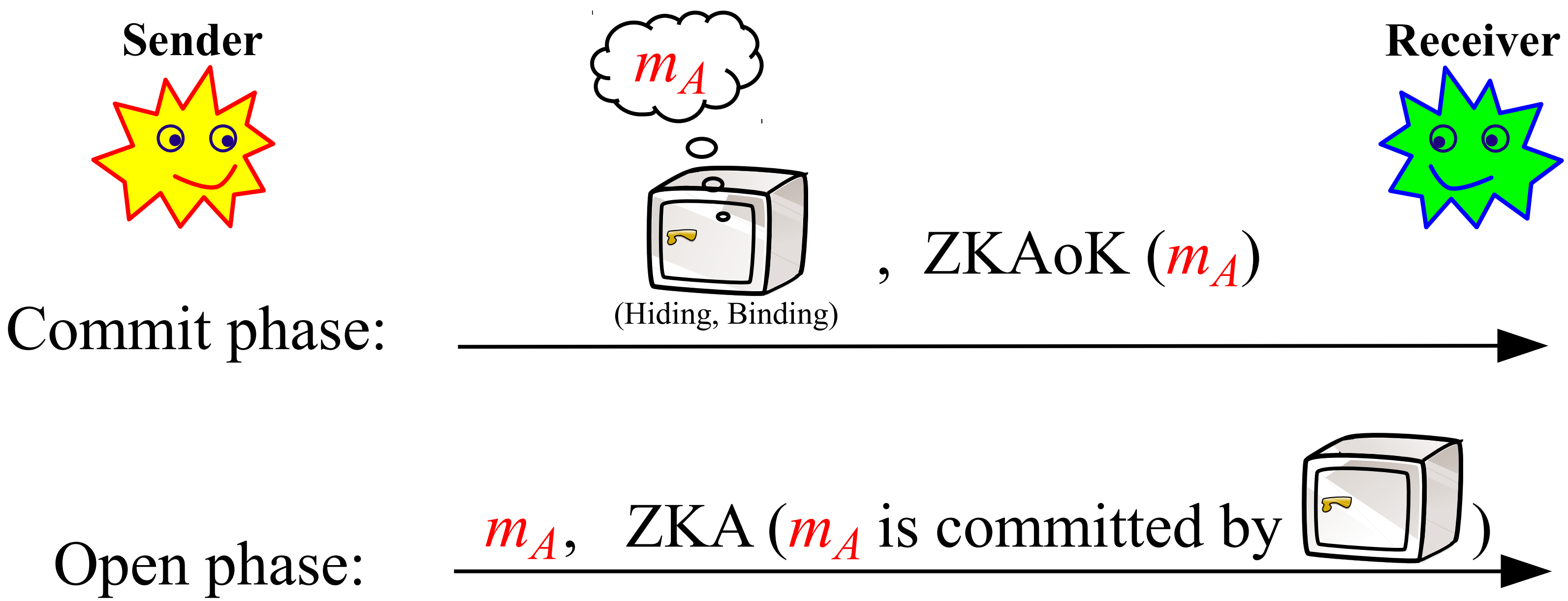

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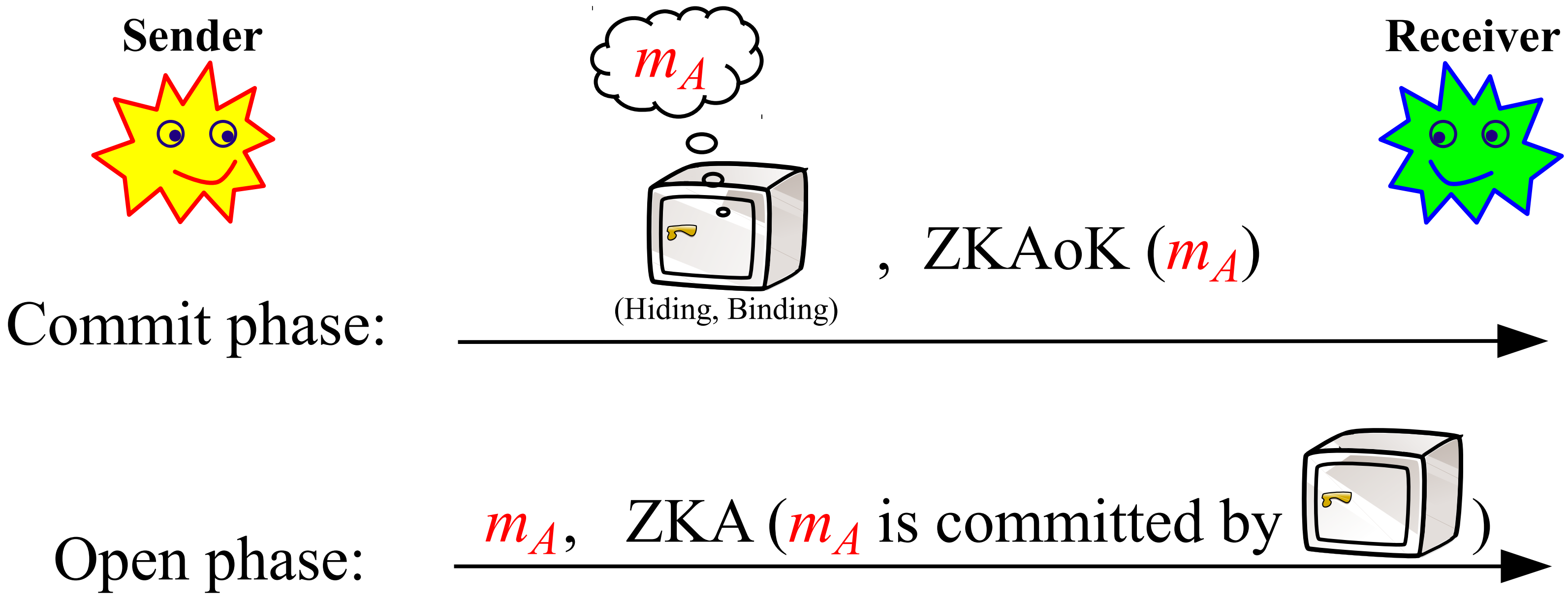

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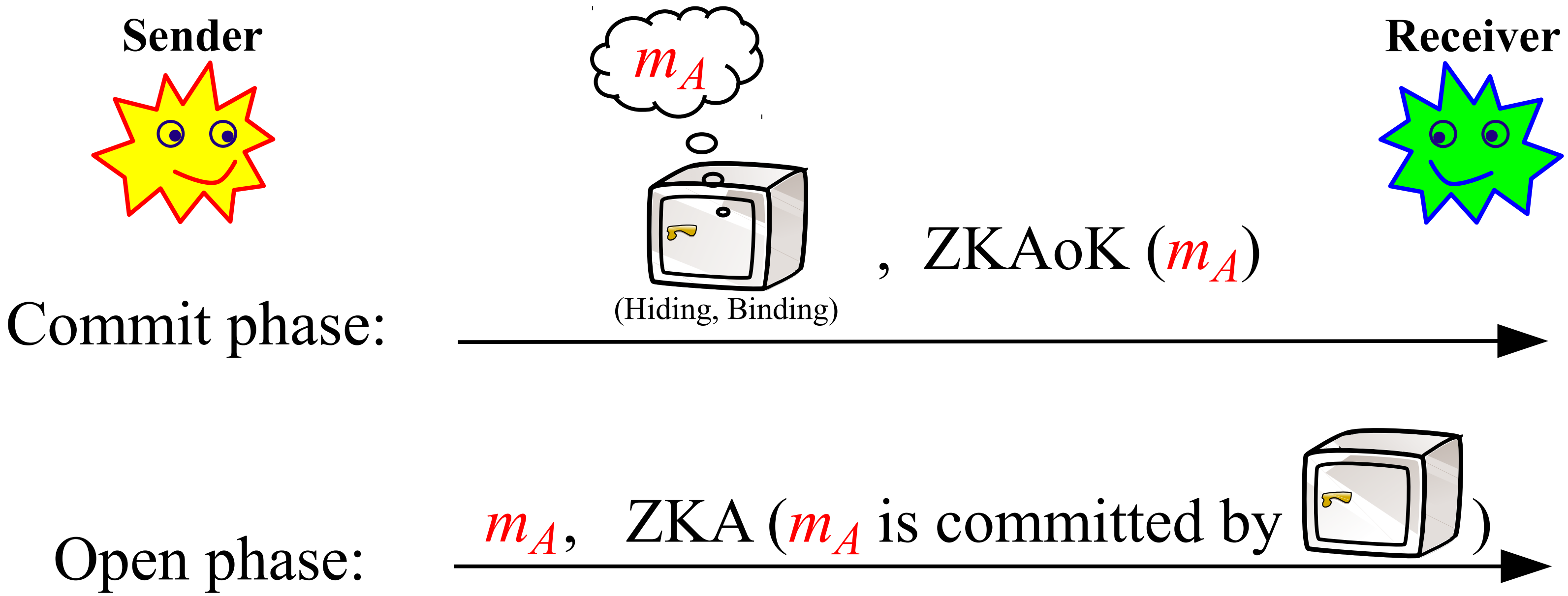

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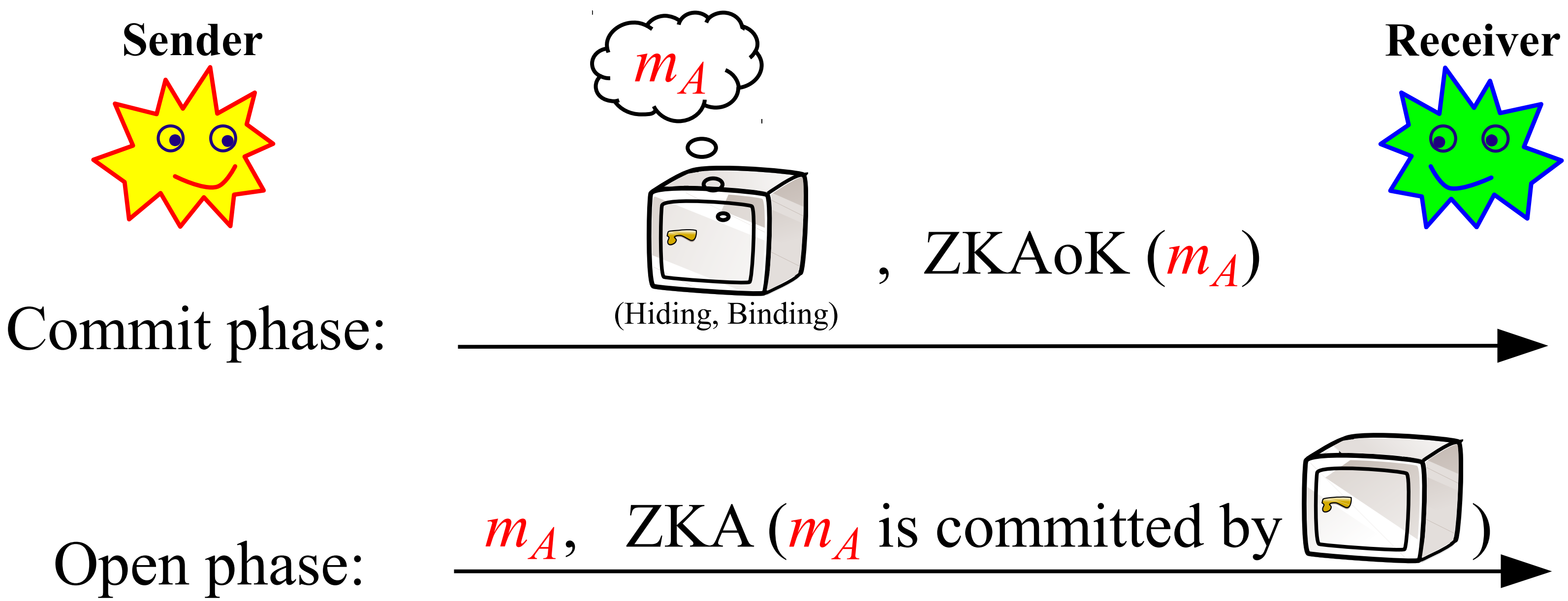

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Note: [Lin03] actually uses this construction in the scope of a more general coin-flipping into a well, where P_A only learns $f(m_A \oplus m_B)$.

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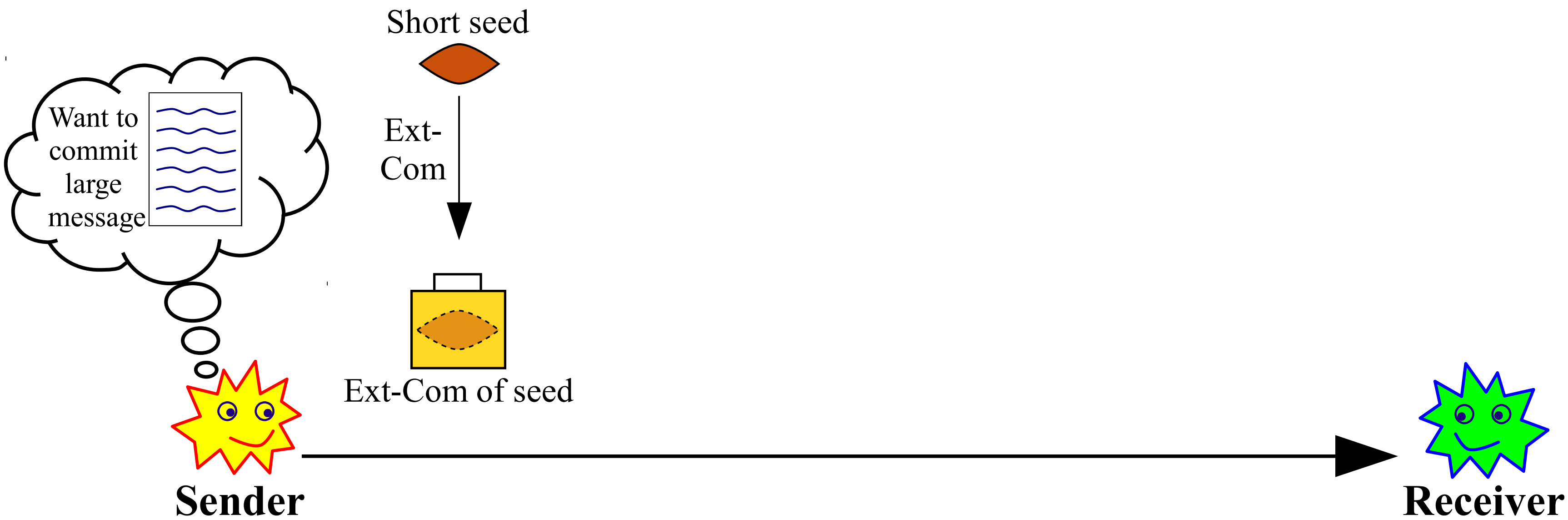

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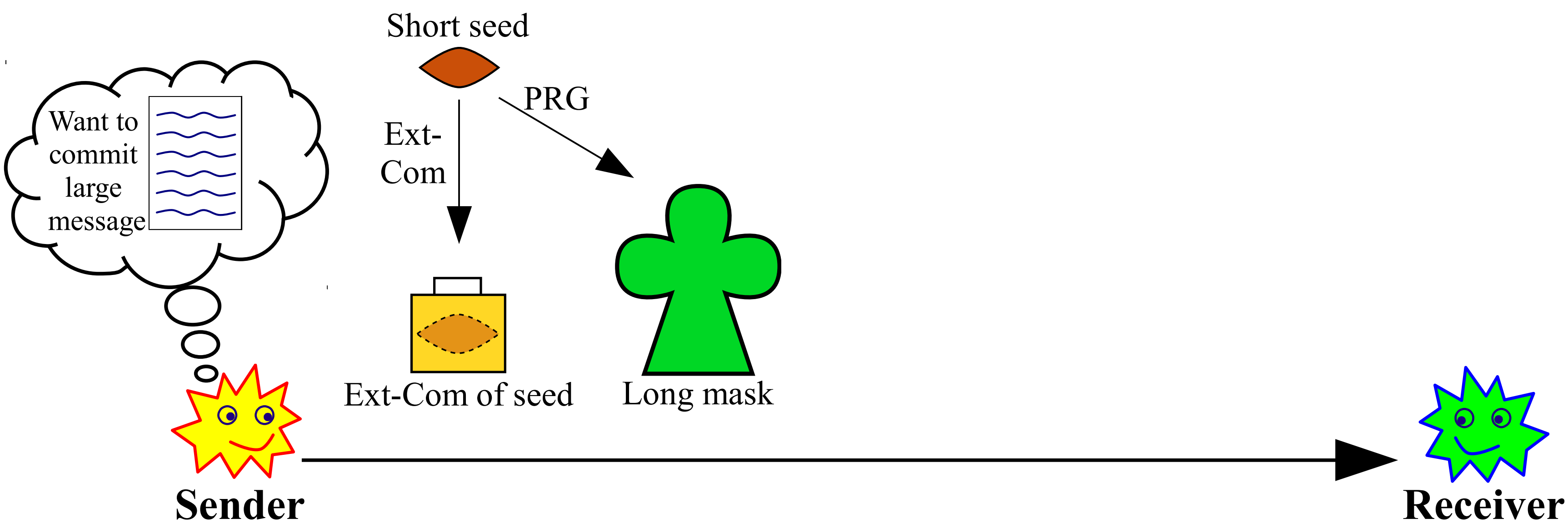

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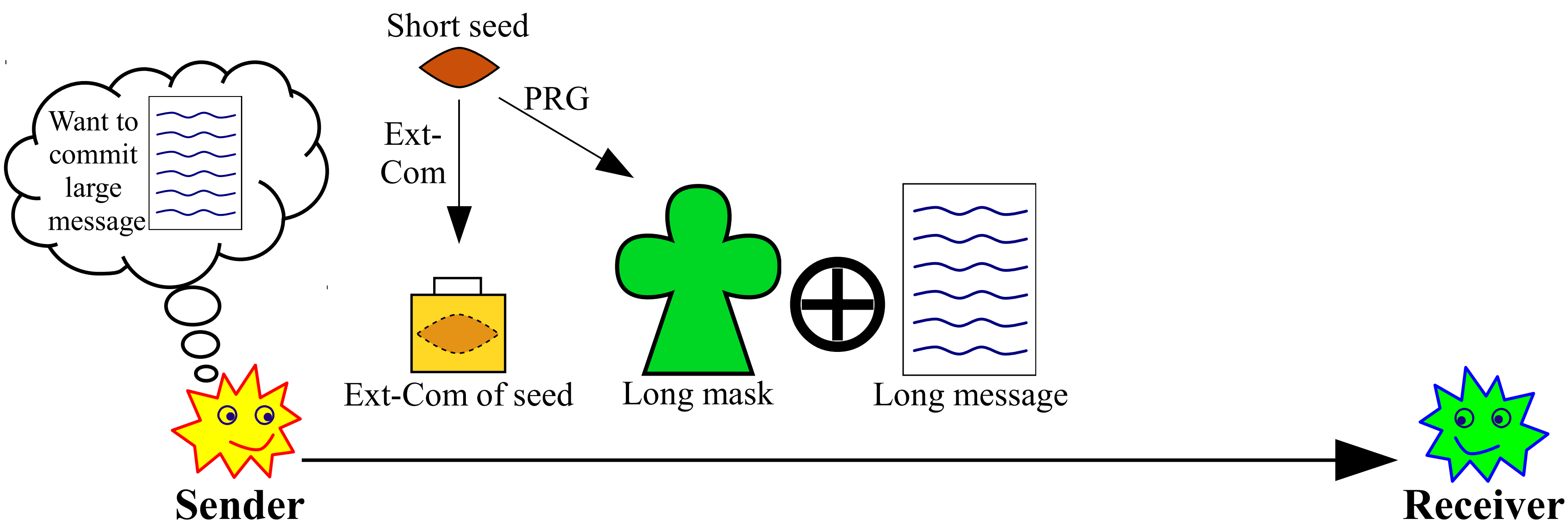

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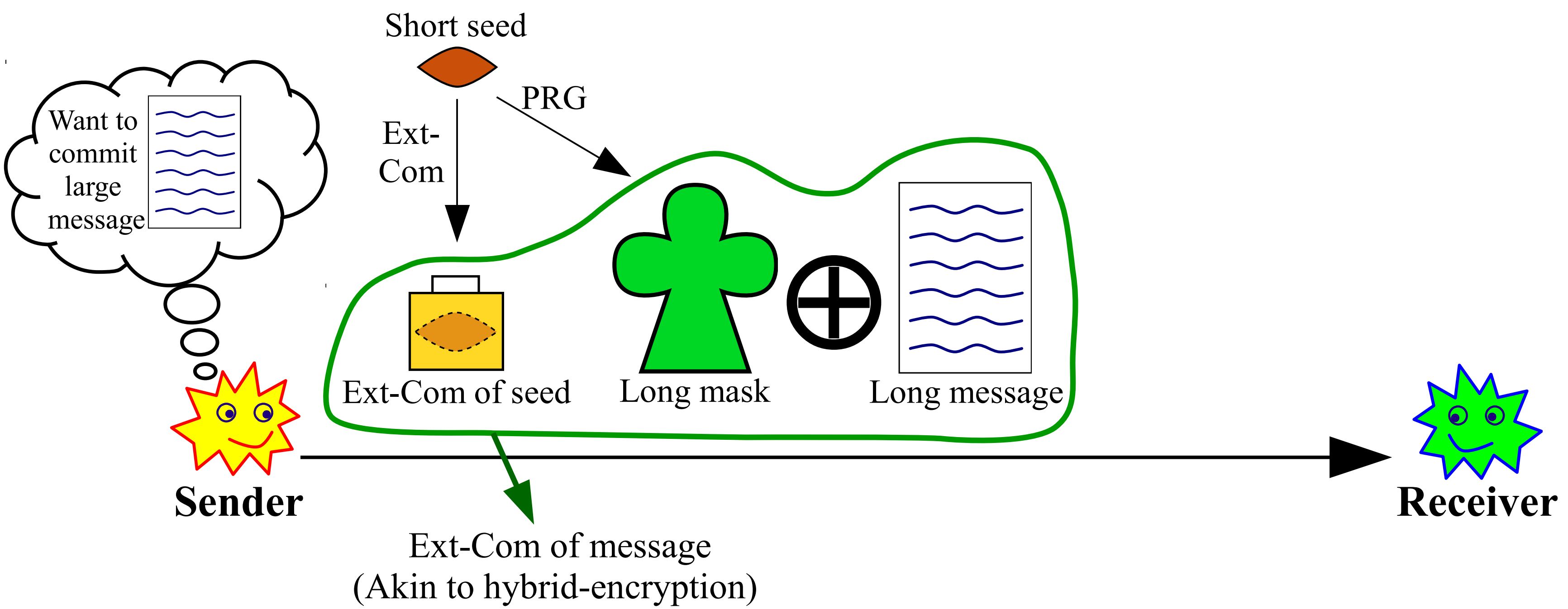

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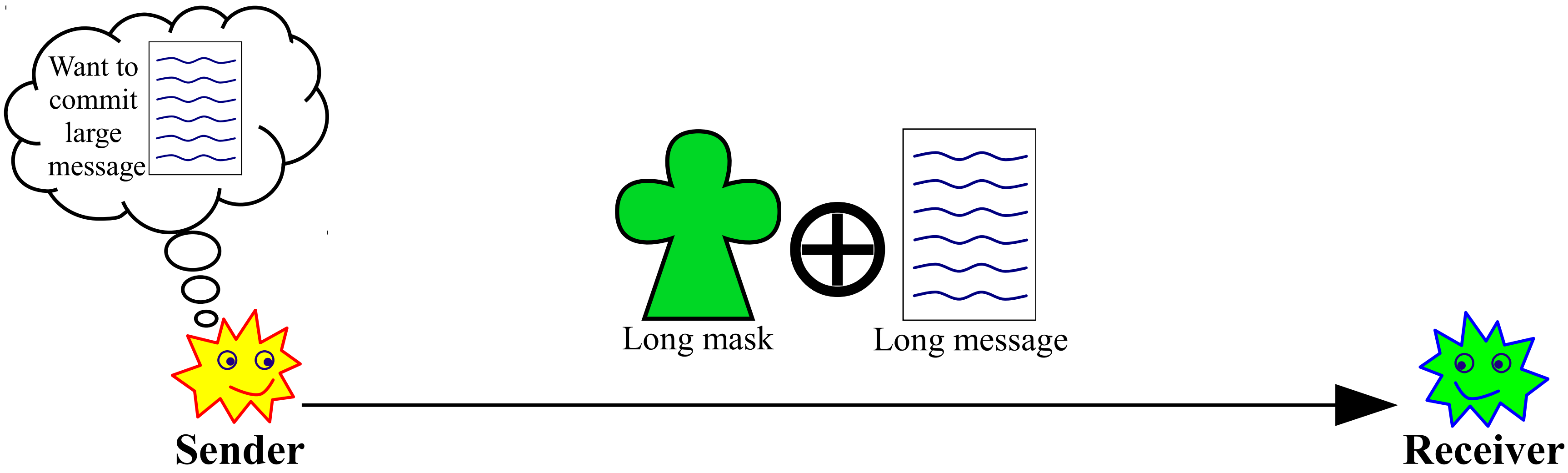

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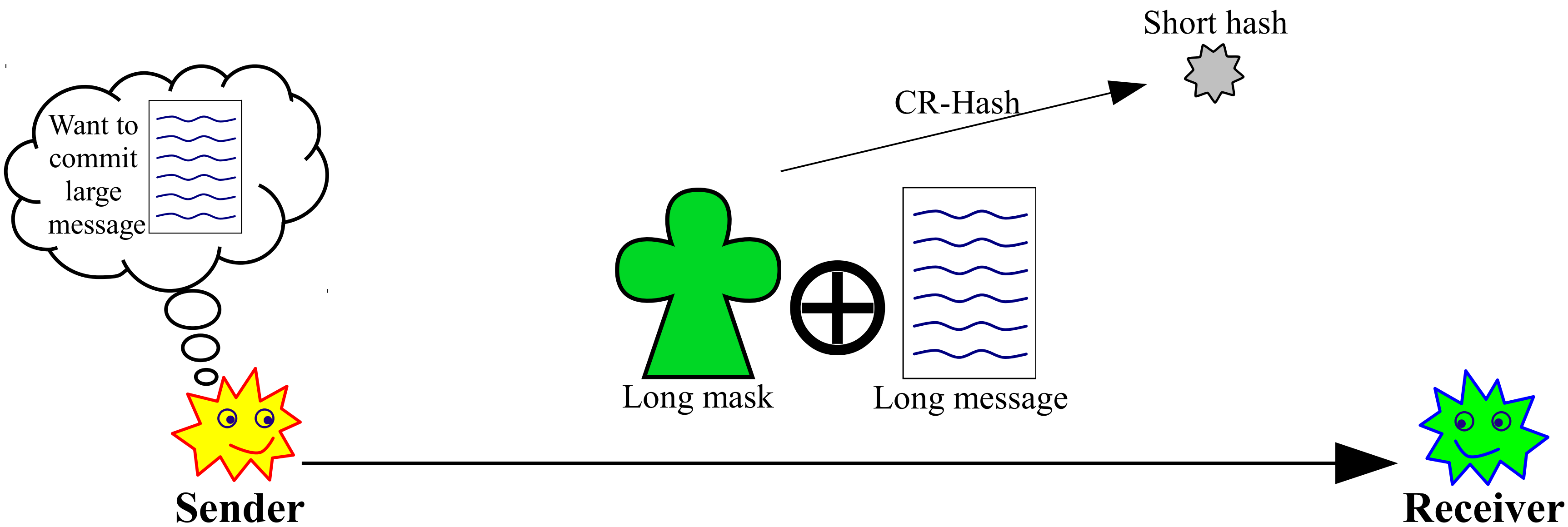

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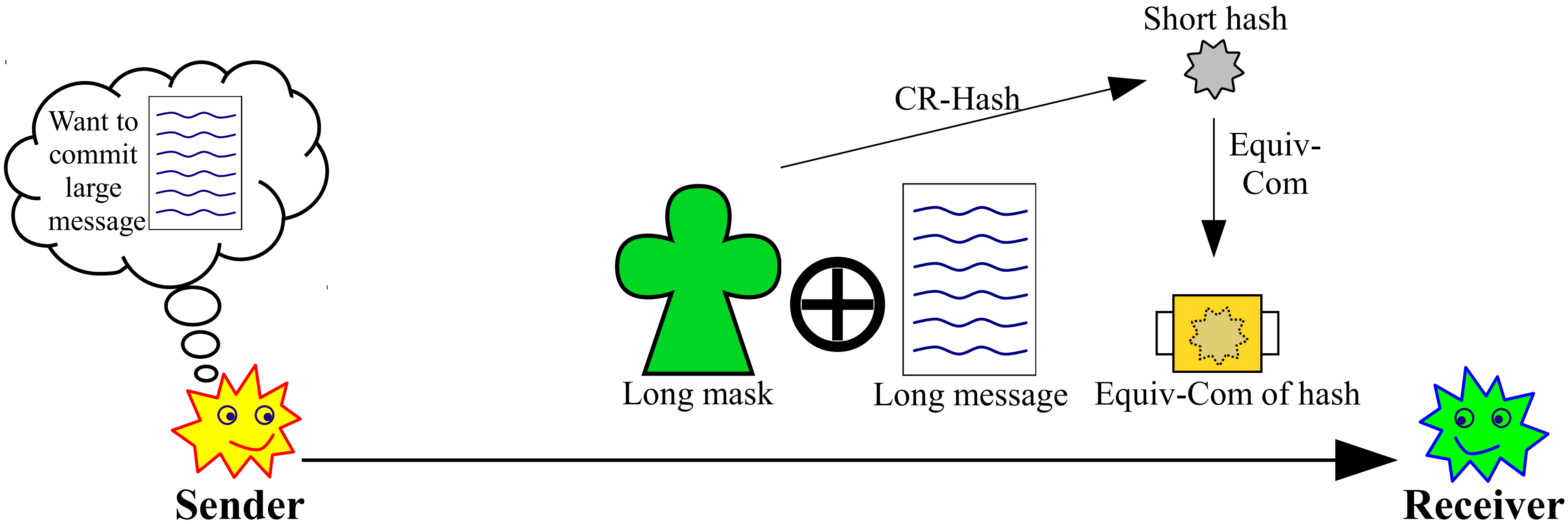

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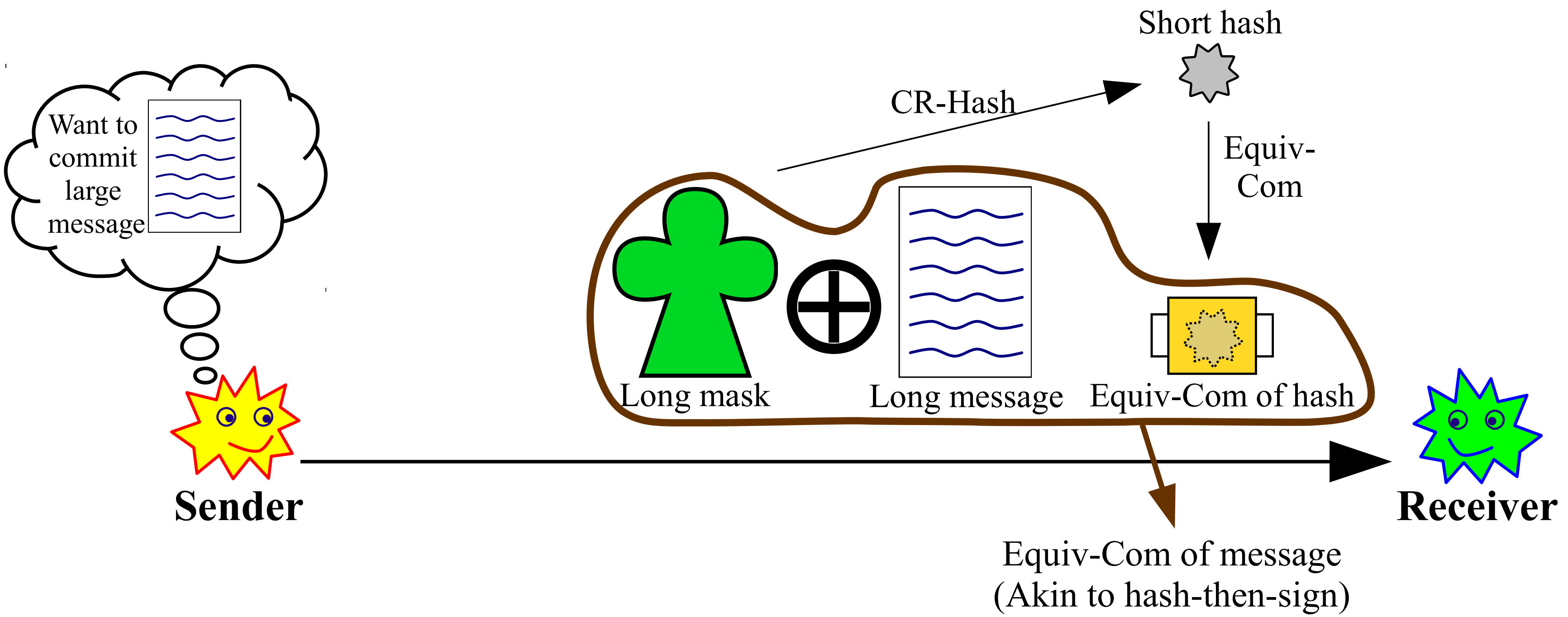

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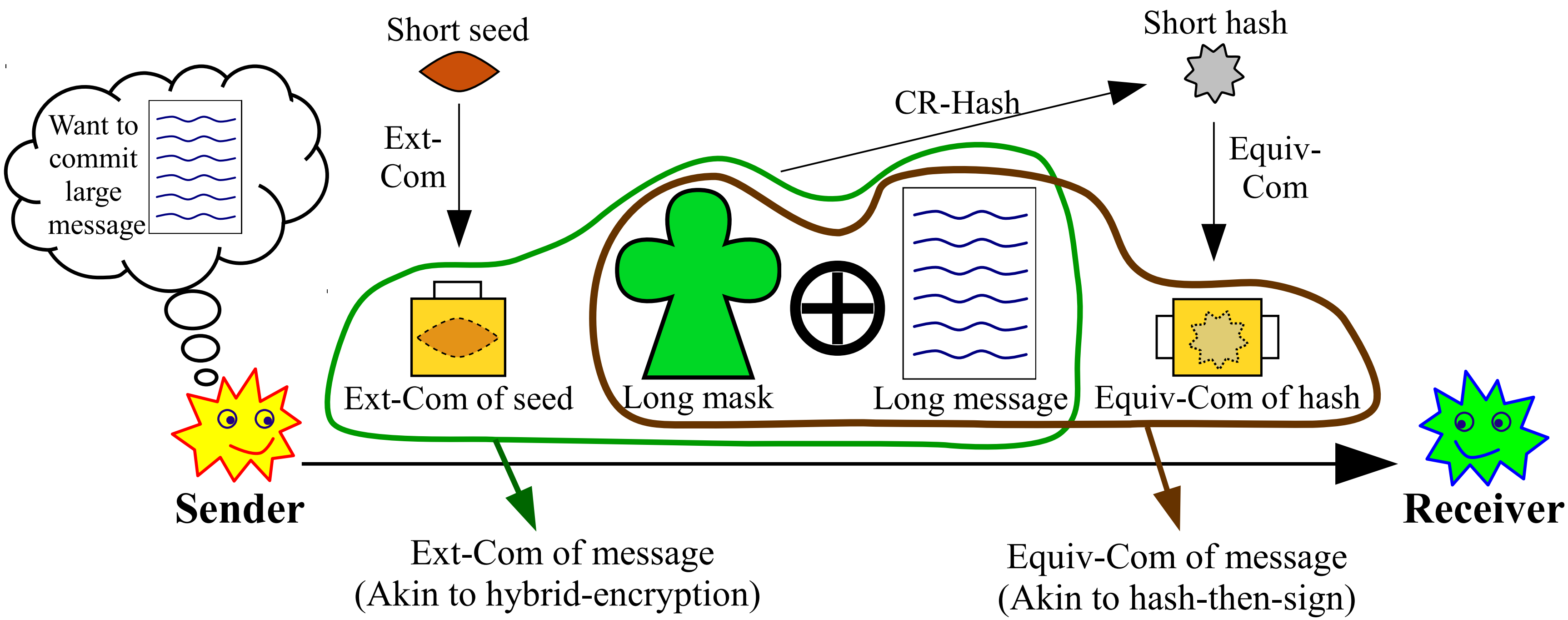

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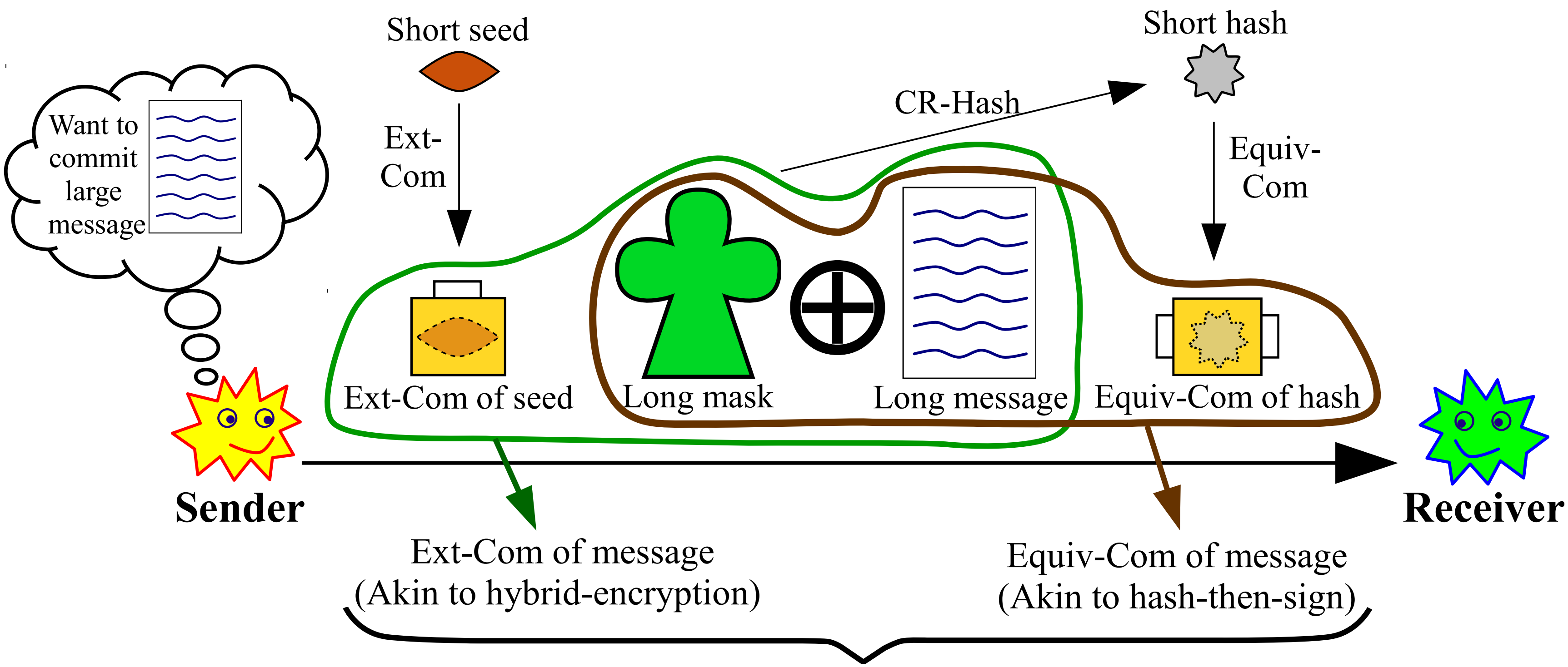

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- Opening the **Ext-Com** of seed does not allow **equivocation**
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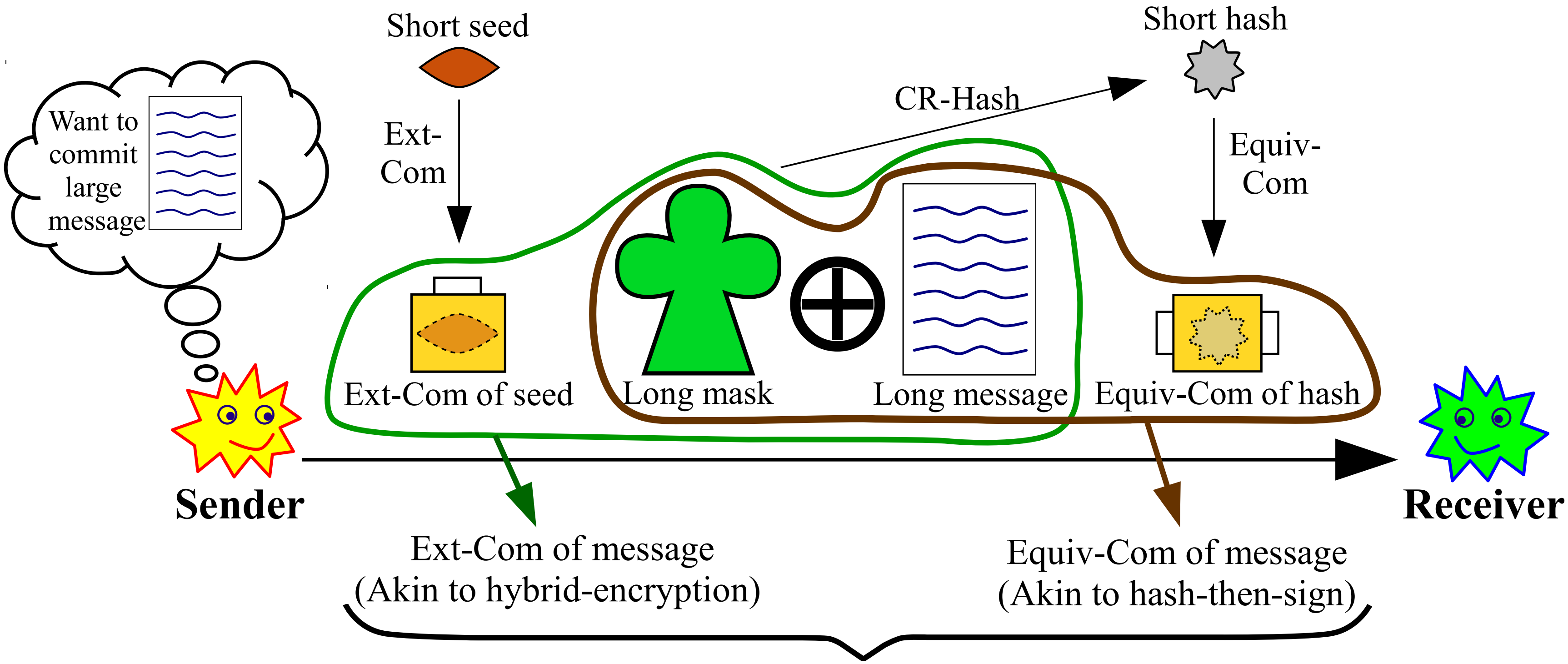

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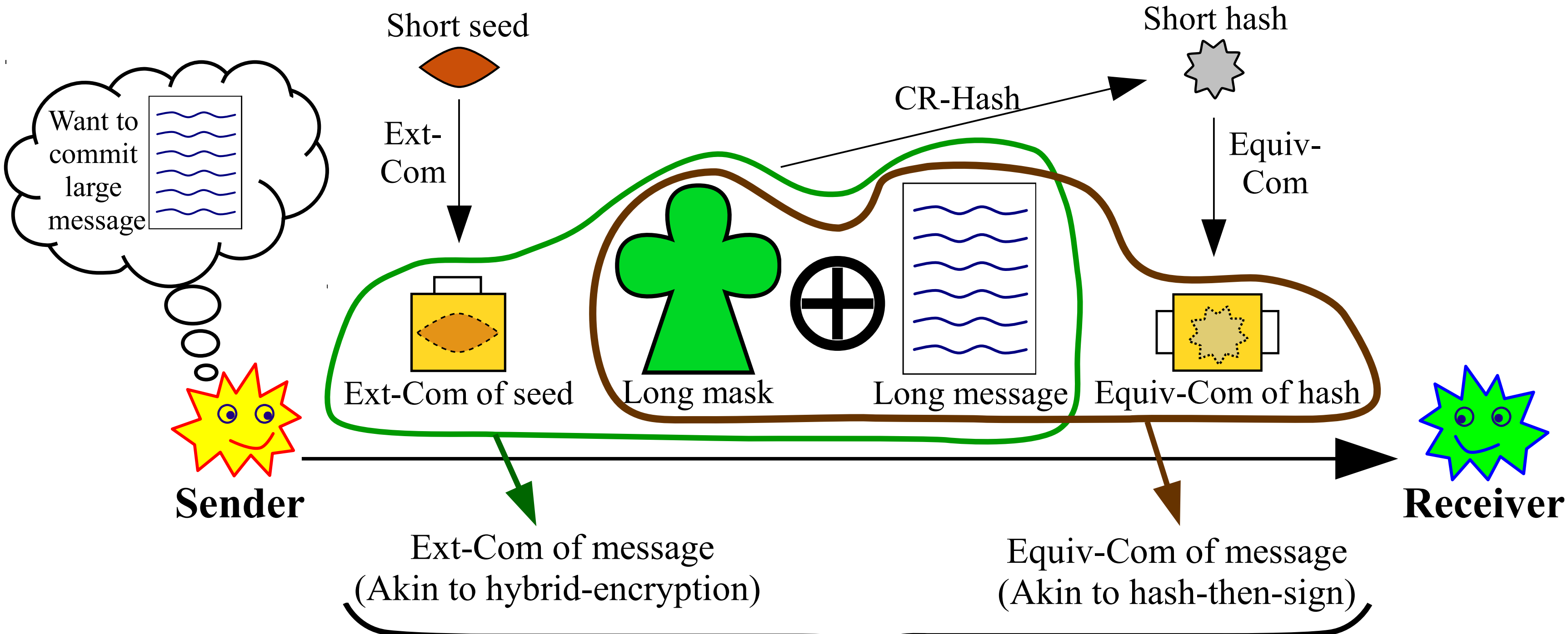

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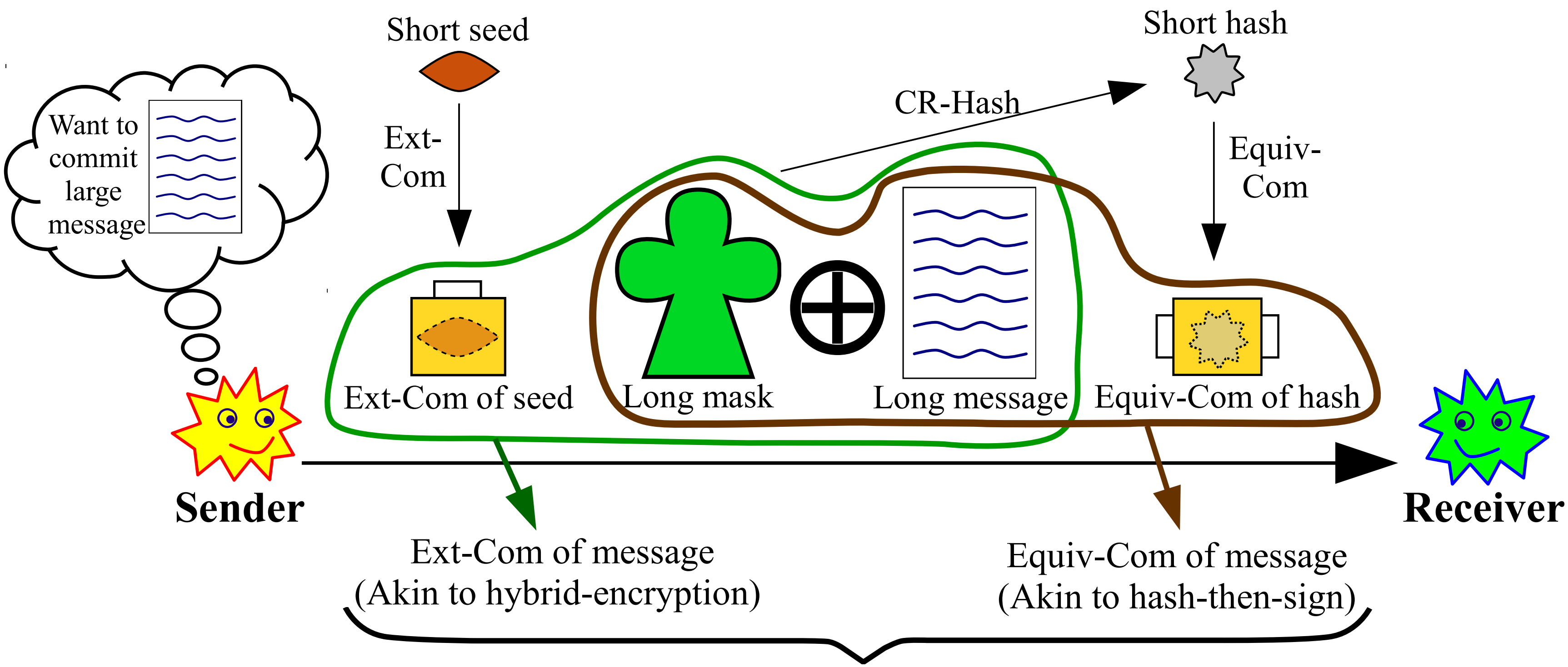

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- Prot #1: **Coin-flipping** simulatable-with-rewinding
- Prot #2: **UC-Com scheme** (namely without rewinding)

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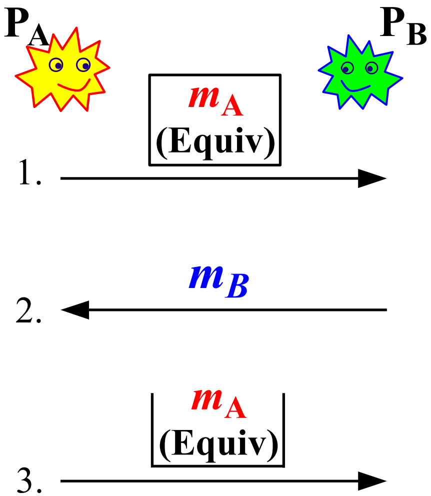

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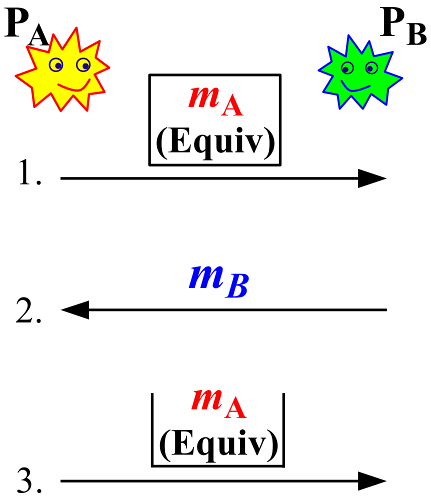

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Legend: RW = rewind; Prob(⊥) = probability of abort.

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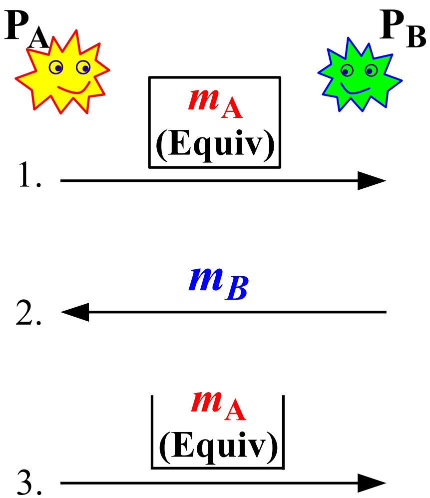

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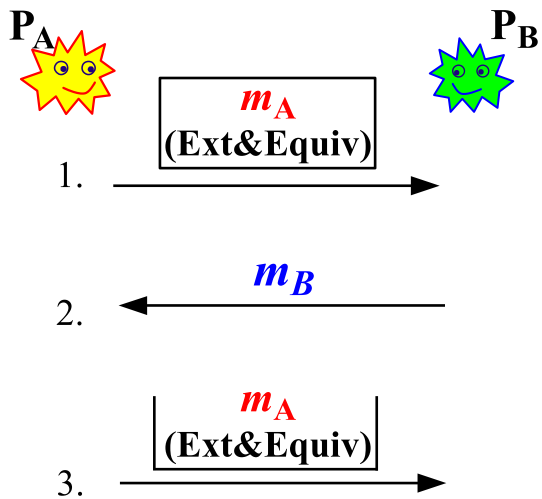

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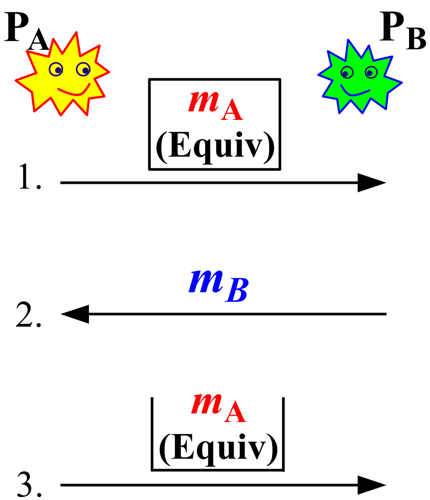

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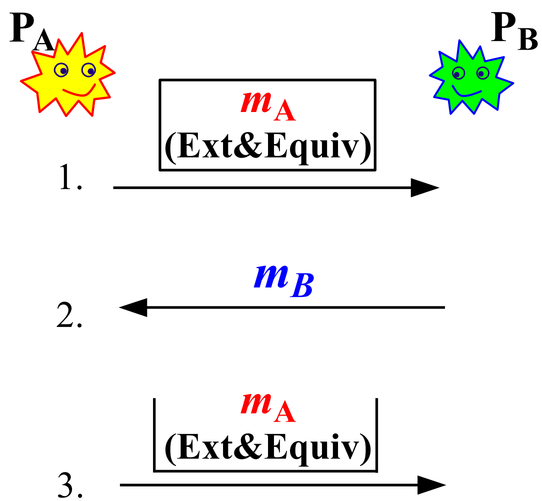

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- Lin03: ZK-based
- PW09: Cut&Choose based

Simulatable, but inefficient for large $|m|$.

This paper

Part 2
Compare
Analyze
Complex

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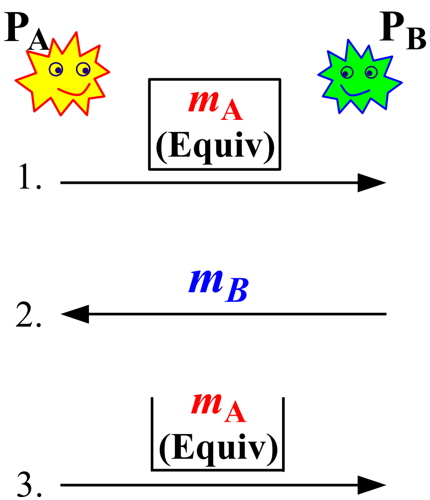

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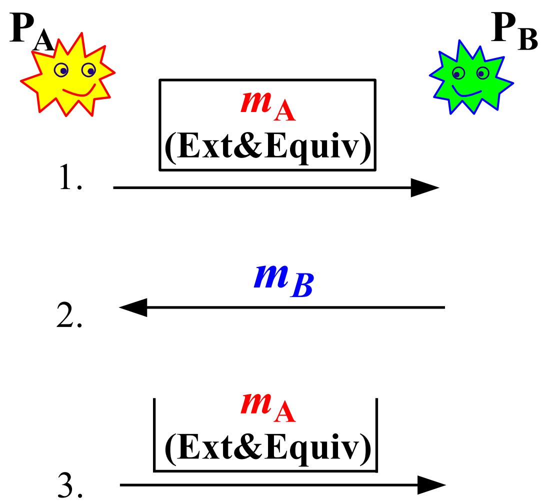

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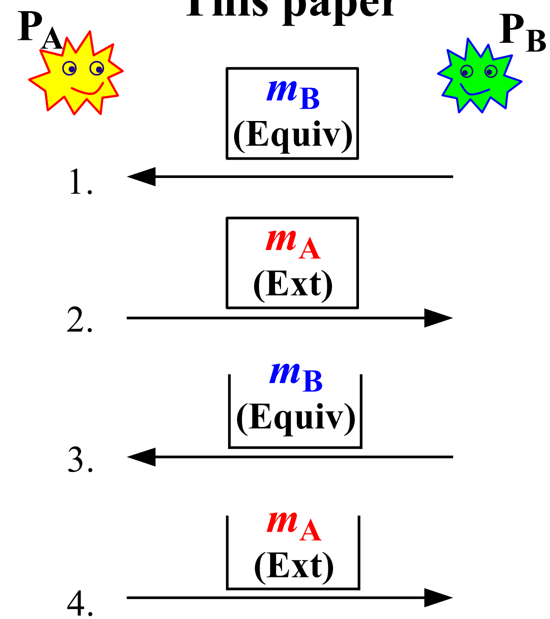

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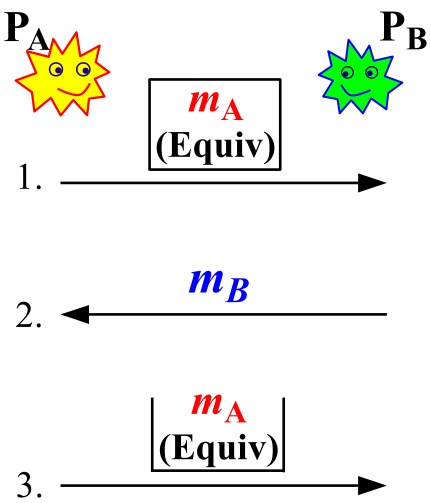
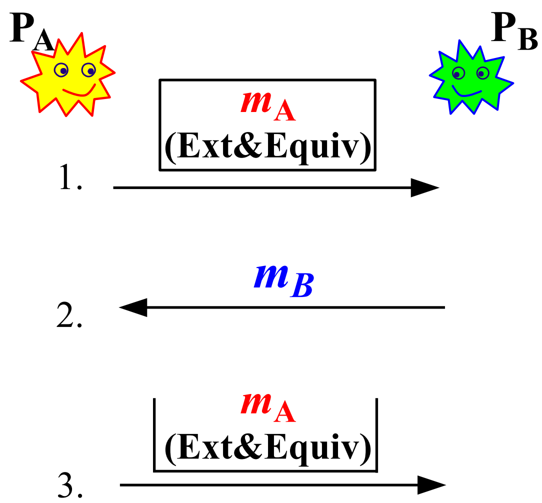


$$m = m_A \oplus m_B$$

Problem: Can't ensure ≈ Prob(⊥) in ideal vs. real world. In step 3, P_A-Prob(⊥) before Sim_B RW may (pathologically) differ from P_A-Prob(⊥) after RW.

[Lin03], [PW09]



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- Lin03: ZK-based
- PW09: Cut&Choose based

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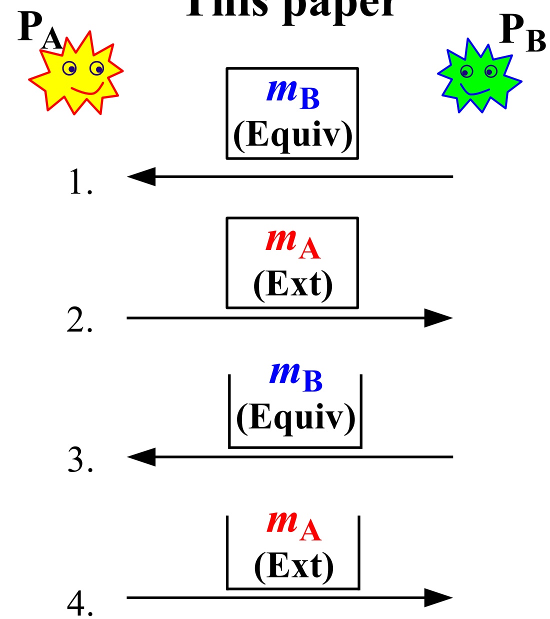
This paper



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Legend: RW = rewind; Prob(⊥) = probability of abort.

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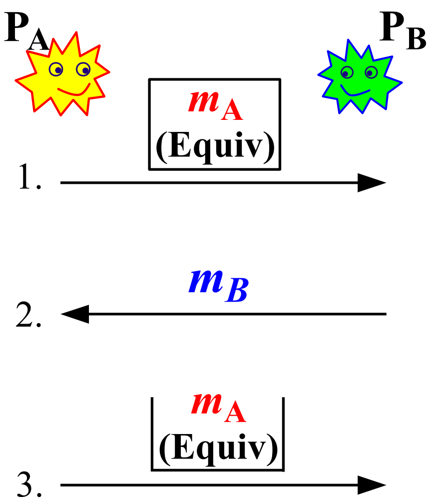

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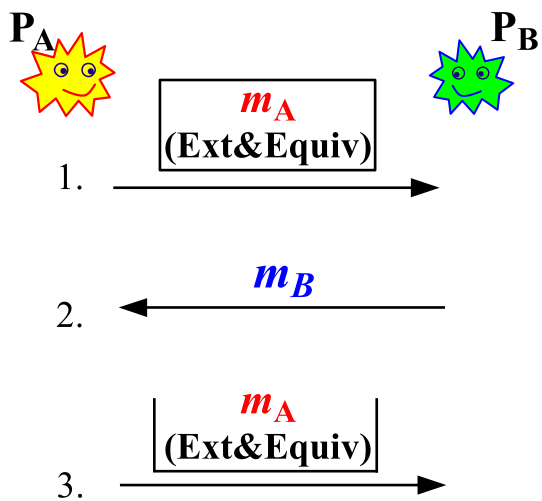

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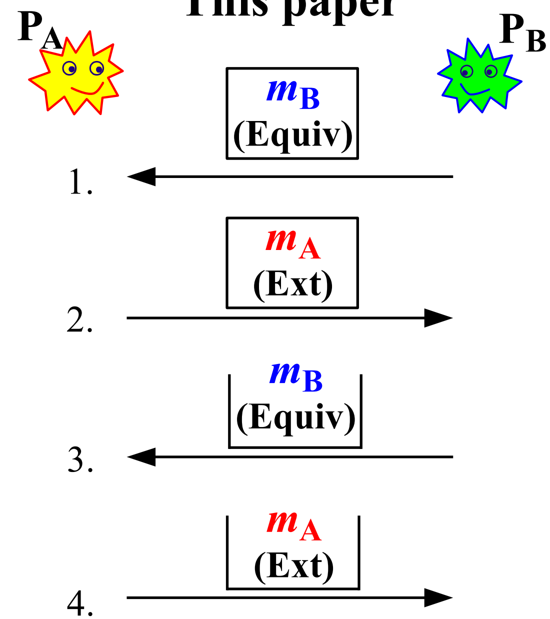

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Ext-Com and Equiv-Com are efficient

Simulatability: In the difficult side, Prob(⊥) by P_B (step 3) may depend on Com(m_A), but not on clear m_A. Can be simulated in Expected-Poly # RWs.

Legend: RW = rewind; Prob(⊥) = probability of abort.

Closer look: possible instantiation and simulation (high level)

Legend: Ped (Pedersen); ElgCom (ElGamal)

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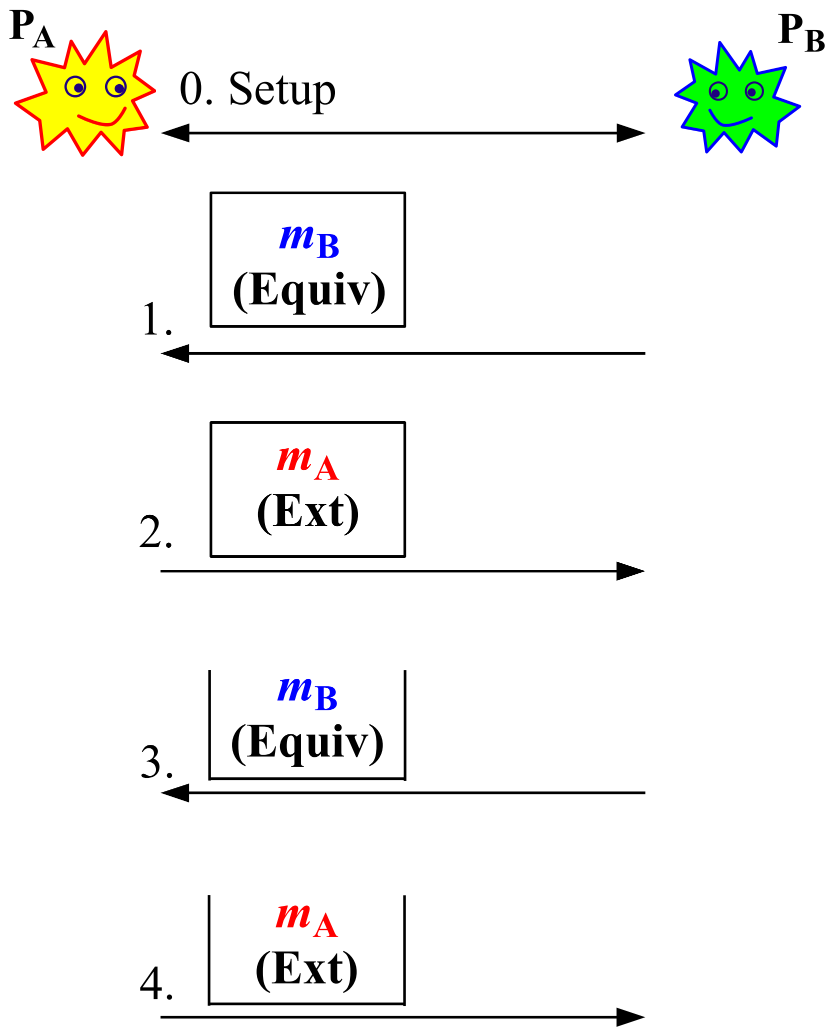

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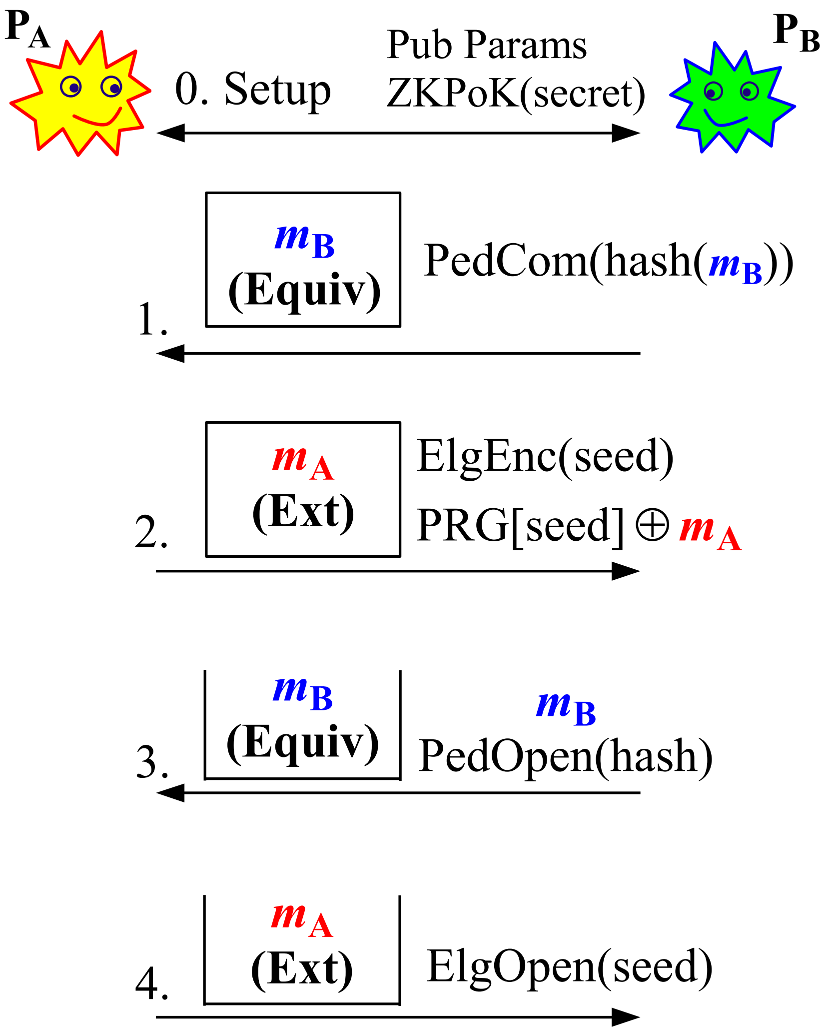

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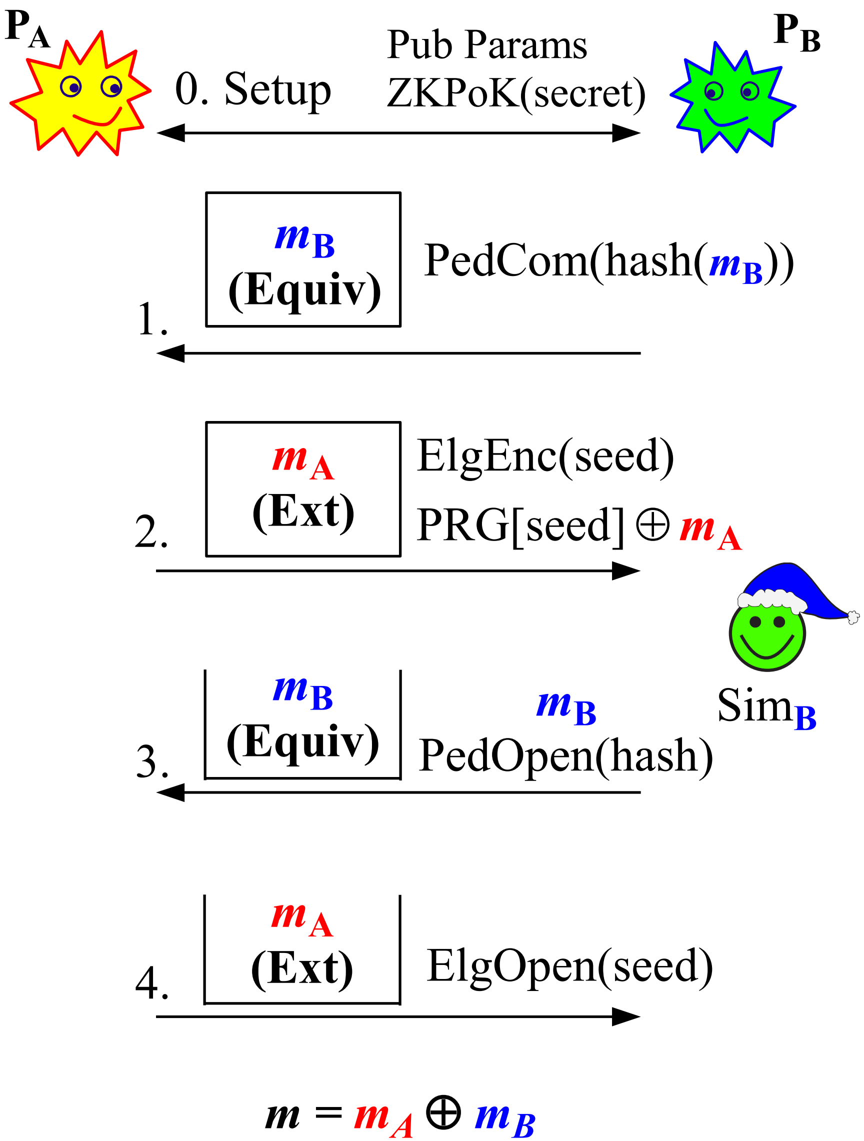

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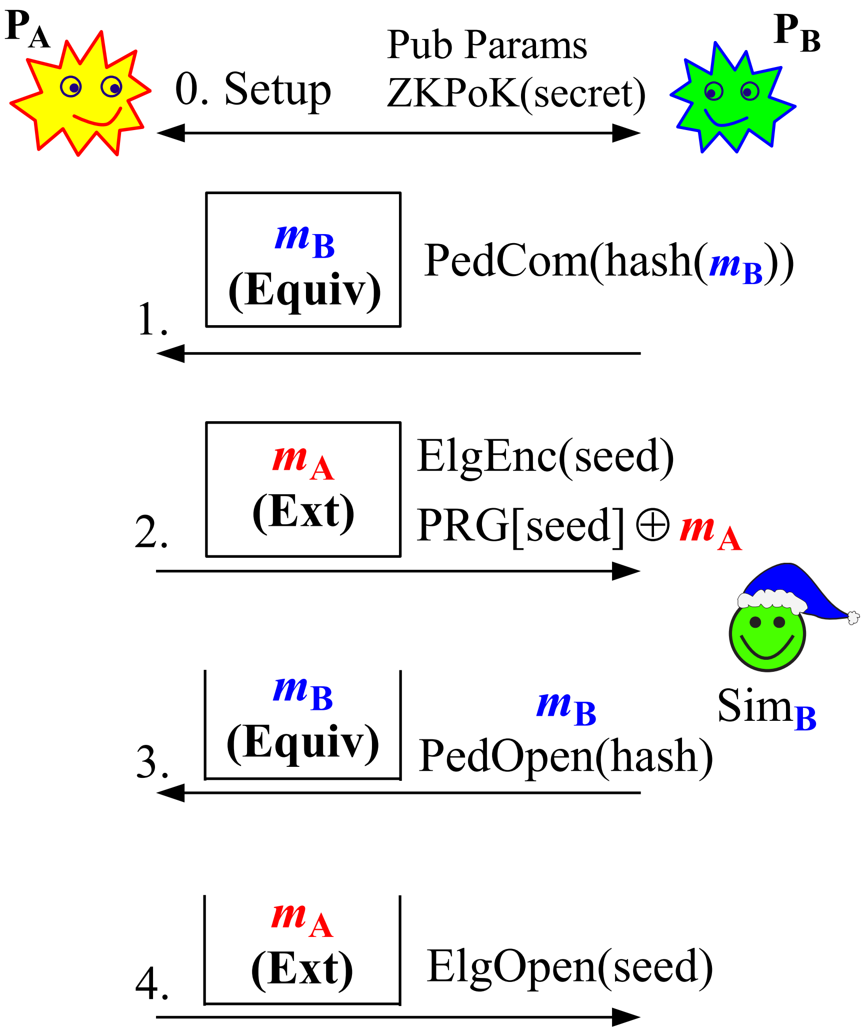

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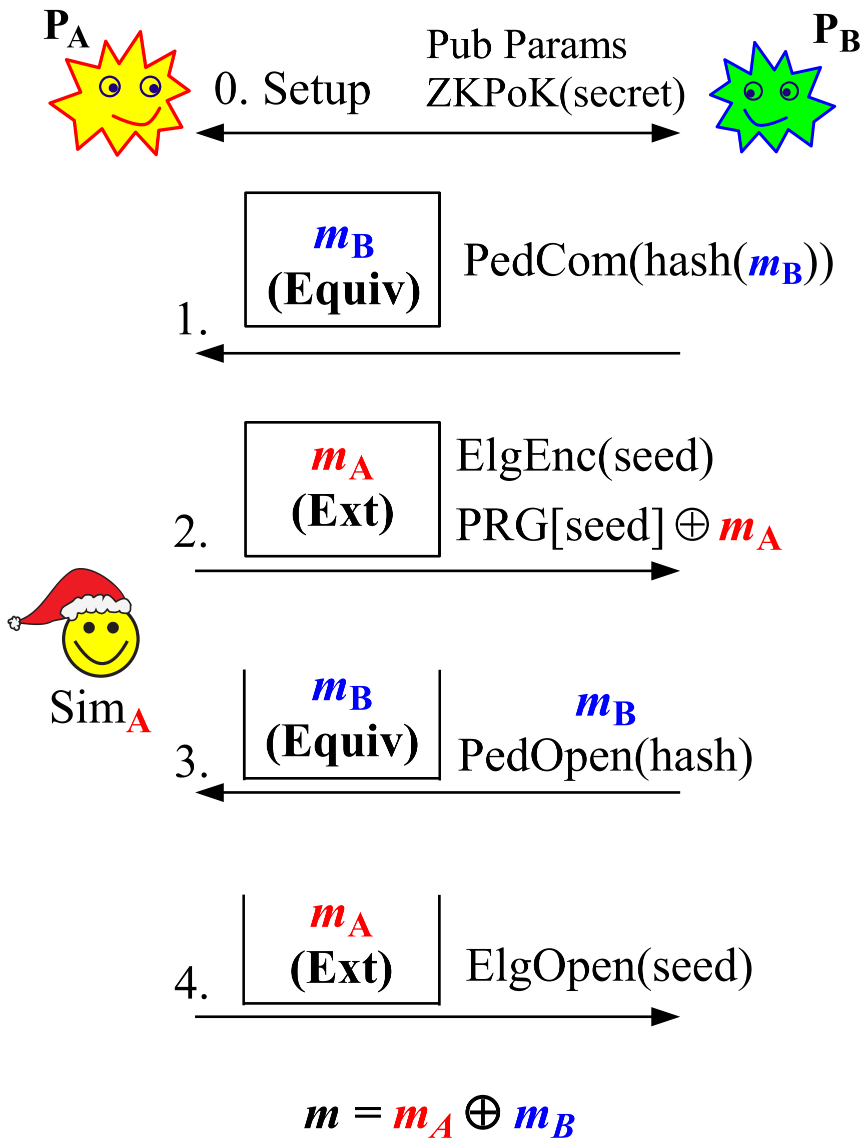

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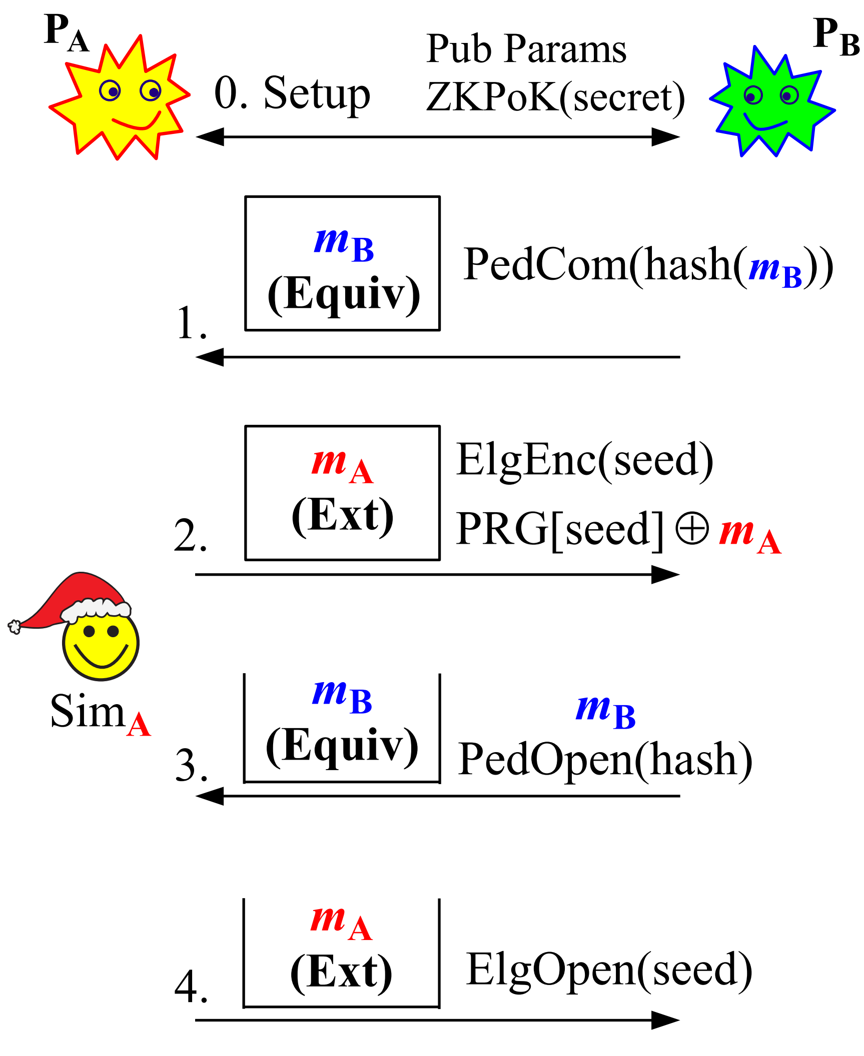

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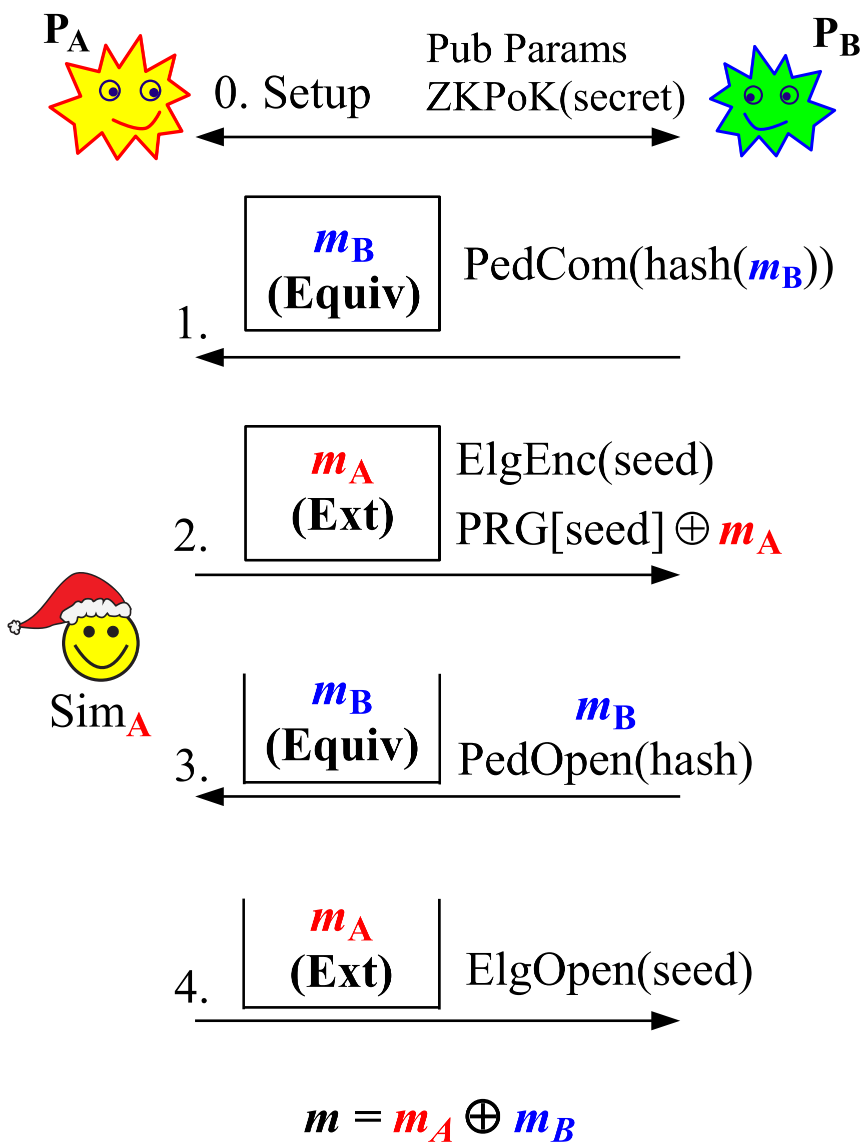

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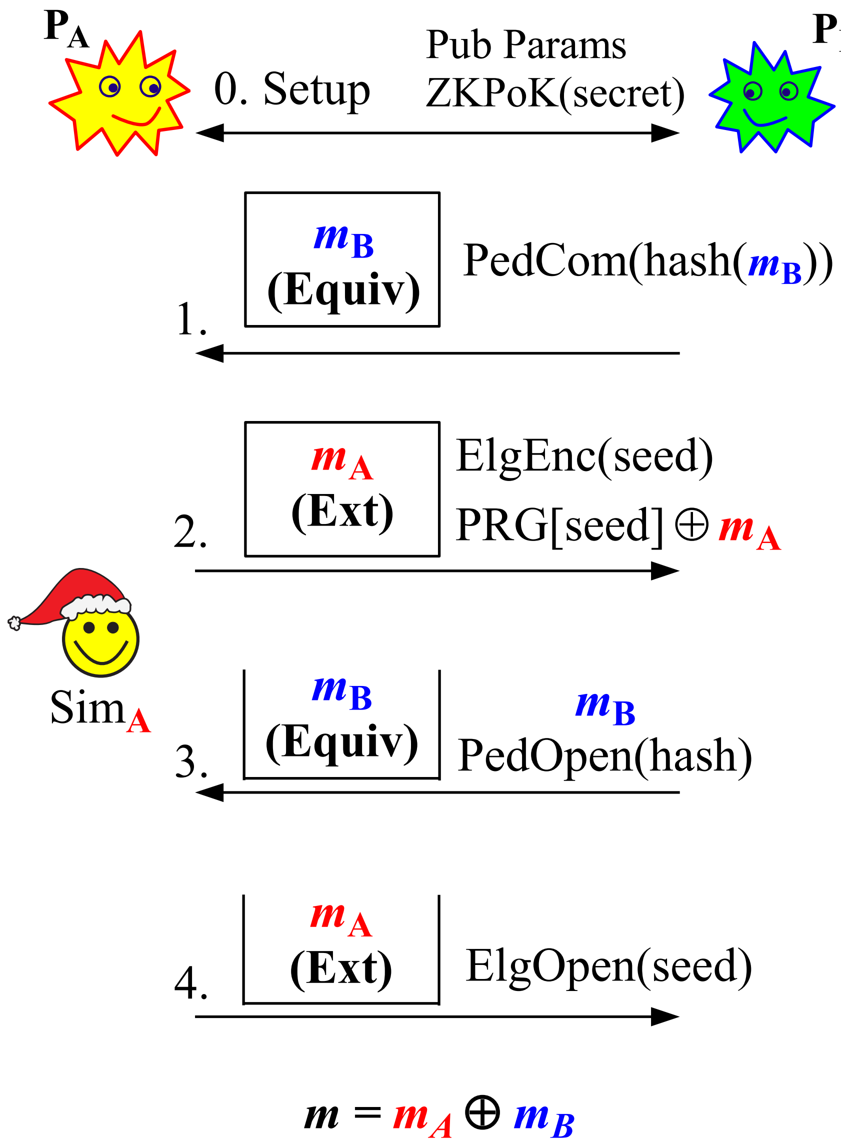

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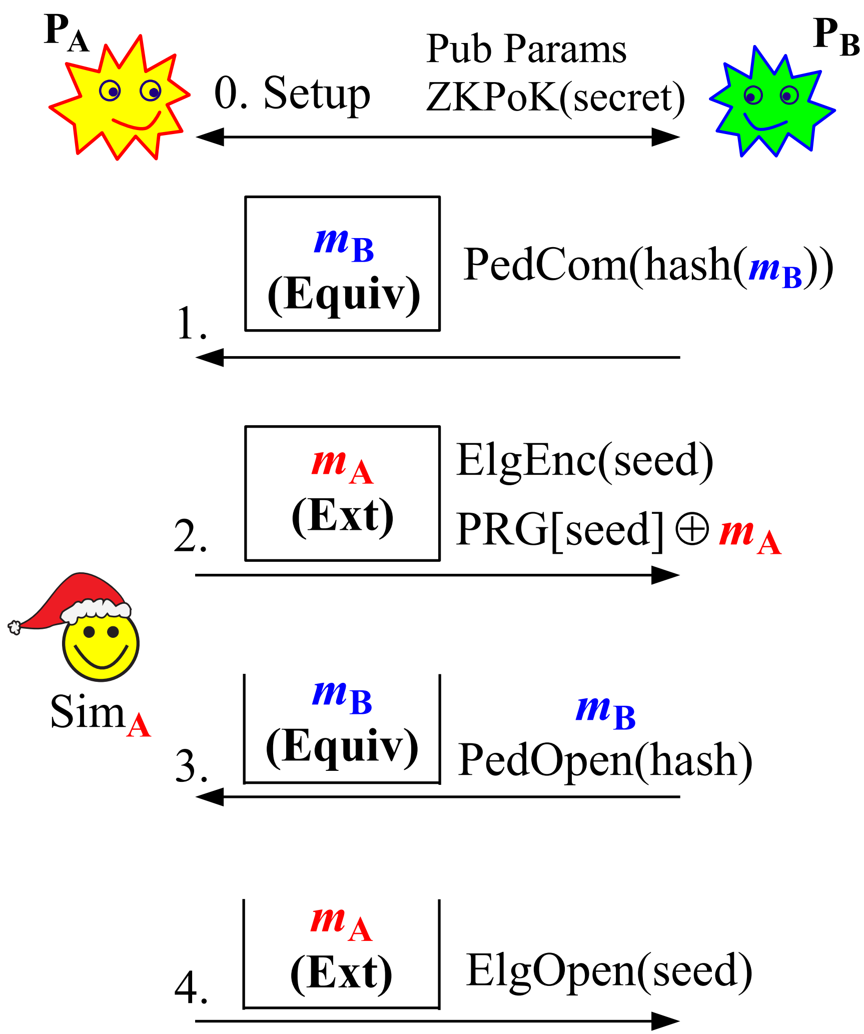

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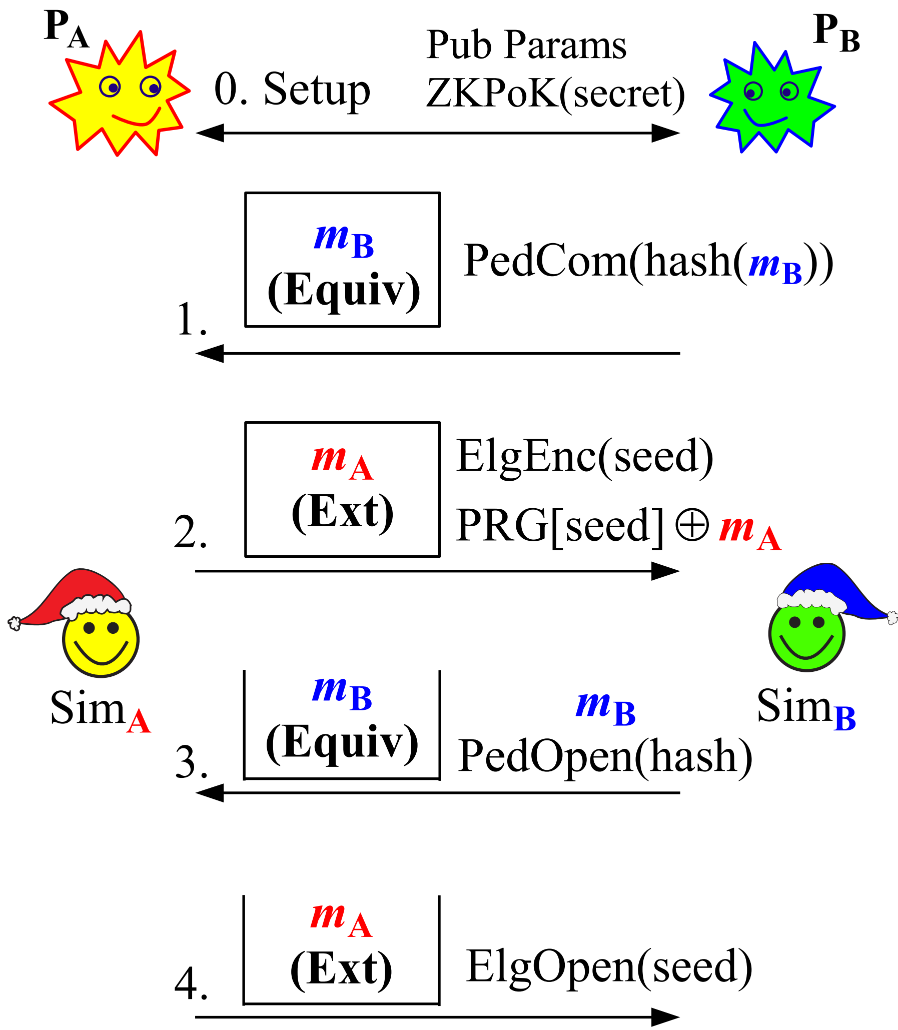

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Legend: $p(k)$ (suitable polynomial of the sec. parameter)

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Fixed offset:

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Amortized for long strings:

- Communication: 2 bits per flipped coin
- Computation (per party): 1 PRG, 1 CR-Hash, 1 XOR

Roadmap

1. Simulatable coin-flipping and commitments

2. Protocol #1: coin-flipping (simulatable with rewinding)

3. Protocol #2: UC Commitment Scheme

4. Open questions / research directions

Toward an efficient UC-Com scheme

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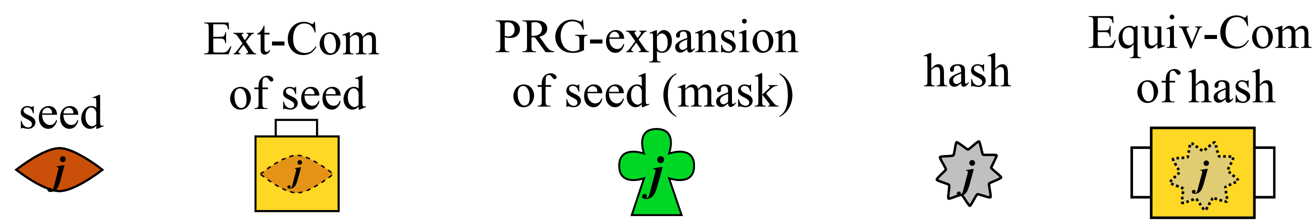
Pictorial notation:



Cut-and-choose warmup

(Warning: heavy slide)

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1. Commit phase

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Legend:
S = Sender; R = Receiver
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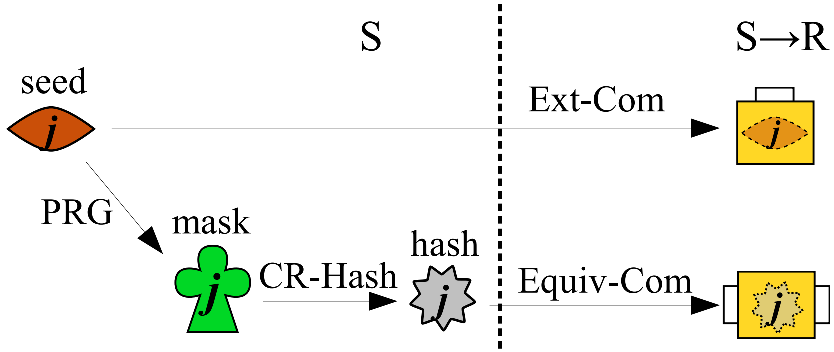
(Commit seeds and hashes) For $j \in \{1,\ldots,n\}$:



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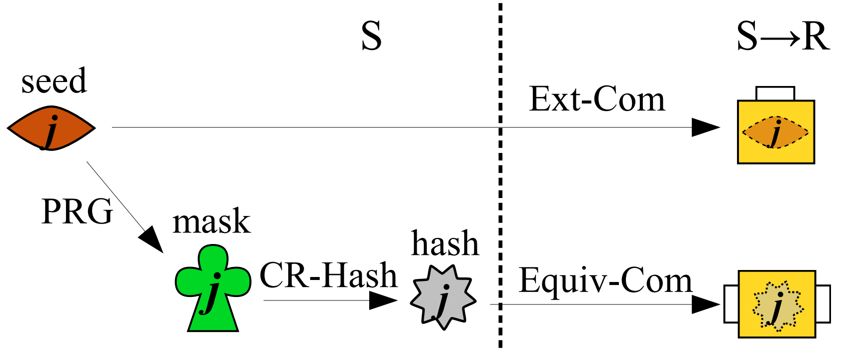

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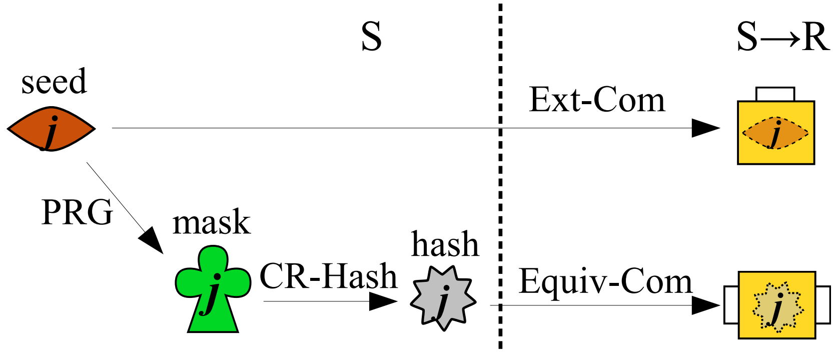

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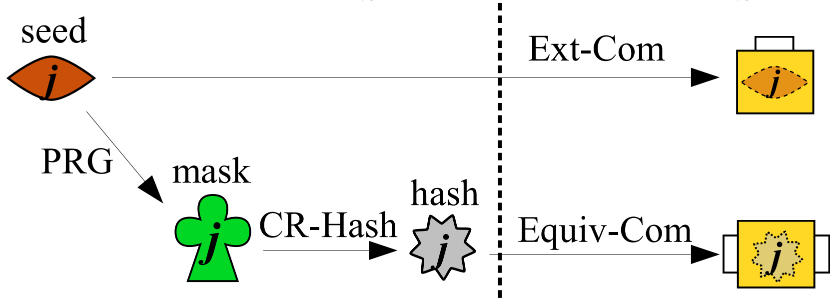

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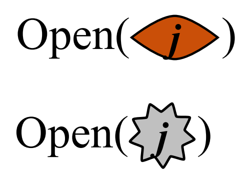

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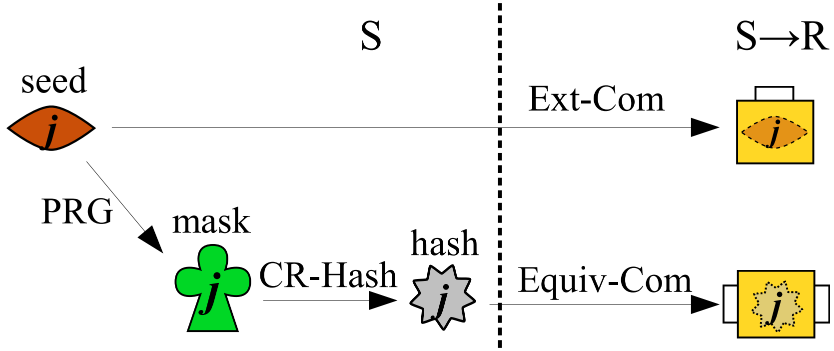

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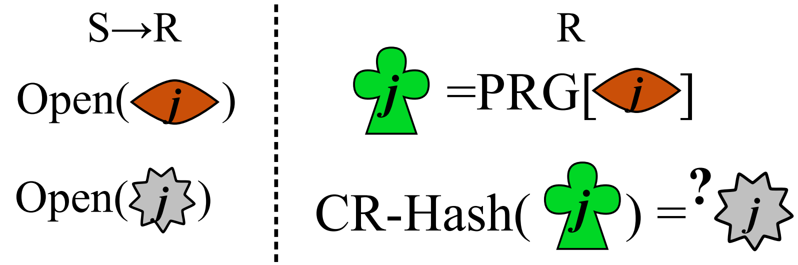

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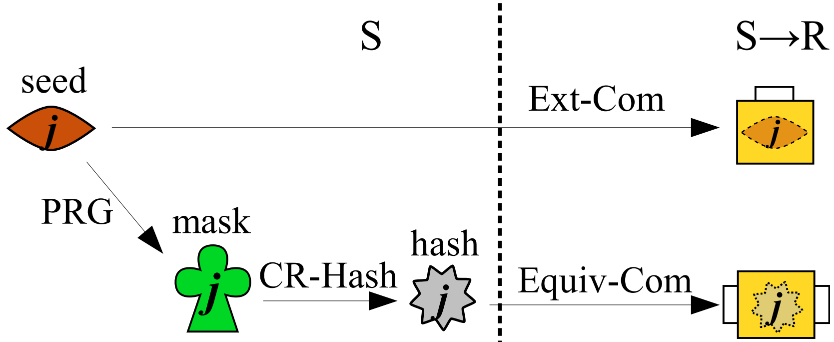

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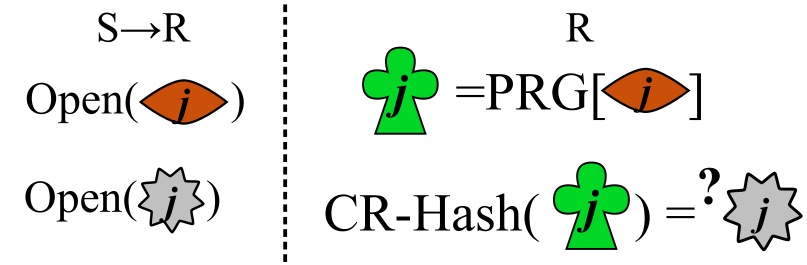

(R believes majority EVAL instances are OK)

Part 3
Outline
Warmup
Improve
Complex
Rel W

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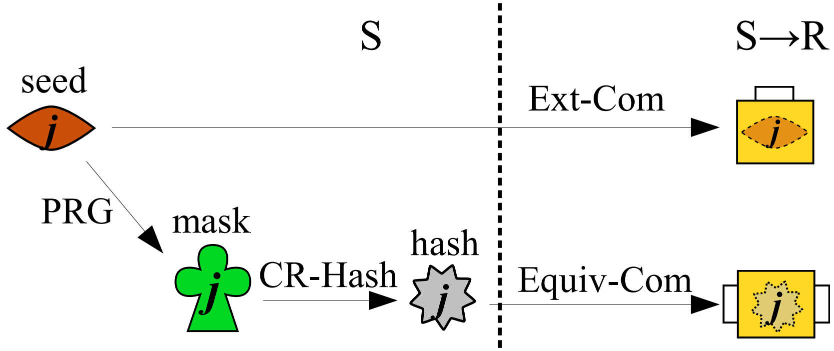

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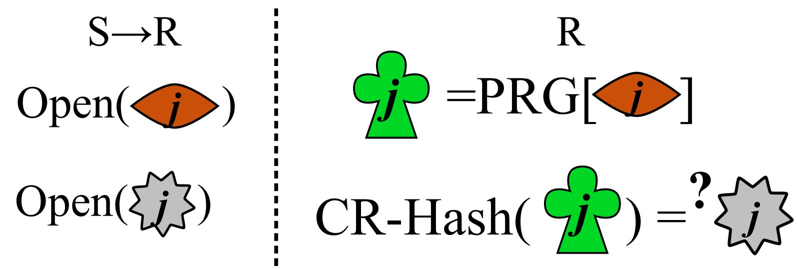

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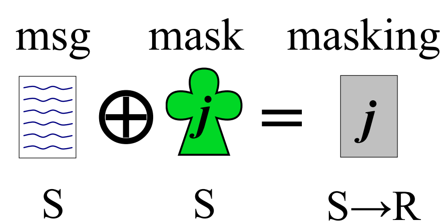

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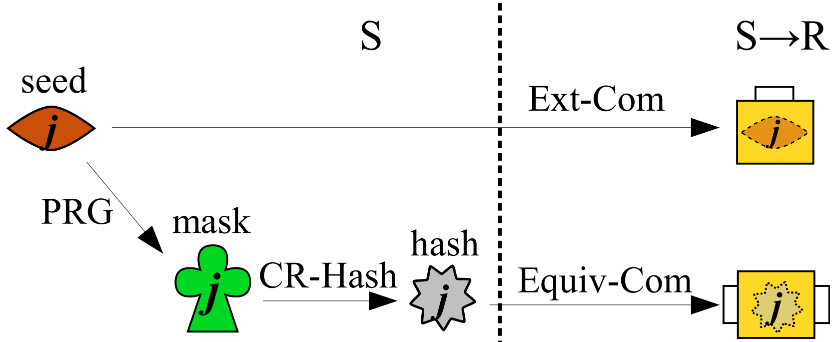

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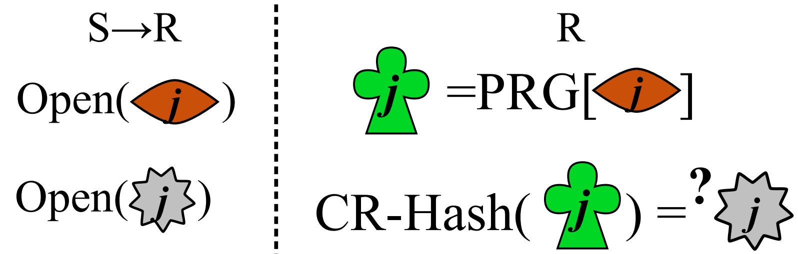

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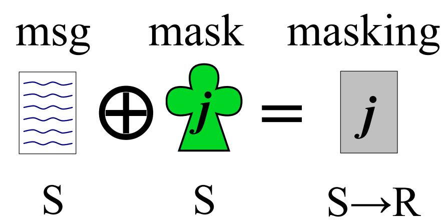

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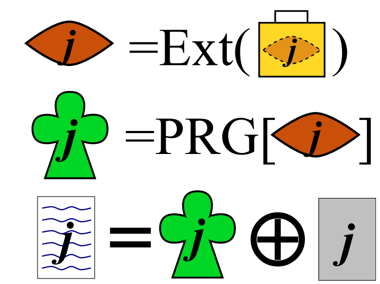

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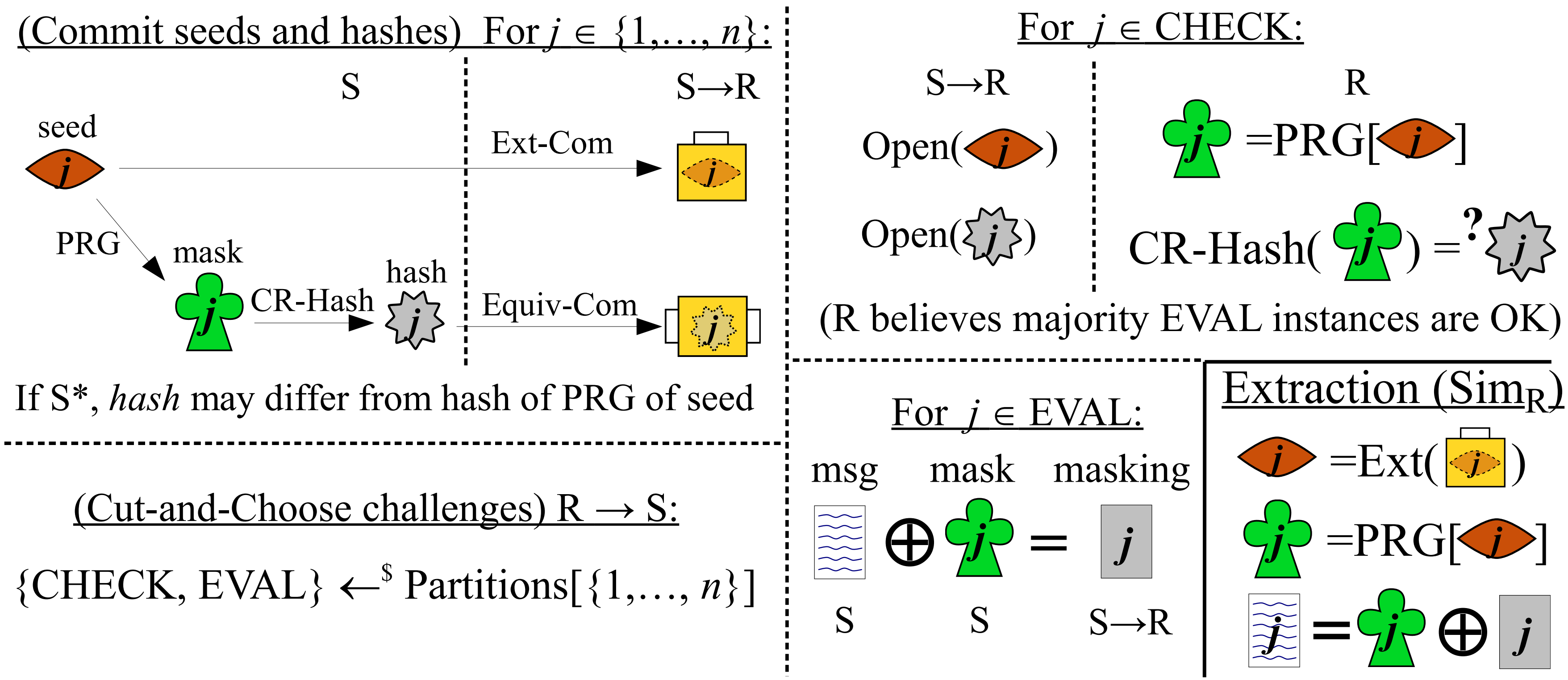

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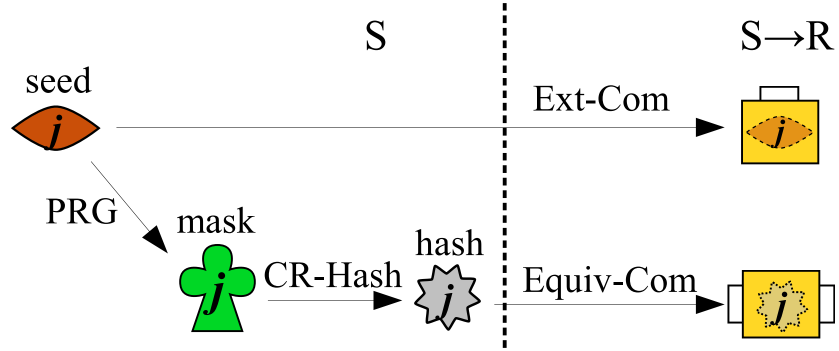

If S*, *hash* may differ from hash of PRG of seed

(Cut-and-Choose challenges) R → S:

$\{\text{CHECK}, \text{EVAL}\} \leftarrow^{\$} \text{Partitions}[\{1,\ldots,n\}]$

For $j \in$ CHECK:

S→R: Open(seed j), Open(hash j)

R: mask j =PRG[seed j]; CR-Hash(mask j) $=^?$ hash j

(R believes majority EVAL instances are OK)

For $j \in$ EVAL:

msg ⊕ mask = masking

S, S, S→R

Extraction (Sim_R):

seed j =Ext(Ext-Com j)

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2. Open phase

S→R:

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Reveal msg

Cut-and-choose warmup

(Warning: heavy slide)

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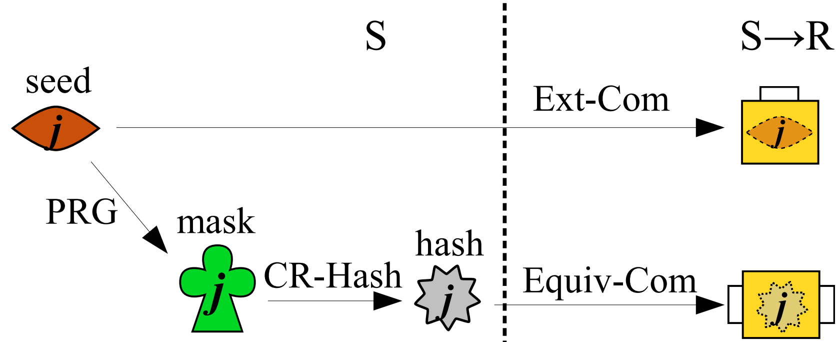

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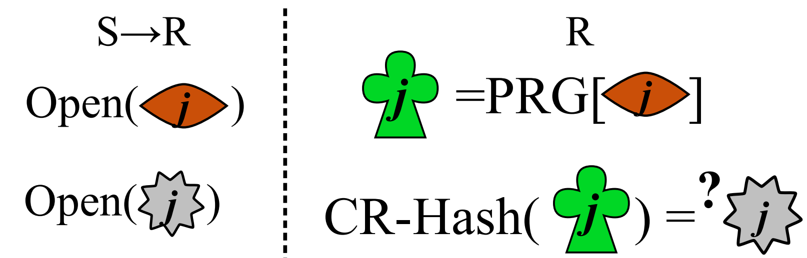

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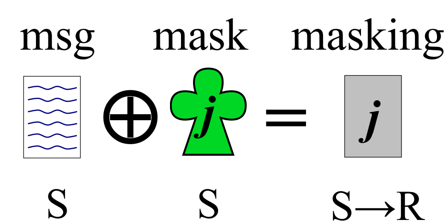

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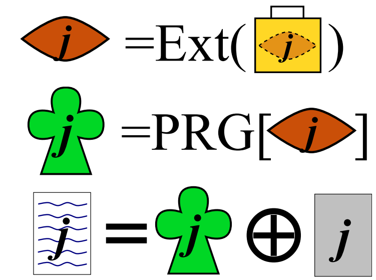

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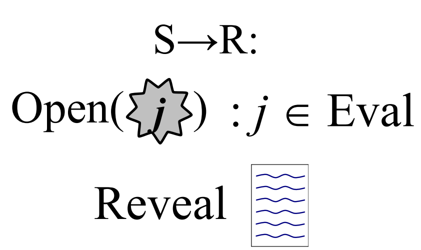

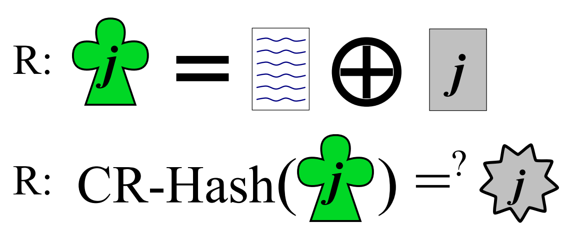

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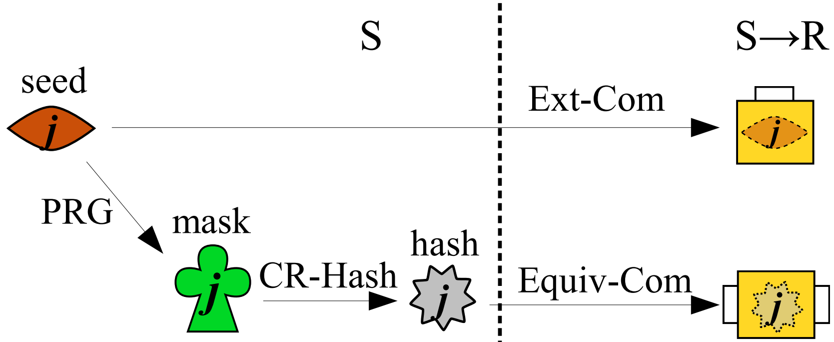

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R: CR-Hash(j) $=^?$ j

Equivocation by Sim_S:

j = ⊕ j

j =CR-Hash(j)

Equiv-Open(j)

Improving communication

Part 3
Outline
Warmup
Improve
Complex
Rel W

Improving communication

Legend:
m = message; n = # instances;
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Problems with the warmup protocol

- Ensure correct extraction of message m implies many instances
- E.g. 40 bits statistical security $\Rightarrow n \geq 123$, e.g. $(n, v, e) = (123, 74, 49)$.
- High communication complexity: $|m| \times e$

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  $\Rightarrow$ Mask each ("authenticated") share, instead of full message m

  $\Rightarrow$ Sim_R extracts m if it extracts enough (t) correct shares out of e shares

  $\Rightarrow$ New params, e.g., (n, v, e, t) = (119, 73, **46**, **23**) $\Rightarrow$ Comm = $|m| \times e / t$

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E.g. C&C and erasure code parameters:

- ***n* = 119** (# instances in cut-and-choose)
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- Interaction due to cut-&-choose can be removed by using Non-Programmable Random Oracle (and increasing statistical security parameter)

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- [FJNT16] (Also OT based):
  - Uses consistency check to allow erasure code instead of ECC
  - Enable homomorphic commitments.

Roadmap

1. Simulatable coin-flipping and commitments

2. Protocol #1: coin-flipping (simulatable with rewinding)

3. Protocol #2: UC Commitment Scheme

4. Open questions / research directions

Possible research directions:

Part 4

Open

Thanks

Refs

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- More efficient UC Coin-Flipping (2 bits / flipped coin & comp. efficient)?

Thank you for your attention

?

Very Efficient Simulatable Flipping of Many Coins into-a-well

luis.papers@gmail.com

https://ia.cr/2015/640

References mentioned in this presentation

(More references in paper)

- [Blu83]: Blum: Coin flipping by telephone – a protocol for solving impossible problems. SIGACT News 15, 23–27 (1983). Appeared also at CRYPTO 1981
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