Easing Coppersmith Methods using Analytic Combinatorics: Applications to Public-Key Cryptography with Weak Pseudorandomness

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Goal

Solve:

$$\begin{cases} f_1(x_1, \ldots, x_n) = 0 \mod N_1 \\ \vdots \\ f_s(x_1, \ldots, x_n) = 0 \mod N_s \end{cases}$$

with

$$|x_1| \leq X_1 \qquad \ldots \qquad |x_n| \leq X_n$$

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Question: which bounds $X_1, \ldots, X_n$ work?

Coppersmith Methods

Overview

1. Construction of polynomials $\tilde{f}_{i,j}$ such that:

$$\tilde{f}_{i,j}(x_1, \ldots, x_n) = 0 \mod N_i^{k_{i,j}}$$

for any original solution $(x_1, \ldots, x_n)$.

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2. Use LLL to find an integer system:

$$\begin{cases} g_1(x_1, \ldots, x_n) = 0 \\ \vdots \\ g_n(x_1, \ldots, x_n) = 0 \end{cases}$$

such that:

1. any original solution is satisfied;
2. it has only a finite number of solutions.

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3. Solve the system (using Groebner basis).

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  - the number of polynomials $\tilde{f}_{i,j}$
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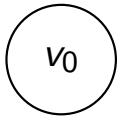
Difficult to compute when s and n non-constant

Our solution

Use analytic combinatorics!

Pseudorandom Generator (PRG)



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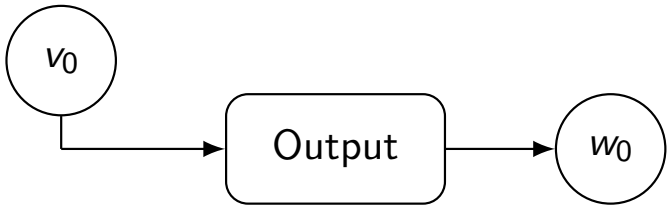

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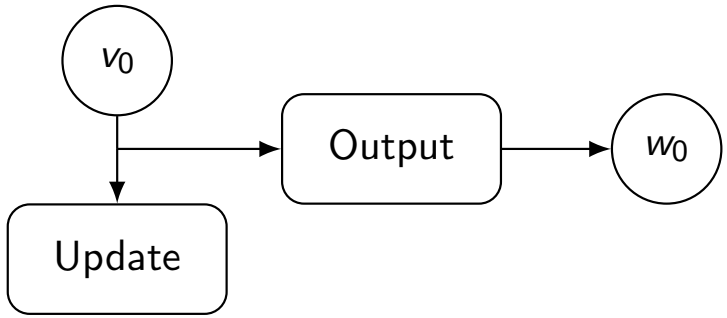

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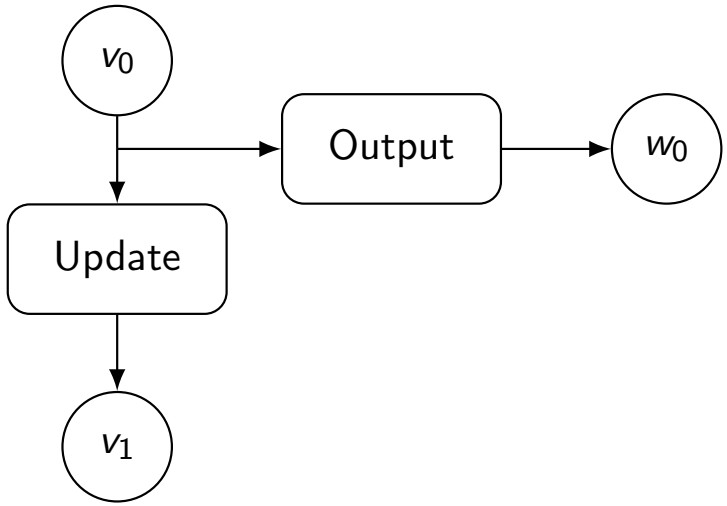

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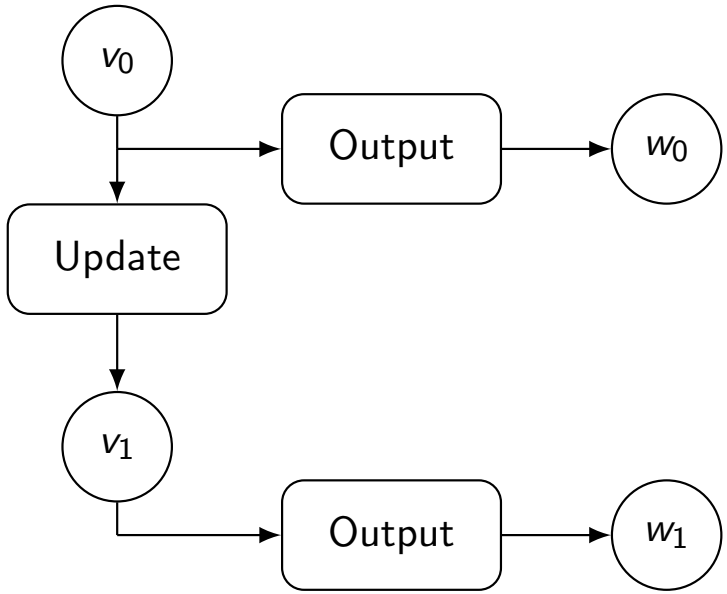

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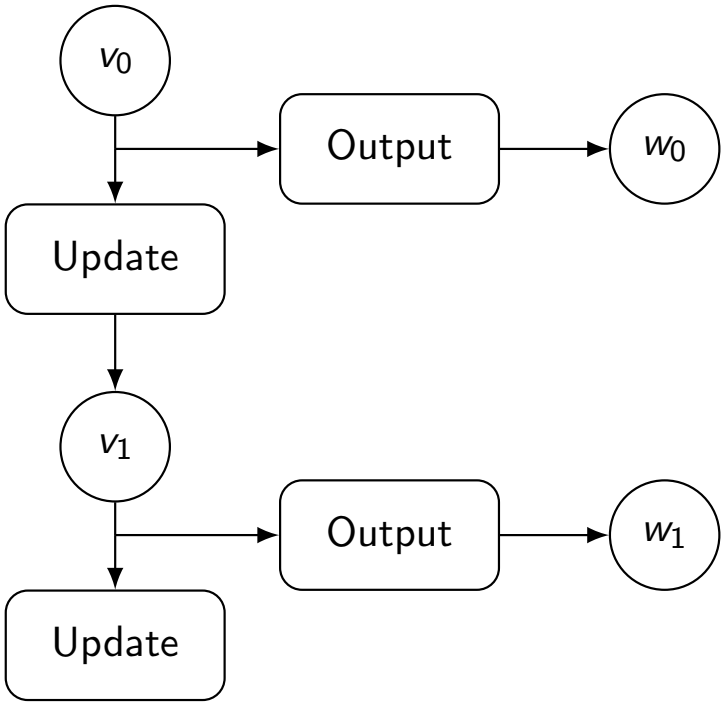

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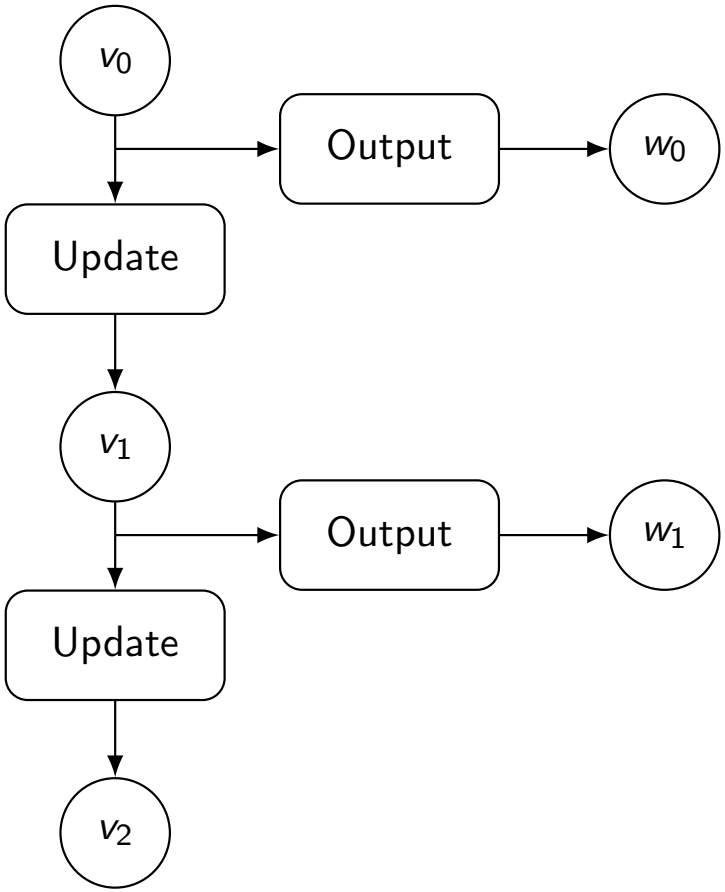

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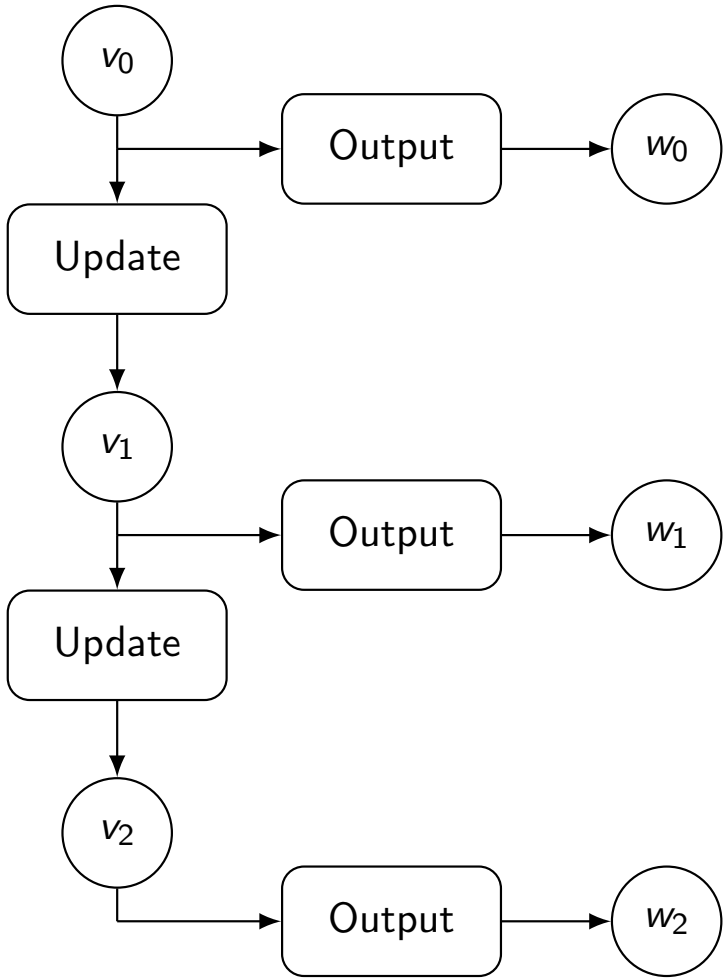

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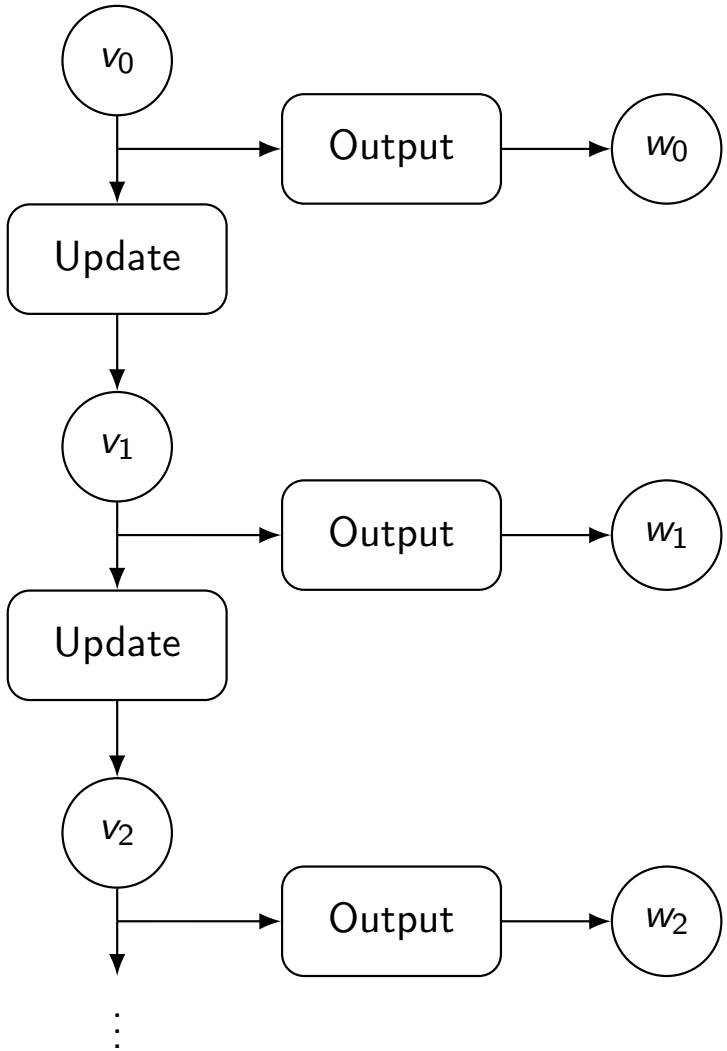

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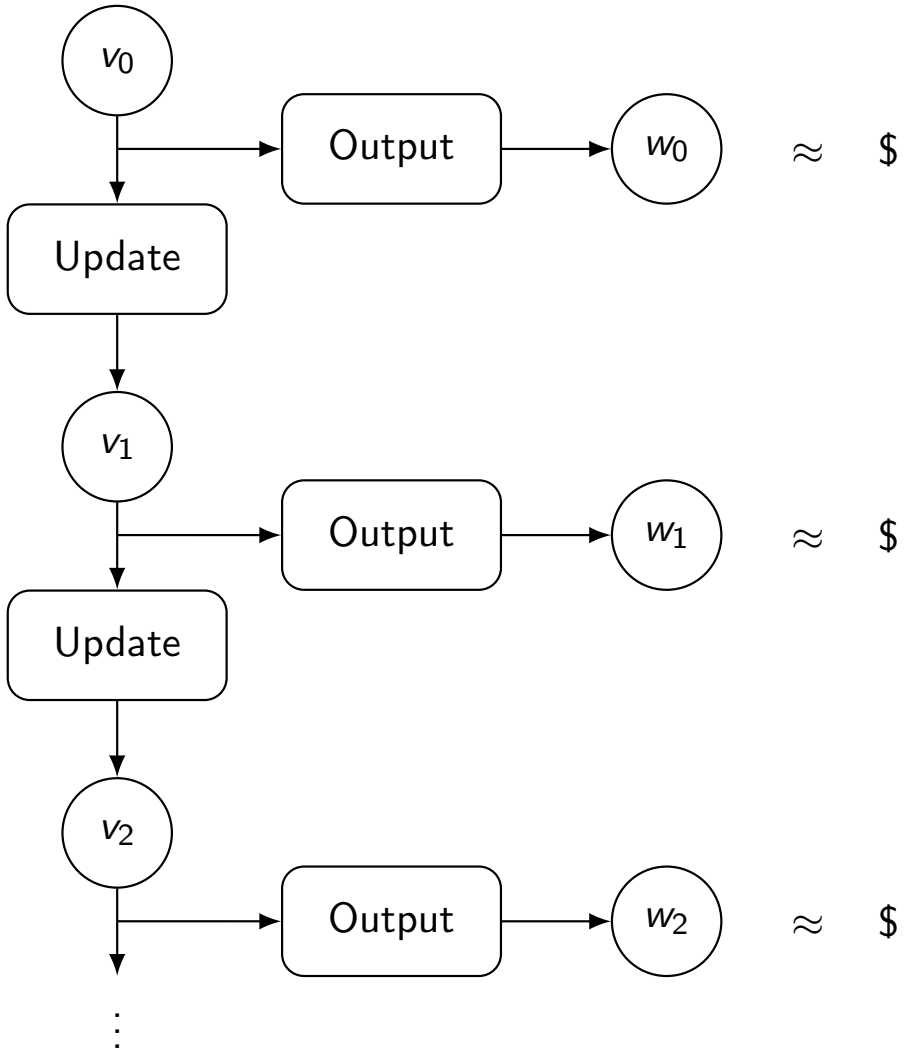

Weak Pseudorandom Generator

Linear congruential generator (LCG)

Parameters $M \in \mathbb{N}$, $a \in \mathbb{Z}_M^*$, $b \in \mathbb{Z}_M$

Seed $v_0 \xleftarrow{\$} \mathbb{Z}_M$

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- can recover v_0, when not truncated ($k > \log_2 M$)
- can predict outputs
  even when $k = 1$ and a, b secret [Ste87]

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Contributions

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    - Compute complexity of [BVZ12];
  - RSA key generation N with LCG
    - Polynomial time (in $\log N$) attacks (instead of exponential [FTZ13]);
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     $\longrightarrow$ manual but easy step (in our case) using a "dictionary"
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Example of Use of Analytic Combinatorics

Number n_d of polynomials of total degree d of the form:

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$$P(z) \underset{z \to 1}{\sim} \frac{1}{4 \cdot (1-z)^2} \qquad \Longrightarrow \qquad n_d \underset{d \to \infty}{\sim} \frac{d}{4}$$

Attack on Algebraic PRG (I)

Parameters $M \in \mathbb{N}$, polynomial F of degree d over $\mathbb{Z}_M$

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We know w_i and we write

$$v_i = w_i \cdot 2^{n-k} + x_i \quad \text{for } 0 \leq i \leq n+1$$

We want to solve:

$$\begin{cases} f_0(x_0, x_1) = w_1 \cdot 2^{n-k} + x_1 - F(w_0 \cdot 2^{n-k} + x_0) \equiv 0 \mod M \\ \vdots \\ f_n(x_n, x_{n+1}) = w_{n+1} \cdot 2^{n-k} + x_{n+1} - F(w_n \cdot 2^{n-k} + x_n) \equiv 0 \mod M \end{cases}$$

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Using our toolbox, the attack works when $m \to \infty$:

$$\frac{\lceil \log_2 M \rceil - k}{\lceil \log_2 M \rceil} \leq \frac{d^{n+1} - 1}{d^{n+2} - 1} \approx \frac{1}{d}$$

RSA Key Generation with LCG (I)

Factorize an RSA modulus $N = pq$ where

$$p = v_0 + Mv_1 + \cdots + M^n v_n \quad \text{and} \quad q = w_0 + Mw_1 + \cdots + M^n w_n$$

and $v_0, \ldots, v_n$ and $w_0, \ldots, w_n$ are non-truncated output of a LCG.

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We have:

$$\begin{cases} f = (v_0 + \cdots + M^n v_n)(w_0 + \cdots + M^n w_n) \equiv 0 \bmod N \\ g_0 = v_1 - (av_0 + b) \equiv 0 \bmod M \\ \vdots \\ g_{n-1} = v_n - (av_{n-1} + b) \equiv 0 \bmod M \\ h_0 = w_1 - (aw_0 + b) \equiv 0 \bmod M \\ \vdots \\ h_{n-1} = w_n - (aw_{n-1} + b) \equiv 0 \bmod M \end{cases}$$

RSA Key Generation with LCG (II)

We construct the polynomials of degree $2t$:

$$\tilde{f}_{i_0,\ldots,i_n,j_0,\ldots,j_n,k} = {v_0}^{i_0} \ldots {v_n}^{i_n} \cdot {w_0}^{j_0} \ldots {w_n}^{j_n} \cdot f^k \bmod N^k$$
$$\text{with } 1 \leq k < t,\ (i_0 = 0 \text{ or } j_0 = 0)$$
$$\text{and } \deg(\tilde{f}_{\ldots}) = i_0 + \cdots + i_n + j_0 + \cdots + j_n + 2k < 2t$$
$$\tilde{g}_{i_0,\ldots,i_n,j_0,\ldots,j_n} = g_0^{i_0} \ldots g_{n-1}^{i_{n-1}} \cdot {v_n}^{i_n} \cdot h_0^{j_0} \ldots h_n^{j_{n-1}} \cdot {w_n}^{j_n} \bmod M^\ell$$
$$\text{with } 1 \leq \ell = i_0 + \cdots + i_{n-1} + j_0 + \cdots + j_{n-1}$$
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$$\text{and } \deg(\tilde{g}_{\dots}) = i_0 + \dots + i_n + j_0 + \dots + j_n < 2t \ .$$

Using our toolbox:
$\forall n \geq 2$, $\exists t$, attack works in time poly in $\log N$.

PKCS#1 v1.5 = padding for RSA: encryption of m:

$$c = \mu(m, r)^e \bmod N \quad \text{and} \quad \mu(m, r) = 0002_{16}||r||00_{16}||m$$

with r random bit string.

Attack similar to the previous one,
when r generated by LCG modulo M.
Work for $M < N^{1/e}$.

Thank you for your attention!
Questions?

- Toolbox for *Coppersmith methods*:
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In Burton S. Kaliski Jr., editor, *CRYPTO'97*, volume 1294 of *LNCS*, pages 277–291. Springer, August 1997.

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