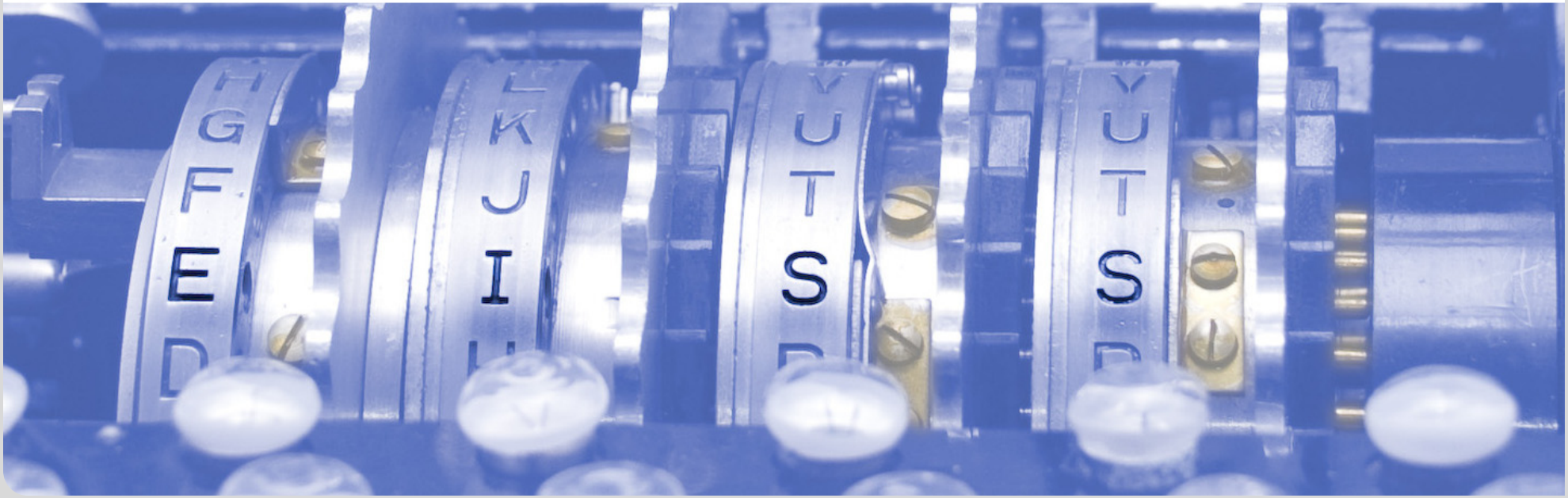
Karlsruhe Institute of Technology

Fault-Tolerant Aggregate Signatures

March, 7th 2016

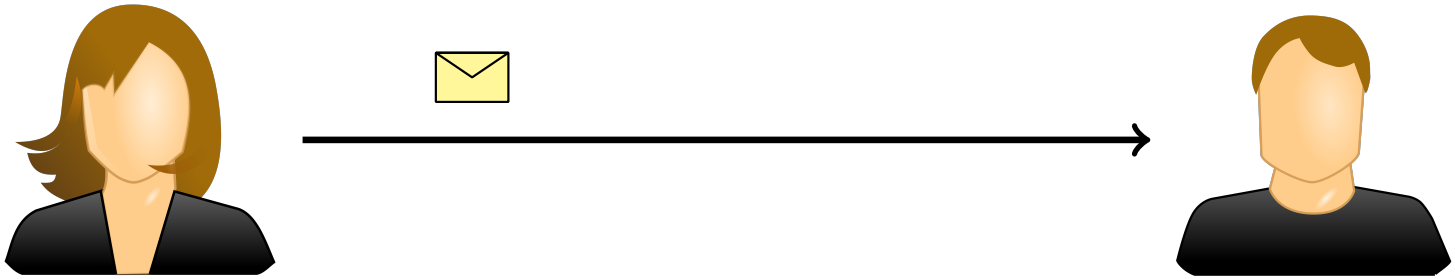
Gunnar Hartung Björn Kaidel Alexander Koch Jessica Koch Andy Rupp

INSTITUTE OF THEORETICAL COMPUTER SCIENCE



Outline

Introduction: Aggregate Signatures



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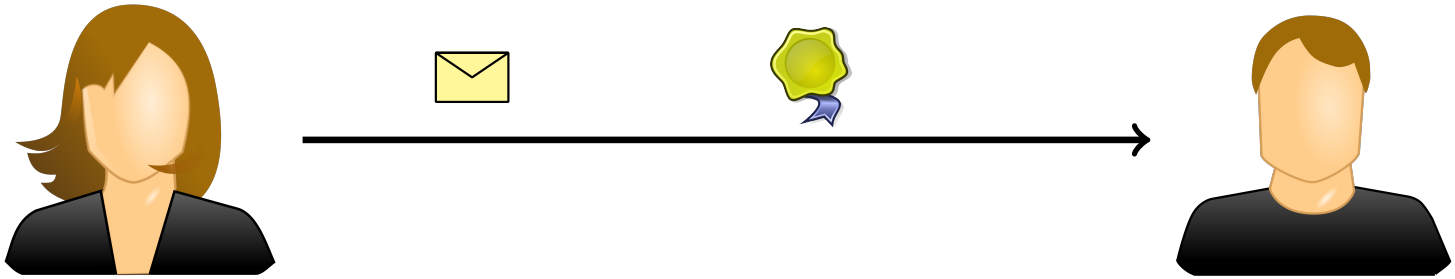

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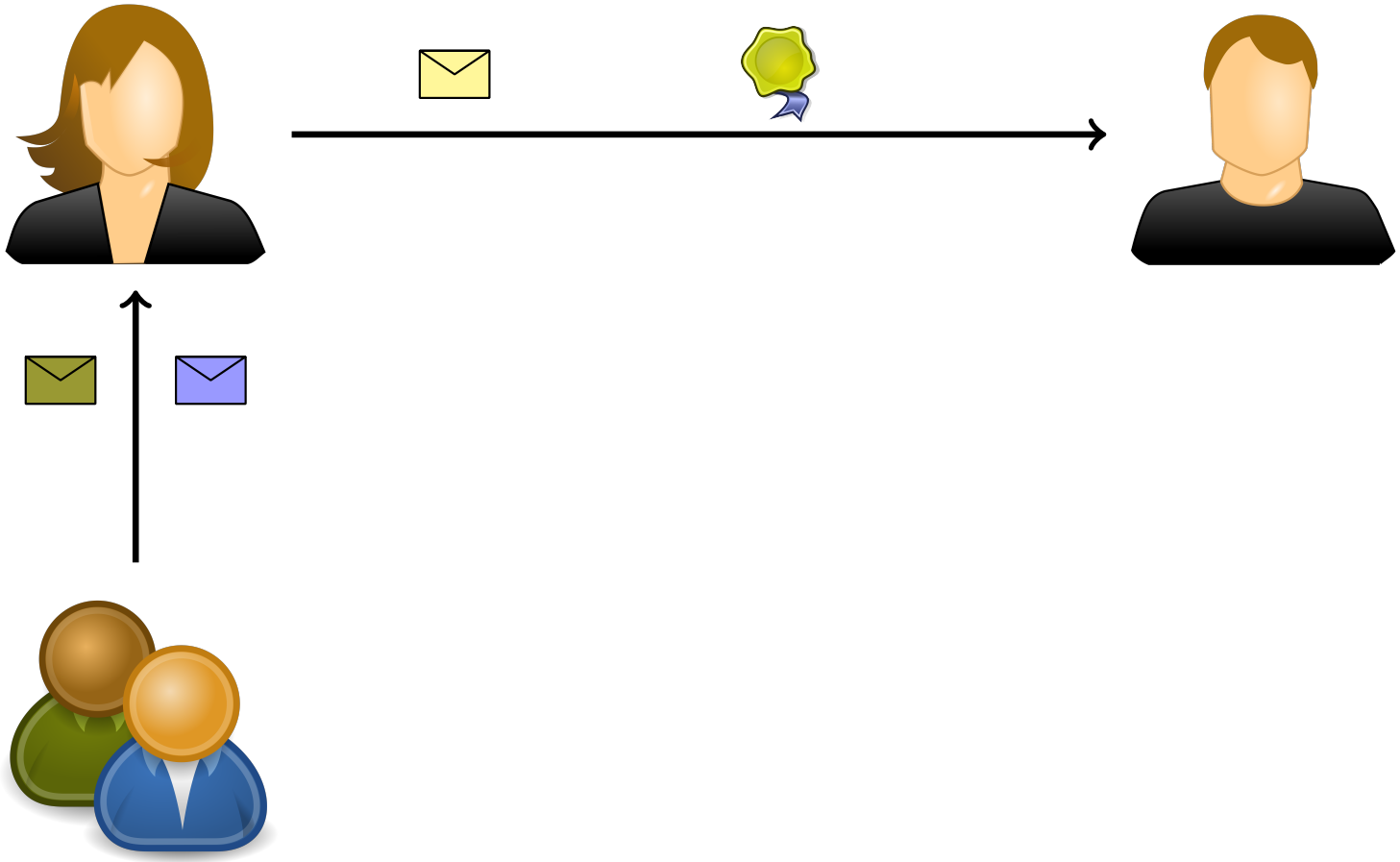

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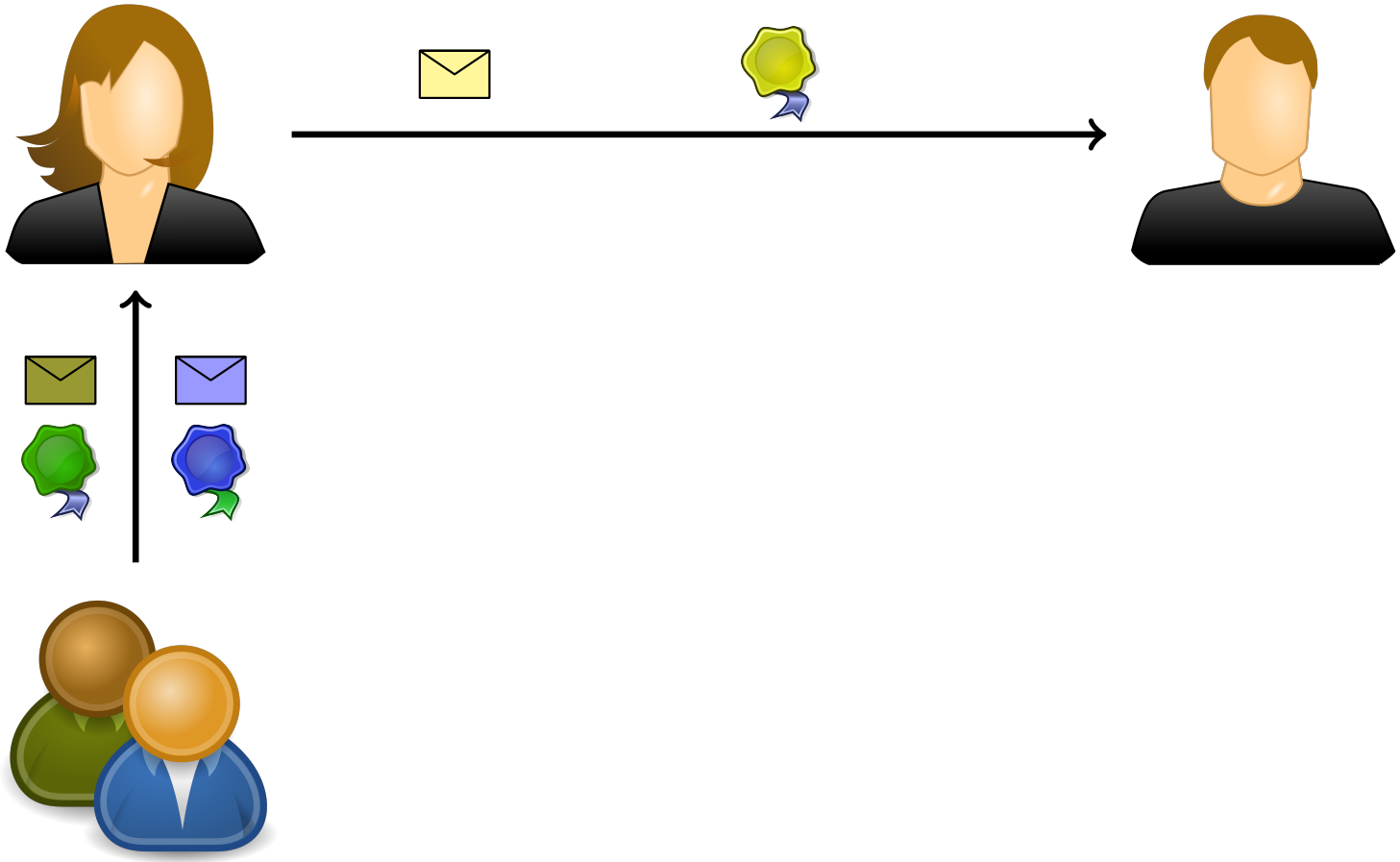

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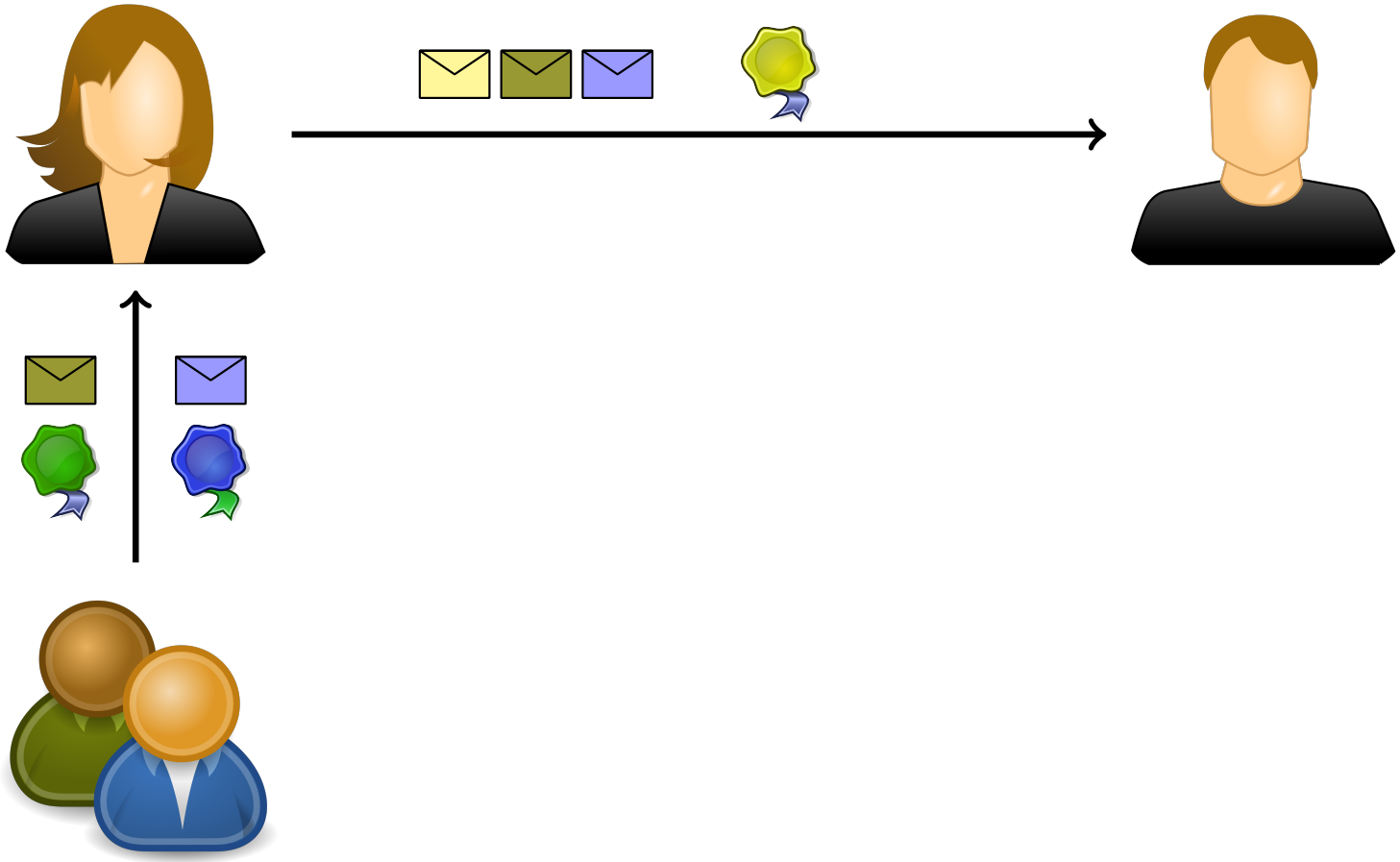

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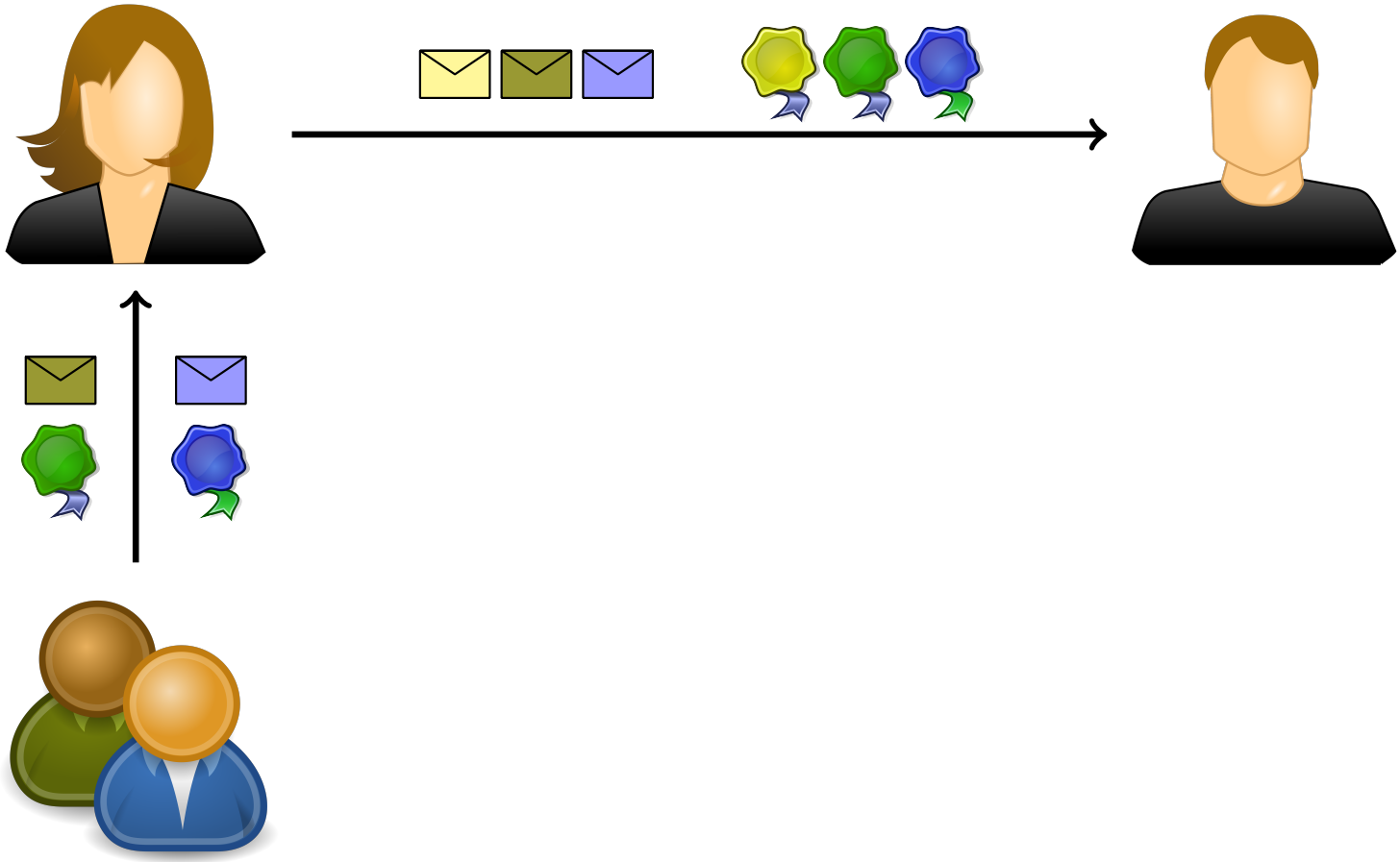

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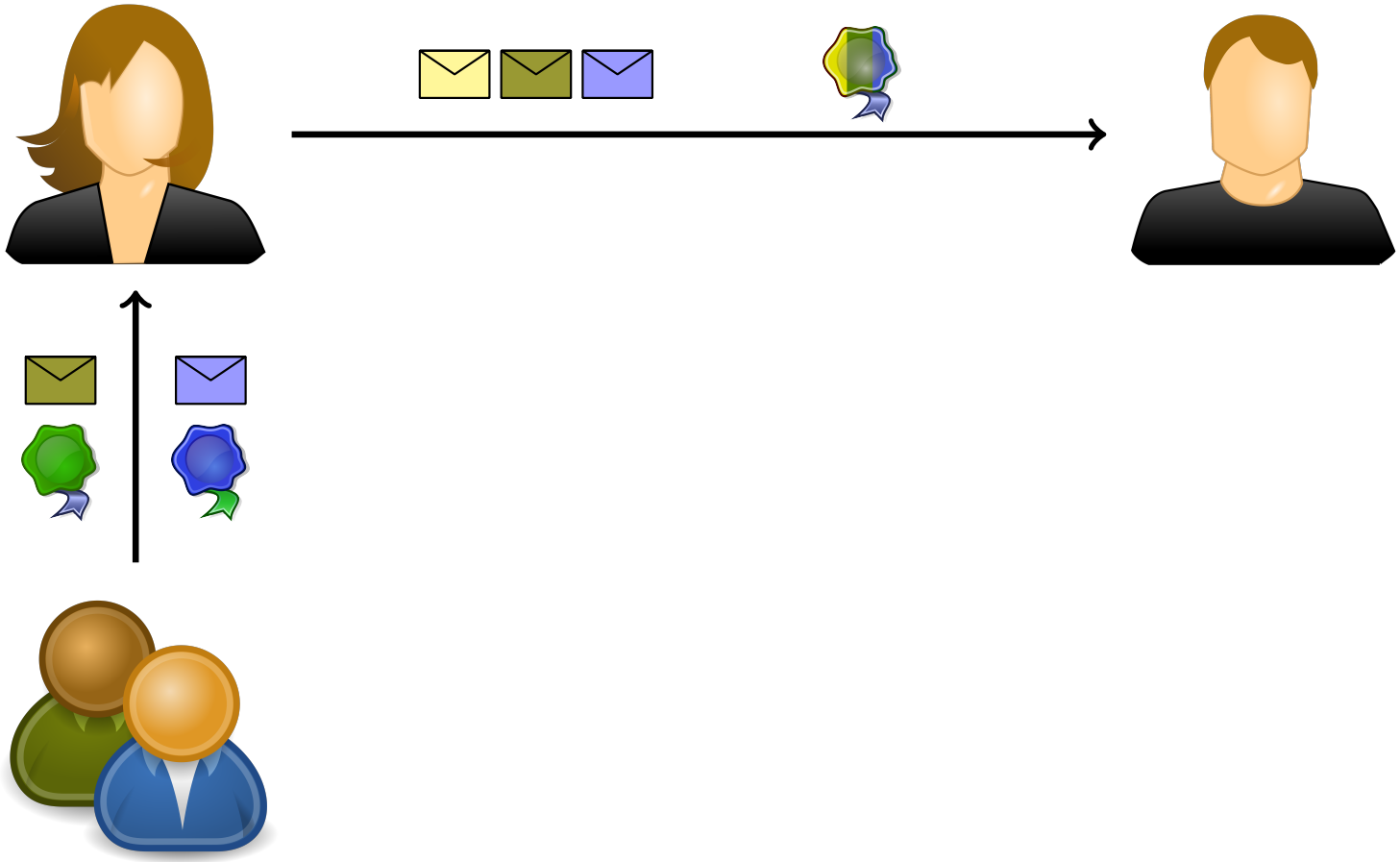

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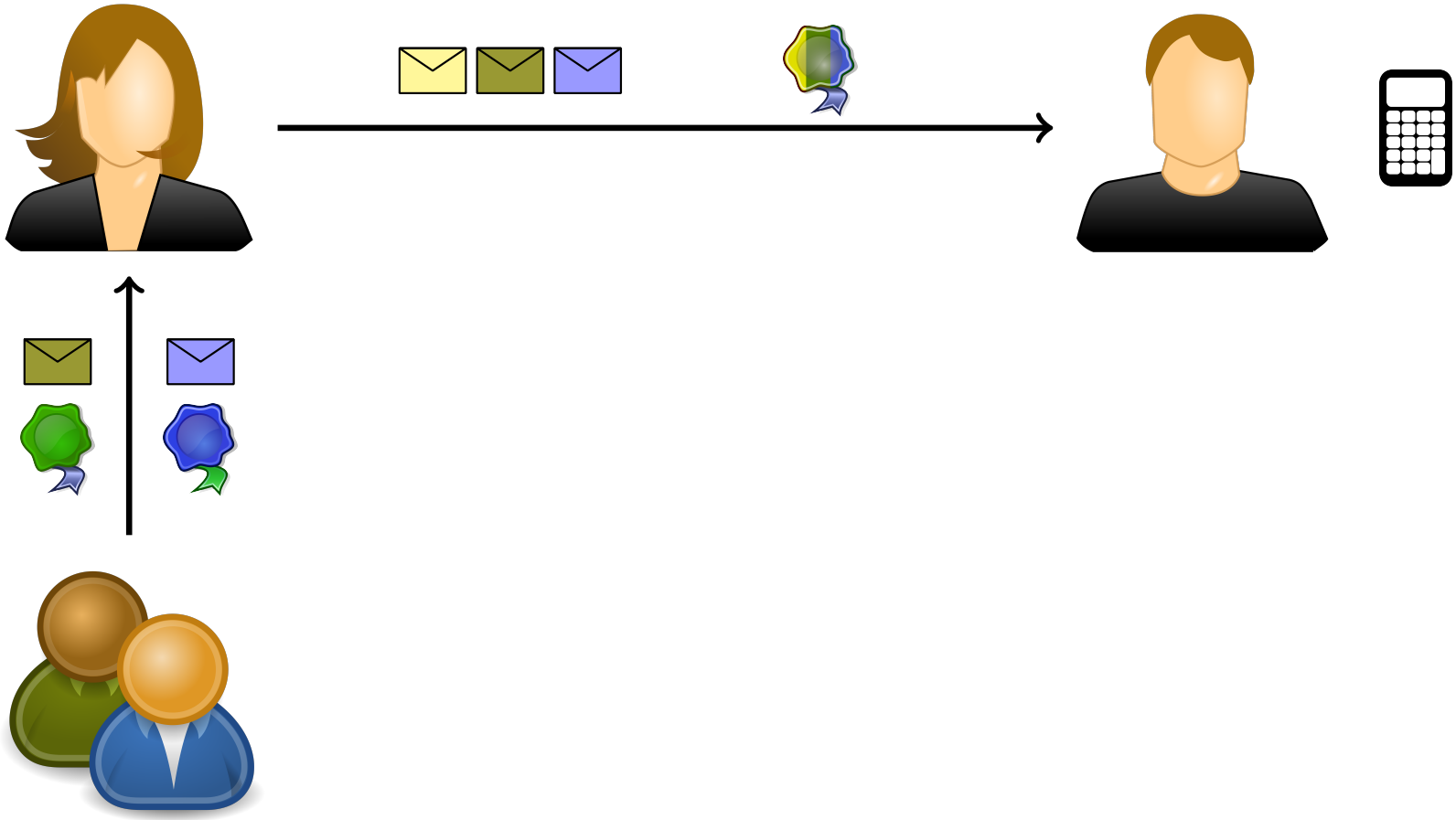

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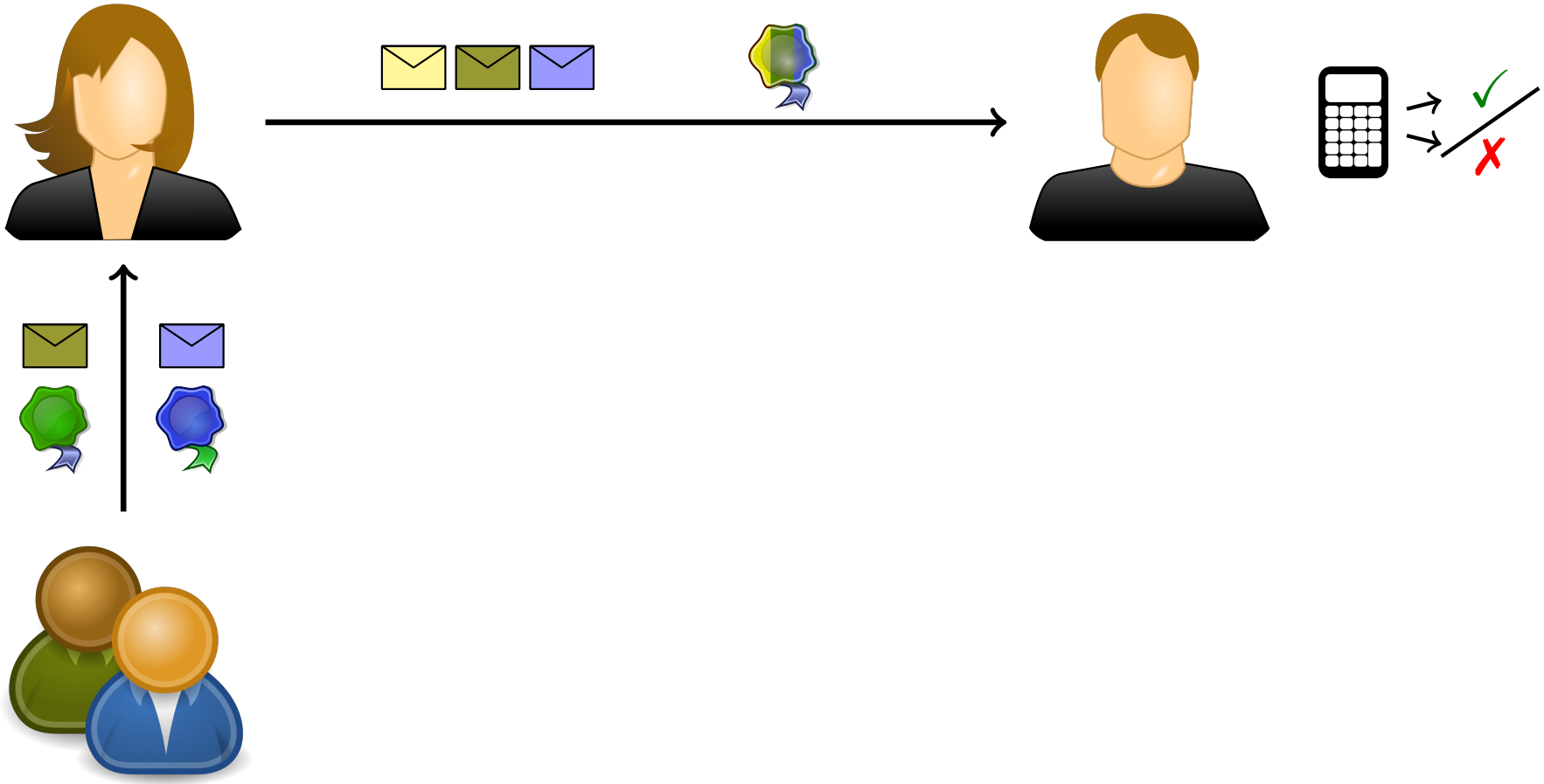

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Aggregate Signatures

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$\Longrightarrow$ Bob can not trust any message.
$\Longrightarrow$ resend data and/or agg. signature

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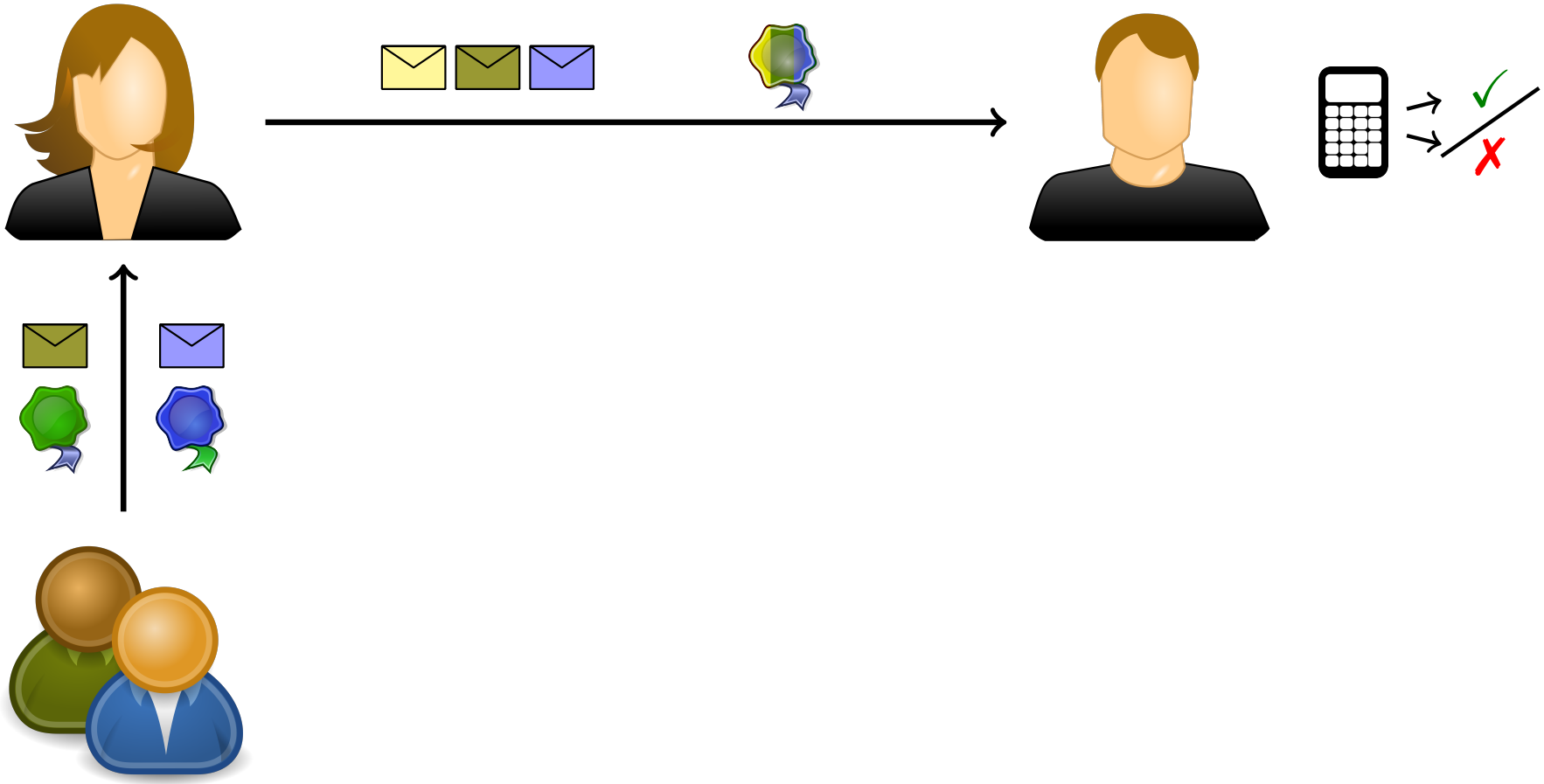

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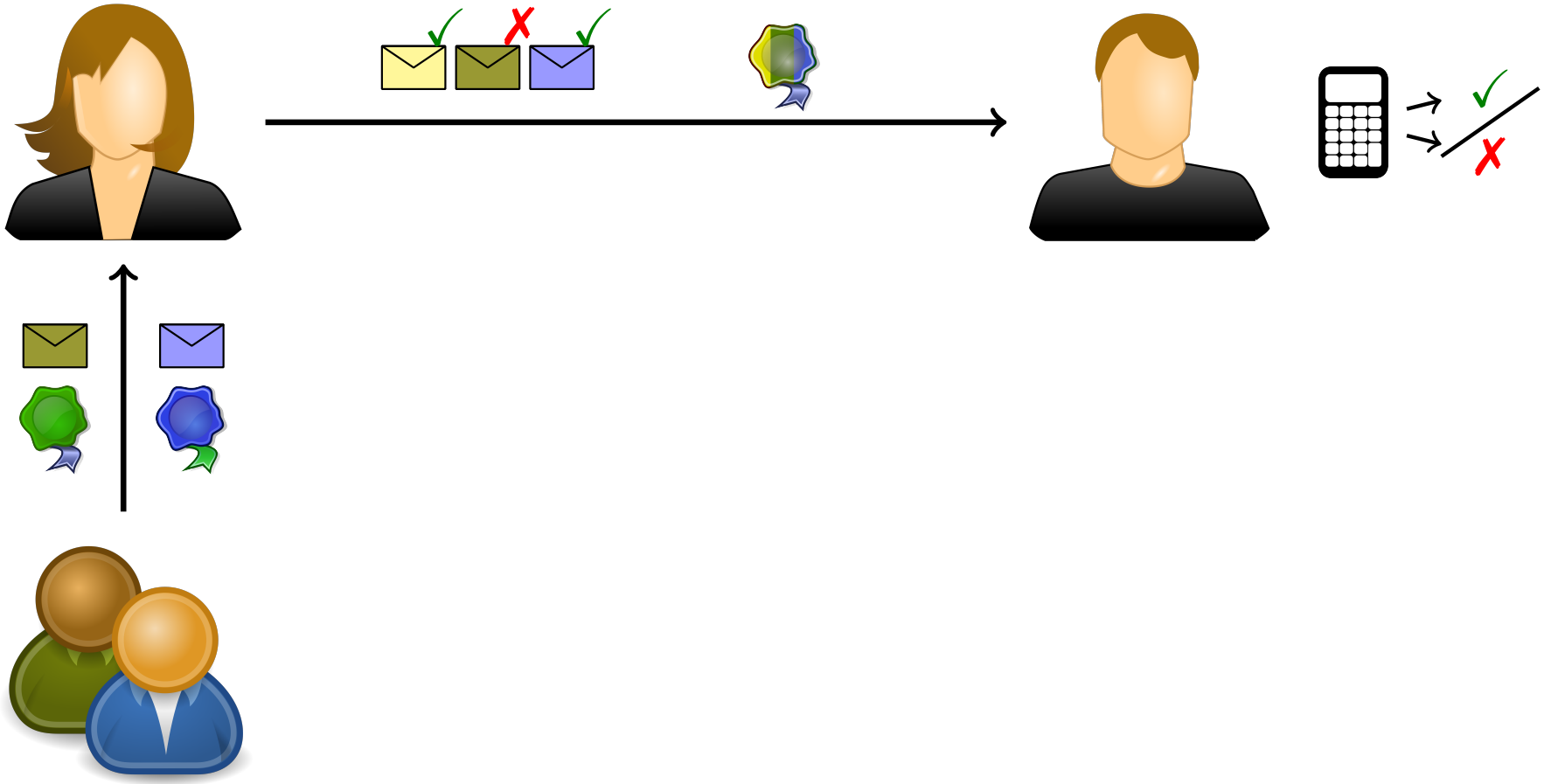

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Fault-Tolerance

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- generalization of correctness

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Fault-Tolerant Aggregate Signature

- We construct an aggregate signature scheme, which can detect a fixed number d of faults
- via a black-box transformation from an arbitrary aggregate signature scheme,
- using Cover-Free-Families [KRS99].

Outline

Our Approach

- signature $\hat{=}$ *vector* of signatures of underlying scheme

$$\left(\qquad \right)$$

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$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} \quad \end{pmatrix}$$

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Incidence matrices of cover-free families [KRS99] allow for more invalid signatures.

Outline

Security

Recall: Verify outputs a list of valid messages

Definition (Security, informal)

A fault-tolerant signature scheme is *secure* iff no PPT adversary (with signing oracle) can forge an (aggregate) signature.

Our Construction:

- Verify outputs the union of messages in valid rows

  $\Longrightarrow$ security inherited from the underlying scheme

Compression Ratio

Definition (Compression Ratio)

Let σ^* be a maximum size aggregate signature for n individual signatures. An aggregate signature scheme has *compression ratio* $\rho(n)$ iff

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Our scheme achieves this bound.

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- Verification of a single message-signature pair (m_j, σ_j)
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- check only relevant rows
- return early on success
- our construction: matrix does not need to be stored, can be generated on demand efficiently

Example Numbers

d	m	n
max.faults	dimension of vector	single signatures
2	25	125
5	121	1331
8	289	4913
4	289	$\approx 1.42 \cdot 10^6$
14	841	24389
26	2809	148877
50	10201	$\approx 1.03 \cdot 10^6$
83	63001	$\approx 3.97 \cdot 10^9$
510	1042441	$\approx 1.06 \cdot 10^9$

Conclusion

- Aggregate signature schemes can “compress” signatures from different signers.

Our Construction:

Pros:

- tolerates a fixed number of errors
- black-box transformation
- asymptotically optimal compression
- selective verification

Cons:

- a-priori upper bound on number of aggregated signatures
- compression-rate limited by fault-tolerance
- requires order among the messages

Thank you.
Questions?

Contact: gunnar.hartung@kit.edu
PGP-Key ID: B1A7 C146
Fingerprint: 4C39 AC36 6FAD 9E52 3144
8352 9E37 381F B1A7 C146
S/MIME Cert: at
https://crypto.iti.kit.edu/?id=hartung&L=2

Security Definition

Definition (Security Experiment)

- *Setup Phase.* Run $(\mathsf{pk}, \mathsf{sk}) := \mathsf{KeyGen}(1^\kappa)$ and start adversary $\mathcal{A}$ with input pk.
- *Query Phase.* $\mathcal{A}$ may adaptively issue signature queries m_i, and receives the responses $\tau_i := \mathsf{Sign}(\mathsf{sk}, m_i)$.
- *Forgery Phase.* $\mathcal{A}$ outputs a claim sequence C^* and a signature τ^*.

Definition (Security)

An aggregate signature scheme with list verification is (t, q, ε)-*secure* if there is no adversary $\mathcal{A}$ running in time at most t, making at most q queries to the signature oracle and winning in the above experiment with probability at least ε.

d-Cover-Free Families

Definition (d-Cover-Free Family [KS64])

A d-CFF $\mathcal{F} = (\mathcal{S}, \mathcal{B})$ consists of:

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- $|B \setminus \bigcup_{k=1}^{d} B_{i_k}| \geq 1$
  i.e. d subsets do not cover a different single subset.

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For a *d*-CFF $\mathcal{F} = (\mathcal{S}, \mathcal{B})$ its *incidence matrix* $\mathcal{M}$ is:

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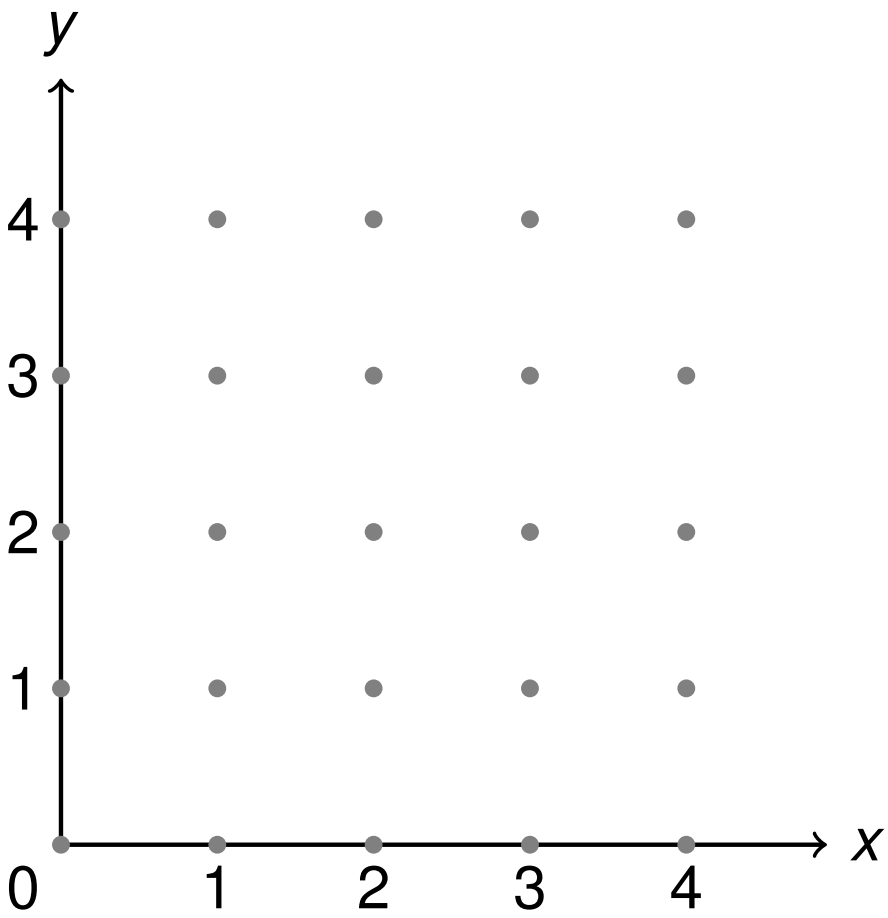
Example



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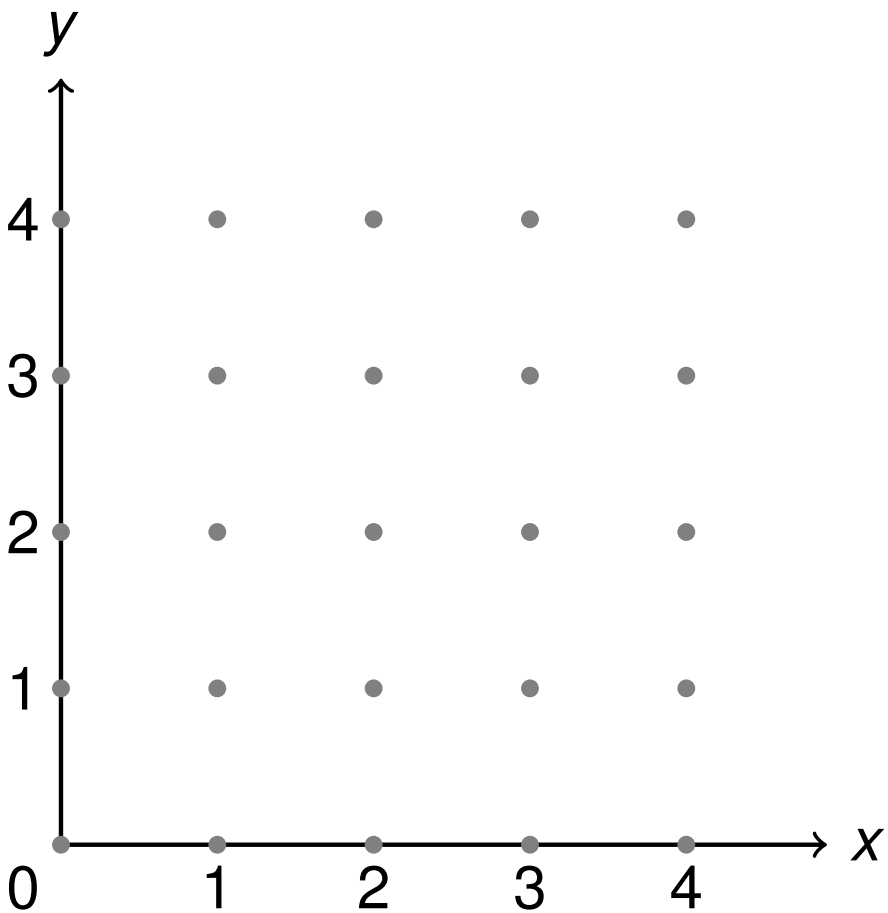

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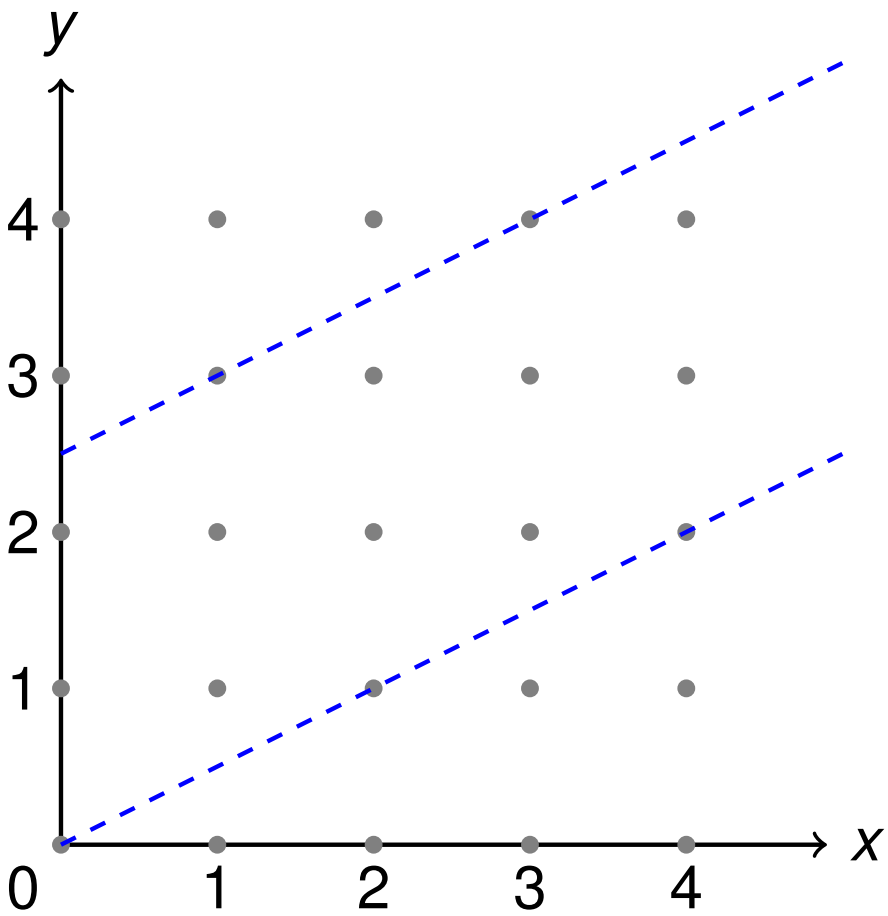

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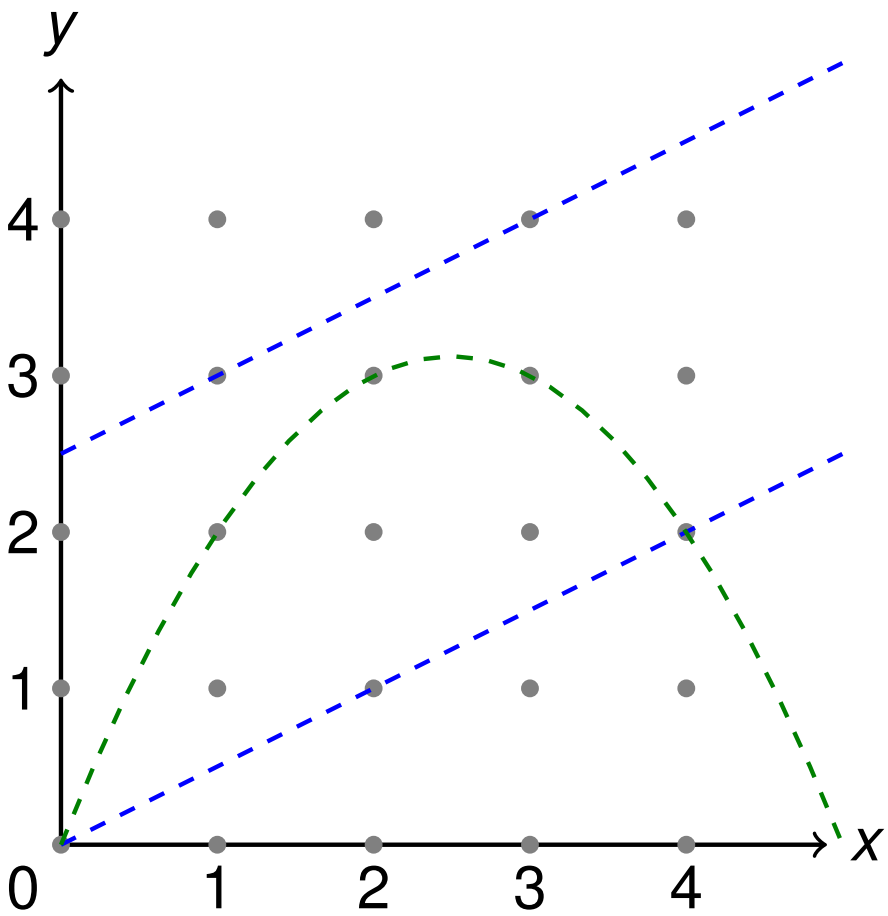

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$$f_2(x) = -\tfrac{1}{2}(x-2.5)^2 + 3\tfrac{1}{8} \;\hat{=}\; 2x^2$$

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Polynomial-Based Construction of a d-CFF

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- required: $2^\ell \geq \binom{n}{n-d} \underset{\text{math}}{\rightsquigarrow} \ell \in \Omega(\log_2(n))$ $\square$

References I

R. Kumar, S. Rajagopalan, and A. Sahai. “Coding Constructions for Blacklisting Problems without Computational Assumptions”. In: *CRYPTO ’99*. Ed. by M. J. Wiener. Vol. 1666. LNCS. Springer, Aug. 1999, pp. 609–623.

W. H. Kautz and R. C. Singleton. “Nonrandom binary superimposed codes”. In: *IEEE Transactions on Information Theory* 10.4 (1964), pp. 363–377. DOI: 10.1109/TIT.1964.1053689.